

Auctions and Information Acquisition: Sealed-bid or Dynamic Formats?*

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Abstract

The value of an asset is generally not known *a priori*, and it requires costly investment to be discovered. In such contexts with endogenous information acquisition, which selling procedure generates more revenues? We show that dynamic formats such as ascending price or multi-stage auctions perform better than their static counterpart. This is because dynamic formats allow bidders to observe the number of competitors left throughout the selling procedure. Thus, even if competition appears strong *ex ante*, it may turn out to be weak along the dynamic format, thereby making the option to acquire information valuable. This very possibility also induces the bidders to stay longer in the auction, just to learn about the state of competition. Both effects boost revenues, and our analysis provides a new rationale for using dynamic formats rather than sealed-bid ones.

Key words: auctions, private value, information acquisition, option value.

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1 Introduction

Assessing the value of an asset for sale is a costly activity. When a firm is being sold, each individual buyer has to figure out the best use of the assets, which business unit to keep or resell, which site or production line to close. The resources spent can be very large when there is no obvious way for the buyer to combine the asset for sale with the assets that he already owns. Similarly, when acquiring a license for digital television, entrants have to figure out the type of program they will have a comparative advantage on, as well as the advertisement revenues they can expect from the type of program they wish to broadcast. Incumbents may also want to assess the economies of scale that can be derived from the new acquisition. All such activities are aimed at refining the assessment of the valuation of the license, and they are costly.

From the seller's perspective, if the assets are auctioned to a set of potential buyers, the better informed the bidders are the higher the revenues, at least when the number of competitors is not too small.¹ However, when information is costly to acquire, a potential buyer may worry about the possibility that he spends many resources, and yet ends up not winning the asset. Providing the bidders with incentives to acquire information is thus key for the seller.

One commonly used format for selling assets is the sealed-bid auction, in which the winner is selected in a single round. Other formats (which are frequently used when the asset is complex) are the multi-stage auction and the ascending auction, in which the number of potential buyers is gradually reduced.² In the sealed-bid format, information acquisition may only take place prior to the auction. In dynamic formats, information acquisition may take place not only prior to the auction, but also in the course of the auction.³

Our objective is to compare dynamic and static auction formats with respect to the buyers' incentives for information acquisition, and with respect to the revenues that they generate for the seller.

¹The reason is that bidders with mediocre valuations may realize, once informed, that their valuation is quite high, hence the increase in revenue. Of course, they may also realize that their valuation is quite low, but if competition is strong, this effect on revenues is small.

²See Ye (2000) for an account of how widespread the practice of multi-stage auctions are.

³Note that if the lapse of time between stages is large enough, or if the ascending price auction is low paced, information acquisition is possible even when acquiring information takes time.

An important insight of this paper is that dynamic auction procedures are likely to generate more information acquisition and higher revenues than their static counterparts.⁴ More precisely, we highlight a significant benefit induced by formats in which bidders gradually get to know the number of (serious) competitors they are facing, which in turn allow them to better adjust their information acquisition strategy.

It should be emphasized that the reason why dynamic formats generate more revenues here is completely different from the classic reason of affiliated values (Milgrom-Weber 1982) in which ascending formats allow the bidders to learn about the information held by others. Here, the valuations of bidders are not influenced by other bidders' information, and yet dynamic auction formats generate higher revenues (by modifying bidders' information acquisition strategy on their own valuations). Our paper thus provides a new rationale - that we believe is of practical importance - for using dynamic auction formats.

To illustrate the claim, suppose there is one good for sale and compare the sealed bid second price auction and the ascending price auction in which each bidder can decide at any time to acquire information.⁵

In the sealed-bid static format, bidders are unlikely to decide to acquire extra information whenever there are potentially many competitors. The point is that the risk of ending up not buying the good (because it turns out that someone else has a higher value) is then so large that bidders prefer not to waste their money (or time) on getting extra information. In contrast, in the ascending price auction format, bidders get to obtain a better estimate of their chance of winning just by observing the number of bidders left. In particular, even if competition appears strong *ex ante*, it may turn out to be weak, and information acquisition may become a valuable option. This has two effects: first, it generates more information acquisition (hence more revenues - at least when the number of bidders is not too small).

⁴Though we focus here on revenues, efficiency may also be higher in the ascending format (see Compte-Jehiel 2000).

⁵For the sake of presentation, we assume below that information acquisition is immediate. The extension to the case where information acquisition takes time is addressed in Section 5. We also discuss other dynamic formats such as multi-stage mechanisms. Also note that when acquiring information takes time, the standard ascending auction format can be amended to allow for information acquisition, by including breaks at pre-determined dates or events (say, each time a bidder drops out), precisely designed so that bidders have enough time to acquire information, or by allowing any bidder to trigger such breaks.

Second, it may induce bidders to wait and remain active in the auction, just to learn more about the state of competition. This latter effect tends to raise the price paid by winners, hence revenues.

In the above discussion we have emphasized the benefit of providing bidders with some estimate of the level of competition (through the number of competitors left). But, not all dynamic formats have the property of conveying such an estimate. For example, in the one-object ascending price auction with secret drop-out, bidders observe the current level of price, but not how many competitors are left.⁶ We show that if in the static auction bidders prefer not to acquire extra information, the ascending price auction with secret drop out does not provide bidders with incentives to acquire information either.⁷ Thus, it is not merely the dynamic nature of the format that is key for our insight, but its property of conveying (for free) an estimate of the intensity of competition (through the information about the number of competitors left).

Related literature:

Our paper is related to various strands of literature in auction theory: the comparison of auction formats (and more precisely here the comparison of the second price and ascending price auction formats), the analysis of information acquisition in auctions and the literature on entry in auctions.

Concerning the comparison between auction formats, we mentioned earlier the work by Milgrom-Weber (1982), who showed that, in affiliated value settings, the ascending and sealed-bid formats differ because the information on others' signals conveyed in equilibrium differ, hence the bidders' assessment of their valuation differ too. In the context of auctions with negative externalities (see Jehiel-Moldovanu 1996), Das Varma (1999) has shown that the ascending format could (under some conditions) generate higher revenues than the sealed-bid (second-price) auction format (in the ascending format a bidder may be willing to stay longer, so as to combat a harmful competitor if he happens to be the remaining bidder).

⁶Rezende (2001) studies this auction format in a symmetric setting.

⁷When drop-outs are secret, bidders only learn that the highest valuation still lies above the current price, which is bad news: bidders' incentives to acquire information do not increase as the auction progresses. In contrast, learning that the number of competitors left is small could increase dramatically the incentive to acquire information.

Concerning information acquisition in auctions, the literature has (in contrast to our work) focused on sealed-bid types of auction mechanisms, and it has essentially examined efficiency issues.⁸

In a private value model, Hausch and Li (1991) show that first price and second price auctions are equivalent in a symmetric setting (see also Tan 1992).⁹ Stegeman (1996) shows that second price auction induces an ex ante efficient information acquisition in the single unit independent private values case (see also Bergemann and Valimaki 2000). However, in Compte and Jehiel (2000), it is shown that the ascending price auction may induce an even greater level of expected welfare.

Models of information acquisition in interdependent value contexts (in static mechanisms) include Milgrom (1981) who studies second-price auctions, Matthews (1977), (1984) who studies first-price auctions and analyzes in a pure common value context whether the value of the winning bid converges to the true value of the object as the number of bidders gets large,¹⁰ Persico (1999) who compares incentives for information acquisition in the first price and second price auctions in the affiliated value setting, and Bergemann and Valimaki (2000) who investigate, in a general interdependent value context, the impact of ex post efficiency on the ex ante incentives for information acquisition.

Our paper is also related to the literature on endogenous entry in auctions, which includes McAfee and McMillan (1987), Harstad (1990) and Levin and Smith (1994).¹¹ In these models,

⁸One exception is an independent contribution by Rezende (2001), that examines the ascending price auction with secret drop out. Building on Compte and Jehiel (2000), Rasmussen (2001) considers a two-bidder dynamic auction with deadline (motivated by internet auctions) in which one bidder can refine her valuation at a cost. He observes that the informed bidder may wait till close to the deadline to make the uninformed bidder believe that he does not need to refine her valuation to win the object.

⁹Engelbrecht-Wiggans (2001) also compares first price and second price auctions, but he examines the case where bidders acquire information on the number of competitors (rather than on their own valuation). These two formats are then not equivalent, since knowing the number of bidders is valuable in the first price auction only.

¹⁰See also Hausch and Li 1993 for an analysis of information acquisition in common value settings.

¹¹These papers analyze the effect of entry fees or reserve prices on the seller's revenue. McAfee-McMillan (1987) show that in contrast with the case where the number of participants is given exogenously, the optimal reserve price may be zero (this insight is related to that of Bulow-Klemperer (1996) about the positive role of competition in symmetric setups). Levin-Smith (1994) (see also Harstad 1990) further analyze this issue by considering (symmetric) equilibria with possibly stochastic participation. They find that restricting

each bidder makes an entry decision prior to the auction, at a stage where bidders do not know their valuation. The decision to enter allows the bidder to both participate to the auction and learn her valuation. These models thus combine the idea of participation costs *and* the idea of information acquisition. This should be contrasted with our model in which there is no participation cost but only a cost to acquire information on the valuation.

Finally, our work is also related to the literature on research contests (Fullerton and McAfee (1998), and more recently Che and Gale (2001)). The main virtue of the ascending price auction identified in this paper is that it increases the incentives to acquire information as, for some realizations of signals, it allows the bidders to realize that competition is less tough than it would have seemed from an *ex ante* viewpoint. Likewise, Fullerton and McAfee (1998) and Che and Gale (2001) identify conditions under which it is a good idea from an efficiency viewpoint to reduce the number of contestants to just a few (in fact two) in an attempt to increase contestants' incentives to exert effort in the contest.¹²

The rest of the paper is organized as follows. Section 2 describes the basic model. In Section 3, only one bidder is uninformed and may acquire information on his valuation. In Section 4, we analyze the case where more than one bidder may acquire information. Further discussion of our model appears in Section 5. We conclude in Section 6.

2 The Model

There is one object for sale, worth 0 to the seller and n potential risk-neutral buyers indexed by $i \in N = \{1, \dots, n\}$. Each bidder $i = 1, \dots, n$ has a valuation θ_i for the object. The valuations θ_i are assumed to be drawn from independent and identical distributions. We assume that this common distribution has a density g with full support on $[0, \bar{\theta}]$.

In our model, there will be two types of buyers: the *informed buyers*, who know the the number of participants to equate the socially optimal number of bidders eliminates the coordination problem that would arise otherwise (if the number of bidders is larger than the socially optimal one, stochastic participation cannot be avoided and may result in no participation).

¹²There are obviously many differences between an auction setup in which the value of winning is determined by the valuation and a contest in which the prize is common to all contestants, but the two setups share a common feature: less fierce competition increases the incentive to acquire information in our setup or to make effort in the contest application.

realization of their own valuation; and the uninformed buyers, who may get informed about their own valuation at some cost c . In the basic version of the model (Section 3), we will consider the case where it is common knowledge that there is a single uninformed buyer. In Section 4, we will consider the symmetric case where initially, each bidder i is informed with some probability $q \in (0, 1)$ and uninformed with probability $1 - q$. We further assume that each bidder's informational state (whether he is informed or not) is private information, and that bidders' informational states are drawn from independent distributions.

One advantage of the basic model is that it allows for an easy characterization of the equilibrium behaviors, in particular regarding the information acquisition decision. When several buyers may acquire information, the decision to acquire information is affected by others' decisions to acquire information. But, our main insights will carry over to this case.

When bidder i is uninformed, his expected valuation from acquiring the object is:¹³

$$v \equiv \int \theta_i g(\theta_i) d\theta_i. \quad (1)$$

When he acquires information, bidder i learns the realization θ_i . Other bidders however do not observe that realization; they do not observe either whether information acquisition occurred. Finally, it will be convenient to denote by $\theta^{(1)}$, $\theta^{(2)}$, ..., $\theta^{(k)}$ the highest valuation, the second highest valuation, and the k^{th} highest valuation, respectively among the (initially) informed bidders. In the basic model, we shall assume that bidder 1 is the uninformed bidder.

In both models (Section 3 and Section 4), the information structure will be assumed to be common knowledge among all bidders.

Remark: The interpretation behind the definition of v (expression (1)) is that in case he wins, bidder i will learn the realization θ_i at no cost. Hence the only motive for spending resources on acquiring information is in checking that it is worthwhile to acquire the object. In some contexts (for example when bidders compete to acquire a firm), the resources that a bidder spends on acquiring information are best thought of as an investment that will have to be made anyway, in case that bidder wins.¹⁴ To cover such applications, one would need

¹³In Compte and Jehiel (2000), we analyze the more general case in which an uninformed bidder i gets an imperfect signal about θ_i prior to the auction. No new insight is gained by doing so however.

¹⁴Such investments for example include the resources spent to assess the synergies with the assets already owned.

to modify the expected valuation from acquiring the asset into

$$\int \theta_i g(\theta_i) d\theta_i - c. \tag{2}$$

Our analysis would extend in a straightforward way to this alternative formulation (by equating v with the expression above); and our main result that the ascending format generates more revenues would even be stronger in that case. We will however stick to the previous formulation in which v is as shown in (1).

Auction formats:

Throughout the analysis, we will be mostly interested in the comparison between static and dynamic auction formats. We will compare the sealed bid second-price auction, the ascending price auction and the ascending price auction with secret drop out, in which bidders do not observe if and when bidders drop out until the auction ends (see below for the formal definition).

In the *sealed bid second-price auction*, bidders decide prior to the auction whether or not to acquire information. Then, each bidder submits a bid. The object is allocated to the bidder whose bid is highest at a price equal to the second highest bid.¹⁵

In the *ascending price auction*, each uninformed bidder may decide to learn his valuation not only before the auction starts, but also at any time during the auction. Learning one's own valuation costs c and takes no time.¹⁶ As mentioned earlier, the decision to acquire information is not observable to other bidders. The rules of the auction are as follows. The price starts at 0, at which each bidder is present. The price gradually increases. At any point in time,¹⁷ each bidder may decide to drop out of the auction. When more than one bidder wishes to drop out of the auction at the same time, the auction stops, one bidder is selected

¹⁵In case of ties, each one of the bidders with highest bid gets the object with equal probability.

¹⁶We will discuss in Section 5 how our result extends to case where learning one's own valuation takes time.

¹⁷We present here the continuous time/price version of the ascending price auction. This raises some technical difficulties regarding the definition of equilibria in undominated strategies. The equilibria we will refer to are the limits as $\varepsilon > 0$ tends to 0 of the equilibria in undominated strategies of the corresponding game in which time is discrete and after each round the price increases by the increment ε (except for rounds following simultaneous decision to drop out).

to exit, each one having an equal chance of being selected; the auction then resumes at the same current price.¹⁸

Bidders are assumed to observe the number of bidders still active in the auction. The auction ends when there is only one bidder left. The object is allocated to that bidder at the current price.

In each of these formats, a bidder (say bidder i) who is informed or who has acquired information has a dominant strategy: bid θ_i in the second-price auction, drop out as soon as the price reaches or exceeds her valuation θ_i in the ascending formats. Similarly, conditional on never acquiring information, an uninformed bidder i has also a dominant strategy: bid v in the second price auction, or drop out at v in the ascending format.

Therefore, the only remaining question is if and when uninformed bidders decide to acquire information. Before going into further details, we introduce functions that will be particularly useful in the analysis.

For any price $p \geq 0$, we denote by $h(p)$ the (ex ante) expected payoff bidder i would obtain if he were to learn (for free) his valuation θ_i and were offered to buy at price p . We have:

$$h(p) = \Pr\{\theta_i > p\}E[\theta_i - p \mid \theta_i > p]. \quad (3)$$

For any price $p \in [0, \bar{\theta}]$ and any number $k \in \{1, \dots, n - 1\}$, we also denote by $H(k, p)$ the expected payoff that bidder i obtains in the ascending format when (i) the current price is p , (ii) bidder i is still active, learns for free his valuation θ_i , and drops out at θ_i (or immediately in case $\theta_i < p$), (iii) there are still k other active informed bidders.¹⁹

¹⁸This assumption ensures that there cannot be jumps in the number of active bidders. It is not essential to our results, but it will make the analysis simpler. Note that bidders who are not selected are not disadvantaged however, because when the auction resumes, the price has not changed and they still have the option to apply to exit at that same price.

An alternative assumption would be that the auction stops when all still active bidders want to exit simultaneously, and that each one of these bidders has an equal chance of being awarded the object. In the absence of an option to acquire information, the two formulations would be equivalent. In the presence of an option to acquire information, the first formulation gives bidders an extra chance to acquire information (after they learn that they were not the only one willing to exit).

¹⁹These informed bidders follow their dominant strategy, as explained above.

Since bidder i gets the object when $\theta_i > \theta^{(1)}$, and then pays $\theta^{(1)}$ for it, we have:

$$H(k, p) = E[h(\theta^{(1)}) \mid \theta^{(k)} > p > \theta^{(k+1)}] \quad (4)$$

As intuition suggests, it is readily verified that the functions h and H are decreasing in their arguments.²⁰

3 The single uninformed bidder case.

In this Section, we assume that a single bidder, say bidder 1, is initially uninformed, and we assume that this is common knowledge.

3.1 A preliminary comparison

We first wish to exhibit simple conditions under which bidder 1 would acquire information with positive probability in the ascending format, whereas he would not in the sealed bid format, illustrating the fact that the second-price auction provides poorer incentives for information acquisition. The derivation of these conditions will also illustrate a key feature of the ascending format, namely that it gives the uninformed bidder an option to remain active in the auction until there are only a few bidders left.

When he acquires information prior to the (second price) auction, bidder 1's expected payoff is equal to

$$H(n-1, 0) - c$$

So if

$$H(n-1, 0) < c, \quad (5)$$

²⁰To see formally why observe that $h'(p) = -\Pr\{\theta_i > p\} < 0$. Now choose $p' > p$ and let $Q^{(k)} = \Pr\{\theta^{(k)} \in (p, p') \mid \theta^{(k)} > p > \theta^{(k+1)}\}$. We have

$$H(1, p) > Q^{(1)}h(p') + (1 - Q^{(1)})H(1, p') > H(1, p'),$$

and, by induction on k ,

$$H(k, p) = EH(k-1, \theta^{(k)}) \mid \theta^{(k)} > p > \theta^{(k+1)} < H(k-1, p)$$

$$H(k, p) \geq Q^{(k)}H(k-1, p') + (1 - Q^{(k)})H(k, p') > H(k, p')$$

for example because n is not small (or because c is significant), bidder 1 does not acquire information in the second price auction.

Assume now that bidder 1 never acquires information in the ascending format. Then in case the price reaches v (which occurs whenever $\theta^{(1)} > v$), bidder 1 drops out and thus gets a profit equal to 0.

In the event where

$$\theta^{(2)} < v < \theta^{(1)}$$

however, there is only one bidder (other than 1) who remains active at price v . Compared to the situation prior to the auction where bidder 1 had n potential competitors, bidder 1 has much higher chances of winning, and may therefore be more inclined to acquire information. Indeed, by acquiring information at v , bidder 1 would obtain an expected payoff equal to

$$H(1, v) - c,$$

which may be positive even when (5) holds (because $H(k, p)$ may decrease a lot with k). Besides, the event $\{\theta^{(2)} < v < \theta^{(1)}\}$ has positive probability. So we have proved the following Proposition:

Proposition 1 *Assume that $H(n-1, 0) < c < H(1, v)$. Then bidder 1 does not acquire information in the second price auction, whereas he acquires information with positive probability in the ascending format.*

To fix ideas about when the conditions of Proposition 1 hold, note that chances of winning get arbitrarily small when the number of bidders increase, hence $H(n-1, 0)$ also gets arbitrarily small. Thus, as soon as the number of bidders is not too small, the conditions of Proposition 1 hold for a significant range of costs. When for example θ_i is uniformly distributed, we have that $v = \frac{1}{2}$ and $H(4, 0) < H(1, v)$, hence it is sufficient that $n = 5$ to ensure that the conditions of Proposition 1 hold for some range of costs c .

3.2 Equilibrium behavior

We now derive in more detail the equilibrium behavior, and in particular the information acquisition strategy of bidder 1 in the ascending price auction. Throughout this Section,

we assume that the conditions of Proposition 1 hold, which ensures that bidder 1 acquires information with positive probability in equilibrium.

As mentioned before, a key feature of the ascending price auction is that it gives bidder 1 an *option to wait* until there are only a few bidders left before deciding whether to drop out or to acquire information. More precisely, we shall consider the strategy that consists in waiting until the first date at which either the price reaches some price p^* (to be defined below), or the number of other bidders still active drops down to 1, and then to decide whether to drop out or to acquire information.

We first define:

Definition 1 *Let p^* be the price such that when there is only one other bidder left, bidder 1 is indifferent between dropping out and acquiring information at p^* .*

By definition, the price p^* satisfies

$$H(1, p^*) = c, \tag{6}$$

which implies, since $H(1, v) > c$ and since $H(\cdot, \cdot)$ is a decreasing function of p ,

$$p^* > v.$$

Note that since $H(1, \cdot)$ is a decreasing function of p , for any $p < p^*$, if there remains only one other active bidder, player 1 derives a positive gain from acquiring information.

Now assume that the current price is p and the number of bidders other than bidder 1 still active is $m \geq 2$. By following the strategy defined above, bidder 1 secures a payoff equal to $H(1, \theta^{(2)}) - c$ when $\theta^{(2)} < p^*$ (by acquiring information once the price reaches $\theta^{(2)}$), and a payoff equal to 0 otherwise (by dropping out). Since by definition of p^* , $H(1, \theta^{(2)}) - c > 0$ whenever $\theta^{(2)} < p^*$, the resulting expected payoff is positive. Hence dropping out before p^* without acquiring information is a dominated strategy. To sum up,

Proposition 2 *In equilibrium, the uninformed bidder either acquires information, or waits until price p^* is reached.*

We now pursue the derivation of equilibrium behavior by showing two claims:

Claim (i) *once price p^* is reached, it is optimal for bidder 1 to drop out, and*

Claim (ii): *as long as there are $m \geq 2$ other active bidders, it is optimal for bidder 1 to wait (and postpone information acquisition).*

Claim (i) follows from the observation for any price $p > p^*$, it is not and will never become optimal to acquire information.²¹ Since $p > v$, it is then optimal for bidder 1 to drop out.

Claim (ii) follows from the observation that instead of acquiring information immediately, bidder 1 may follow the strategy proposed earlier (postponing information acquisition until $m = 2$ or dropping out once $p = p^*$). It will yield the same outcome in events where $\theta^{(2)} < p^*$, but it permits to avoid suboptimal information acquisition whenever $\theta^{(2)} > p^*$.

To complete the derivation of equilibrium behavior, we now examine the events where there is only one other bidder left ($m = 1$) and bidder 1 has not acquired information yet. The issue is whether *bidder 1 chooses to acquire information immediately or else to postpone his decision.*

To answer this question, we define a new threshold:

Definition 2 *Let p^{**} be the price for which if bidder 1 were given the option to buy at p^{**} , he would be indifferent between acquiring information before deciding to buy the object, and buying the object unconditionally.*

Such a price p^{**} is uniquely defined by²²

$$h(p^{**}) - c = v - p^{**}. \quad (7)$$

and satisfies

$$p^{**} < v.$$

Besides, for prices above p^{**} , bidder 1 would rather acquire information before deciding to buy the object, while for prices below p^{**} , he would rather buy unconditionally.²³

We will now show a third claim:

²¹This follows again from the monotonicity of $H(\cdot, \cdot)$ with respect to k and p implying that for all $k \geq 1$ and $p > p^*$, $H(k, p) < H(1, p^*) = c$.

²²Indeed, it is readily verified that the function $h(p) + p$ is strictly increasing in p on $[0, \bar{\theta}]$. Besides, for $p = 0$, $h(p) + p = v$, and for $p = v$, our assumption that $H(1, v) > c$ implies (since $h(v) > H(1, v)$) that $h(p) + p > c + v$, thus guaranteeing that p^{**} is uniquely defined and satisfies $p^{**} < v$.

²³Under the alternative formulation where v is set equal to $E\theta - c$, for any price $p > 0$, bidder 1 always prefers to acquire information. So we would set $p^{**} = 0$.

Claim (iii): *Assume that there is one other active bidder (say bidder 2) and that the current price p is below p^* . Then it is optimal for bidder 1 (a) to postpone information acquisition if $p < p^{**}$ and (b) to acquire information if $p > p^{**}$.*

Assume that bidder 1 postpones information acquisition until the price reaches $p + \varepsilon$ (rather than acquiring information at p), where ε should be thought of as a small increment. Under the event where the bidder 2 does not drop out before $p + \varepsilon$, bidder 1 obtains the same expected payoff. Under the event where bidder 2 drops out before $p + \varepsilon$, bidder 1's expected payoff is equal to

$$E[h(\theta_1) - c \mid p < \theta_1 < p + \varepsilon]$$

if he acquires information at p , while it is equal to

$$v - E[\theta_1 \mid p < \theta_1 < p + \varepsilon]$$

if he had planned to acquire information at $p + \varepsilon$.

Since ε can be chosen arbitrarily small, we get that when $p > p^{**}$ the former option (immediate acquisition) is preferable, and that when $p < p^{**}$, the later option (delayed acquisition) is preferable.

We may now gather all three claims above and conclude the derivation of equilibrium behavior:

Proposition 3 *When there is more than one other active bidder, bidder 1 does not acquire information. When bidder 1 has not acquired information yet and there is only one other active bidder, bidder 1 acquires information if only if the current price lies between p^{**} and p^* where p^* and p^{**} are defined by (6) and (7), respectively. Bidder 1 drops out as soon as price p^* is reached.*

As a Corollary of Proposition 3, we get the following Proposition, which makes precise under what events information acquisition occurs in the ascending-price auction.

Proposition 4 *Assume that $H(n-1, 0) < c < H(1, v)$. Then, in the ascending price auction, bidder 1 acquires information whenever $\theta^{(1)} > p^{**}$ and $\theta^{(2)} < p^*$.*

3.3 Revenues

What is the effect of information acquisition on the seller's revenues? In general, the effect of information acquisition on revenues is ambiguous.

When $\theta^{(1)} < v$ for example, the seller would prefer that bidder 1 does not acquire information. Without information acquisition the selling price is $\theta^{(1)}$, while with information acquisition, the selling price never exceeds $\theta^{(1)}$ and sometimes gets below $\theta^{(1)}$ (when bidder 1 learns that $\theta_1 < \theta^{(1)}$).

In contrast, when $\theta^{(2)} > v$, the seller would prefer that bidder 1 acquires information. Without information acquisition the selling price is $\theta^{(2)}$, while with acquisition, the selling price never gets below $\theta^{(2)}$ and sometimes exceeds $\theta^{(2)}$ (when bidder 1 learns that $\theta_1 > \theta^{(2)}$).

When the number of bidders is large enough, the event $\{p^* > \theta^{(2)} > v\}$ is much more likely than the event $\{\theta^{(2)} < v\}$, and as a consequence, the second effect dominates:

Proposition 5 *When the number of bidders is large enough, the ascending price auction generates more revenues than the sealed-bid second price auction.*

Proof: Define $\Delta(p)$ as the gain in revenues generated by information acquisition under the event where information acquisition by bidder 1 occurs at price p , with only one other bidder still active, say bidder 2. We have:

$$\Delta(p) = E[R(\theta_1, \theta_2, p) - R(\theta_1, v, p) \mid \theta_2 > p],$$

where $R(x, y, z)$ denotes the second largest element among x , y and z . Let $\underline{\Delta}$ be a lower bound on $\Delta(p)$. Note that for any $p \geq v$, $\Delta(p)$ is bounded away from 0, by some scalar $\Delta > 0$. Under the conditions of Proposition 4, we obtain that, conditional on information acquisition, the increase in revenues generated by the ascending price auction is equal to

$$E[\Delta(\max(\theta^{(2)}, p^{**})) \mid \theta^{(2)} < p^*, \theta^{(1)} > p^{**}].$$

Let $P = \Pr\{\theta^{(2)} < v \mid \theta^{(2)} < p^*, \theta^{(1)} > p^{**}\}$. The expression above is at least equal to $P\underline{\Delta} + (1 - P)\Delta$, hence it is positive when the number of bidders get large because P gets small.

To illustrate, we quantify this effect in a simple numerical example.

3.4 A numerical example

We assume that each θ_i is drawn from the uniform distribution on $[0, 1]$. We choose a cost of information acquisition equal to $c = 0.025$. For this cost, the conditions of Proposition 1 are satisfied as soon as there are at least 5 bidders (in addition to bidder 1). To fix ideas, we first give, as a function of the number of bidders, the probability that bidder 1 acquires information in the ascending price auction.

$n - 1$	5	6	7	8	9
$\Pr\{\text{bidder 1 acquires information}\}$	0.36	0.25	0.16	0.12	0.08

Information acquisition is therefore significant, at least when the number of bidders is not too large. Bidder 1 does not acquire information with probability one however, essentially because there are events where $\theta^{(2)} > p^*$, in which case bidder 1 prefers to drop out without acquiring information: competition is too tough.²⁴

The next table gives, as a function of the number of bidders, the percentage of increase in revenues (denoted $\bar{\Delta}$) that would be generated if bidder 1 were to acquire information with probability 1. These numbers give an idea of the range of increase in revenues that we can expect.

$n - 1$	5	6	7	8	9
$\bar{\Delta} = \frac{R^{\text{acquisition}} - R^{\text{no acquisition}}}{R^{\text{no acquisition}}}$	4.3%	3.7%	3.1%	2.6%	2.1%

Finally, we give, as a function of the number of bidders, the percentage of increase in revenues (denoted Δ) generated by the ascending price auction (as compared to the second price auction). Because bidder 1 does not acquire information with probability one however, these numbers do not match the maximum percentage of increase $\bar{\Delta}$. Still, compared to $\bar{\Delta}$, the percentage of increase Δ is significant, as the following table shows.

$n - 1$	5	6	7	8	9
$\Delta = \frac{R^{\text{ascending}} - R^{\text{second price}}}{R^{\text{second price}}}$	2.5%	1.9%	1.3%	0.9%	0.6%
$\Delta/\bar{\Delta}$	57%	50%	42%	35%	28%

Note that $\Delta/\bar{\Delta}$ is significant even though the probabilities of information acquisition are not very large. The reason is that bidder 1 acquires information precisely in those events where

²⁴In our numerical example, p^* is equal to 0.6.

$\theta^{(2)}$ is not very large, hence in events where information acquisition is most likely to have a big impact on revenues.

We shall return to this numerical example after examining the case where more than one bidder can acquire information, in which case the increase in revenues generated by the ascending price auction is even larger.

4 Information acquisition by multiple bidders

We now turn to our more general model in which more than one bidder can acquire information (remember that each bidder i is initially assumed to be informed with probability q , see Section 2). Finding equilibrium behavior is difficult in general, because the incentives of a bidder to acquire information may depend on the other bidders' decisions to acquire information. Our main insight however that ascending price auctions generate more revenues than second price auctions will carry over to that more general model. For two reasons: first, because as in the previous setting, ascending price auctions generate more information acquisition. Second, because uninformed bidders may find it optimal to delay their decision as to whether to drop out or to acquire information, and when several bidders do that, this tends to push up the price paid by the winner.

We start with some preliminary notation. We shall denote by n_1 the number of bidders who are initially informed, and by n_2 the number of bidders who are initially uninformed.²⁵ Consider now the second price auction. If a bidder, say bidder 1, is initially uninformed and decides to acquire information, he obtains an expected payoff at most equal to $H(n_1, 0)$ when there are n_1 informed bidders. Thus, taking expectations over n_1, n_2 , whenever

$$E[H(n_1, 0) \mid n_2 \geq 1] < c,$$

that bidder will have no incentives to acquire information (whatever the information acquisition strategy of the uninformed). As in the previous section, if the number of bidders is large enough, ex ante chances of winning are so small in the second price auction that it is not worthwhile acquiring information.

We now move to the ascending price auction. In the previous section, we considered the following strategy: wait until there are only two bidders left, acquire information, and drop

²⁵These are the realizations of random variables as determined by q (see Section 2).

out if the valuation turns out to be low. Thanks to this drop out option, there were almost no cost to waiting, because when the other bidder was informed there was only a negligible chance that the other bidder would want to drop out precisely at the same price.

In the general model, the situation is different, because there is a chance that the other remaining bidder is also uninformed, hence that he will also be willing to drop out as soon as there are only two bidders left. Staying active above one's own expected valuation may thus involve a cost. Still we will show below that for not too large information acquisition costs, dropping out at v without acquiring information is a dominated strategy. This effect will in turn imply the superiority of the ascending price auction in generating more revenues.

Define $G(p)$ as the payoff that bidder 1 would obtain in the following scenario: (i) The current price is p , (ii) there is only one additional bidder left, say bidder 2, who drops out at p , (iv) bidder 1 is uninformed and acquires information (for free) at p .

Since bidder 2 is willing to drop out, bidder 1 has a chance $1/2$ of getting the object even if he is willing to drop out. So we have:

$$\begin{aligned} G(p) &= \int_{\theta_1 \leq p} \frac{\theta_1 - p}{2} f(\theta_1) d\theta_1 + \int_{\theta_1 > p} (\theta_1 - p) f(\theta_1) d\theta_1 \\ &= \frac{v - p + h(p)}{2} \end{aligned}$$

where $h(p)$ is defined by (3) (see Section 2).

Next define

$$\tilde{H}(p) = \min(G(p), H(1, p)),$$

where $H(k, p)$ is defined by (4) (see Section 2).

In what follows, we shall consider information acquisition costs that are not too large, i.e. no larger than $\tilde{H}(v)$. This will ensure that the price \tilde{p} for which

$$\tilde{H}(\tilde{p}) = c$$

lies above v . Note that by definition, \tilde{p} lies below p^* (where $H(1, p^*) = c$, see Section 3), but for some distributions and cost parameters, \tilde{p} coincides with p^* exactly.²⁶

²⁶For example, when θ_i is uniformly distributed on $(0, 1)$, we have that $h(p) = \frac{(1-p)^2}{2}$ and $H(1, p) = \frac{(1-p)^2}{6}$. Thus, $h(p)/2 > H(1, p)$ and clearly for some range of c (sufficiently close to $H(1, v)$) we have that $\tilde{p} = p^*$.

The following Proposition shows that in equilibrium, and so long as the price remains below \tilde{p} , uninformed bidders do not drop out: they either acquire information or they remain active.

Proposition 6 *In equilibrium, for any current price $p \in [v, \tilde{p})$, an uninformed bidder (say bidder 1) derives a positive gain from following the strategy that consists in waiting until the first date where either only one other bidder remains active (at which point bidder 1 acquires information) or where the price reaches \tilde{p} (at which point bidder 1 drops out).*

We first observe that in any event where there remain only two bidders left and the current price p lies above v , it is a dominant strategy for uninformed bidders *not to postpone* information acquisition.²⁷ So uninformed bidders compare immediate information acquisition with immediate drop out without information acquisition. We distinguish two events: event A , where the other bidder is already informed, or where the other bidder acquires information and obtains a realization $\theta > p$; and event B , where the other bidder is uninformed and decides to drop out, or where the other bidder is uninformed, acquires information and obtains a realization $\theta < p$.

If he acquires information, bidder 1 obtains $G(p)$ in event B , and $H(1, p)$ in event A . If he does not acquire information and drops out immediately, he obtains $(v - p)/2$ in event B . He thus prefers to acquire information whenever

$$(\Pr A)H(1, p) + (\Pr B)(G(p) - (v - p)/2) > c$$

Since $G(p) - (v - p)/2 > G(p)$ whenever $p > v$, the inequality above always holds for $p \in (v, \tilde{p})$. It follows that in equilibrium, for any current price $p \in [v, \tilde{p})$, if there are only two remaining bidders, uninformed bidders acquire information immediately.

The analysis above also implies that bidder 1 gets at least $\tilde{H}(p) - c$ in the event where all other bidders but one drops out before \tilde{p} . Note that this event has positive probability because there is always a chance that all other bidders are informed and drop out before

²⁷Compared to Section 3, the argument is slightly different because the other bidder's distribution over exit price may include mass points. When there are mass points, the argument is as follows. At a mass point, consider the event where the other bidder (say bidder 2) exits. Bidder 1 obtains $v - p$ if he postpones, $G(p) - c$ if he acquires information at p , and $h(p) - c$ if he could acquire information slightly before. Thus when $p > v$, postponing information acquisition is a dominated strategy.

\tilde{p} . In the event where the price reaches \tilde{p} with at least three active bidders, bidder 1 drops out. If he is the only one willing to exit, he becomes inactive and obtains 0. Otherwise, he can always re-apply to exit, until either he is selected to exit, or until there remain only two active bidders, at which point he may acquire information and derive an expected payoff at least equal to $\tilde{H}(\tilde{p}) - c = 0$.

One Corollary of Proposition 1 is the following:

Proposition 7 *In equilibrium, uninformed bidders either acquire information, or drop out after price \tilde{p} is reached.*

Note that this Proposition is weaker than that obtained for the case of a single uninformed bidder, because we cannot guarantee that bidders will wait until there are only two bidders left to acquire information. And indeed, a more detailed analysis would show that uninformed bidders may have an incentive to acquire information before others, so as to avoid the event where all remaining bidders want to drop out of the auction simultaneously.

Nevertheless, this Proposition will be sufficient to prove that with a not too small number of bidders, the ascending price auction generates more revenues than the second price auction. Intuitively, Proposition 7 implies that either uninformed bidders acquire information, and then revenues increase essentially for the same reason as that of Section 3; or uninformed bidders do not acquire information, but then the strategy of waiting until price \tilde{p} is reached raises the price paid by the winner on average.

Proposition 8 *If n is large enough, revenues in the ascending auction are larger than in the second price auction.*

Proof: Let $\hat{p} = (v + \tilde{p})/2$, and let m_2 denote the number of uninformed bidders who acquire information in equilibrium, and let $\bar{\theta}_u$ be the second largest realization of the valuations of the uninformed bidders acquiring information on their valuation. We also define

$$\Delta(p) = E[R(\theta_1, \theta_2, p) - R(\theta_1, v, p) \mid \theta_1 > p],$$

where $R(x, y, z)$ denotes the second largest element among x , y and z . $\Delta(p)$ corresponds to the increase in revenues generated by the fact that one uninformed bidder acquires information

in the event $\theta^{(2)} = p$. Note that for any $p \geq v$, $\Delta(p)$ is bounded away from 0. We set $\Delta = \min_{p \geq v} \Delta(p)$.

We first restrict attention to the event A where the number of uninformed bidders n_2 is between 1 and n^* (assumed to be sufficiently large, see below) and the second largest valuation among informed bidders is smaller than \hat{p} :

$$A = \{1 \leq n_2 \leq n^* \text{ and } \theta^{(2)} < \hat{p}\}.$$

If $m_2 \leq n_2 - 1$, then (by Proposition 7) the price must increase to \tilde{p} at least, hence generating an increase in revenue at least equal to $\tilde{p} - \hat{p}$; If $\theta^{(2)} \geq v$ and $m_2 = n_2$, then at least one uninformed bidder acquires information in equilibrium, generating an increase in revenue at least equal to Δ . So revenues may only decrease in events where $\{\theta^{(2)} < v \text{ and } m_2 = n_2 \leq n^*\}$, but conditional on A , this event has probability at most equal to $\Pr\{\theta^{(2)} < v \mid \theta^{(2)} < \hat{p}, n_1 = n - n^*\}$, which is small when n is large. Since revenues increase by an amount bounded away from 0 in the complement event, revenues increase conditional on A .

We now consider the event $B = \{n_2 > n^* \text{ and } \theta^{(2)} < \hat{p}\}$.²⁸

If $m_2 \leq n_2 - 1$, then the price must increase to \tilde{p} at least, hence generating an increase in revenue at least equal to $\tilde{p} - \hat{p}$; If $m_2 = n_2$ and $\bar{\theta}_u > \tilde{p}$, revenues increase by $\tilde{p} - \hat{p}$ at least. So revenues may only decrease in events when $\bar{\theta}_u < \tilde{p}$ and $m_2 = n_2$. Conditional on B , this event has small probability for n^* large enough. Since revenues increase by a significant amount in the complement event, revenues increase conditional on B .

Finally conditional on event $\theta^{(2)} > \hat{p}$, whether uninformed bidders acquire information or not, revenues cannot decrease in the ascending price auction. So overall, revenues must increase. **Q.E.D.**

Deriving the equilibrium in a modified ascending price auction.

When bidders observe the number of bidders left, the information acquisition strategy may

²⁸In case $n_1 < 2$, we set $\theta^{(2)} = 0$.

be quite complex in the ascending price auction.²⁹ Yet, even without deriving explicitly the equilibrium, we were able to prove that the ascending price auction must generate more revenues than the sealed-bid auction when the number of bidders is large enough.

In a variant of the ascending price auction, we now provide a complete characterization of the equilibrium. Specifically, assume now that participants of the auction are informed of when there are two bidders left, but they receive no information about the number of participants left prior to that date. (Other than that, the same specifications as for the ascending price auction described in Section 2 hold. That is, the price raises exogenously and the auction stops when there is one bidder left: this bidder wins the object and pays the current price for it.³⁰)

Equilibrium behavior is easily derived. Choose a price $\hat{p} > v$ that solves:

$$r(p)H(1, p) + (1 - r(p))G(p) - c = 0, \quad (8)$$

where $r(p) = (1 - F(p))/(1 - qF(p))$, and assume that³¹ $H(1, \hat{p}) > G(\hat{p})$. (This is the case when c is not too large). Then it is an equilibrium for uninformed bidders to drop out when the price reaches \hat{p} and there are still more than two bidders left, and otherwise to acquire information as soon as there are two bidders left and the price is below \hat{p} .

To see this, observe that $r(p)$ is the probability of facing a bidder with valuation above p , conditional on having two bidders left and the uninformed bidder acquiring information at p .³² The left hand side of (8) corresponds to the expected payoff an uninformed bidder (say bidder 1) would obtain if he remained active and happened to become selected at price

²⁹For example, as mentioned earlier, it is no longer the case that bidders find it optimal to wait until there is only one other bidder left to learn their valuations because of the risk that these two bidders simultaneously decide to drop out as a result of learning bad news on their valuations.

³⁰To ensure that at some point, exactly two bidders will remain active, we assume that in case there remain m active bidders and $m - 1$ or m bidders want to exit at the same date, respectively one or two of them are denied the possibility to exit (each one of them having an equal chance of being selected).

³¹In the alternative case where $H(1, \hat{p}) < G(\hat{p})$, the equilibrium is more complex and involves mixed strategies.

³²Indeed, conditional on having two bidders left, the other bidder has a chance $(1 - q)/(1 - qF(p))$ of being uninformed, hence a chance

$$r(p) = 1 - F(p) \frac{1 - q}{1 - qF(p)}$$

of having a valuation above p .

p . So as long as $p < \hat{p}$, he is willing to remain active, and for any $p > \hat{p}$, he is willing to drop out. Now at $p = \hat{p}$, if he drops out, the probability of facing a bidder with valuation above \hat{p} conditional on being selected is not $r(\hat{p})$ but some $\hat{r} > r(\hat{p})$ (when there are more bidders around being selected is informative that the other bidder was more likely to be informed with valuation greater than \hat{p}).³³

5 Discussion

We discuss below various extensions of our model. First, we examine the case (common in practice) where acquiring information takes time. Then we examine other types of dynamic selling mechanisms, which are frequently used in practice: multi-stage mechanisms, and auctions where price increases by increments. We next discuss another setting where information acquisition would seem to be an important issue: multi-object auctions. Finally, we discuss a dynamic format that would not lead to an increase in revenues: the ascending price auction with secret drop out.

(i) *When information acquisition takes time.*

Although we have focused on the case where information acquisition is immediate, the extension to the case where information acquisition takes time is straightforward: we may compute the option value for a bidder to remain active until there are only two bidders left, and acquire information at that date. The benefit from this strategy will not be as high as it is under immediate information acquisition, because it may be that the other remaining bidder will exit before information is actually learned. Nevertheless, if learning does not take too much time, this option will remain valuable.

More formally, let us consider the simple situation in which only one bidder, say bidder 1,

³³Indeed, consider the event where there are m bidders left at \hat{p} . An uninformed bidder will be selected with probability 0 if there are at least 2 informed bidders, with probability $1/(m-1)$ in the event where there is only one informed bidder (event A), and with probability $2/m$ in the event where there are no informed bidders (event B). Conditional on the event "being selected and there were m bidders left at \hat{p} ", the probability of facing uninformed bidder is

$$\frac{(2/m) \Pr B}{(2/m) \Pr B + (1/(m-1)) \Pr A}.$$

Since $\Pr A / \Pr B = (m-1)q(1-F(p))/(1-q)$, the probability of facing an uninformed type is equal to $(1-q)/(1-q + (m/2)q(1-F(p)))$, hence is smaller than $(1-q)/(1-qF(p))$, which implies that $\hat{r} > r(\hat{p})$.

may acquire information, and assume that in the lapse of time that is needed for information acquisition, price increases by an amount equal to Δ at most. Assume that the current price is p and only one other bidder, say bidder 2, remains active. If bidder 1 decides to acquire information, the probability that bidder 2 exits before bidder 1 actually learns his valuation is $q(p) = (F(p + \Delta) - F(p))/(1 - F(p))$. Bidder 1's expected gain from acquiring information at p is therefore at least equal to

$$q(p)(v - (p + \Delta)) + (1 - q(p))H(1, p + \Delta) - c$$

If Δ is not too large, then $q(p)$ will be small, and under the assumption of Proposition 1 (i.e. $H(1, v) > c$), the expression above will be positive for a range of prices above v ; the consequence being that it remains a *dominated* strategy for bidder 1 to drop out at these prices without acquiring information.

(ii) *Multi-stage mechanisms*

Given the prevalence of multi-stage auctions in practice, it is natural to wonder whether our analysis extends to such settings. It should be clear that it does: the key issue is whether contestants get information about the state of competition in the course of the selling procedure, and have enough time to use that information to acquire information.

One possible design would consist in (i) having the number of selected contestants reduced from one stage to the next, (ii) letting enough time for information acquisition in between stages, and (iii) having the highest losing bid at a particular stage serve as a reserve price for the next stage.³⁴

For example, our main insight would carry over to a selling procedure that consists of the following two stages: a first stage in which the price rises exogenously until only two bidders remain active, later followed by a second stage in which the two remaining contestants are required to participate in a standard ascending price auction in which the reserve price is set to the highest price reached in the first stage.³⁵

Our insight may thus account for the prevalence of multi-stage mechanisms in situations

³⁴This last feature ensures that bidders bid seriously in the first stage.

³⁵In the case where only bidder 1 may acquire information, the analysis would be identical to that of Section 3. In the case of information acquisition by multiple bidders, equilibrium behavior would be similar to that derived at the end of Section 4, for the modified ascending price auction, because in both formats, the bidders do not get information about the number of remaining bidders until only two bidders are left.

where information acquisition costs are significant (so that information acquisition is not undertaken in sealed-bid mechanisms), and where information takes a significant time to acquire (since it is easy to design multiple stages that leave enough time for information acquisition between stages).

(iii) *Auctions with discrete price increments.*

As is often the case in practice, prices do not increase continuously, but by small (or not so small) discrete increments. Assuming that the price increment is at most equal to Δ , and that the lapse of time between two price increments is sufficient to acquire information, we can once again derive the benefits of the strategy that consists in waiting until there are only two bidders left. As for the case where information acquisition takes time, if Δ is not too large, we obtain that as long as the current price is not too much above v , it is a dominated strategy to exit without acquiring information.

(iv) *Multi-object auctions.*

While our formal analysis has been based on a single object problem, we show in Compte-Jehiel 2002a that our basic insight carries over to multi-object auctions. Besides in these auctions, not only do bidders have to decide whether or not to acquire extra information; but when they do so, they have to decide *on which object* to acquire information. Ascending formats that generate information on which object a bidder has better chances of winning are then likely to perform better than formats such as static ones that do not generate such information: when a bidder is not guided as to which object(s) to focus on, he takes poor information acquisition decisions, which in turn is likely to discourage him from acquiring any information. In contrast, when a bidder is guided as to which objects he should focus on, he makes good information acquisition decisions, which in turn leads him to acquire extra information more often. (See Ausubel and Milgrom 2001 for an account of why this may be of practical importance in package auctions.)

(v) *The ascending-price auction with secret drop-out*

An important feature of the ascending price auction is that in the course of the auction, bidders learn about the number of bidders who are still interested in the object. (This is also true, though to a lesser degree, in the variant studied at the end of Section 4 in which bidders are informed when there are two bidders left.) To illustrate why this feature of the ascending format is important, we now briefly examine the ascending price auction with secret drop

out. The auction is identical to the ascending auction, except that bidders do not observe whether and when other bidders drop out until the auction gets to a complete end (i.e. until there is only one bidder left). Consider the situation of Section 3 in which all bidders except bidder 1 are ex ante informed of their valuation and this is common knowledge. Bidder 1 still has the option to postpone information acquisition, but in general, this option is not as valuable as in the ascending price auction with public drop out, as we now show.

Indeed, for any current price p (at which the auction is still going on), bidder 1 now only learns that $\theta^{(1)}$ is larger than p . Hence the expected payoff that bidder 1 would obtain by acquiring information is

$$E[h(\theta^{(1)}) | \theta^{(1)} > p] - c$$

This expression coincides with $H(n-1, 0) - c$ at $p = 0$, and since h is decreasing in p , it is also decreasing in p . So, if acquiring information yields a negative profit at 0, it also yields a negative profit at any other price. Intuitively, learning that $\theta^{(1)}$ is larger than p is bad news for bidder 1, hence this decreases his expected payoff from information acquisition. It thus follows that if $H(n, 0) < c$, then bidder 1 does not acquire information in the second-price auction. Nor does he acquire information in the ascending price auction with secret drop out.

Other variants of the ascending price auction include the possibility (see Harstad and Rothkopf 2000 and Izmalkov 2003) that bidders might re-enter after dropping out.³⁶ In such formats, the observation that there are few bidders around may not be as reliable as in the ascending format analyzed in Sections 2 to 4.³⁷

6 Conclusion.

We conclude by taking a broader perspective, and we consider the more general issue of how one should organize screening among agents who may acquire further information about their types.

From a general Principal-Agent perspective, we have dealt with a setting in which a principal (the seller) attempts to screen among agents (potential buyers) who may acquire information about their types (valuations), or invest so as to affect their types (as in the

³⁶Alternatively, bidders may sometimes have the possibility to hide that they are still around.

³⁷This would be the case if in equilibrium bidders sometimes use the option to drop out and re-enter.

alternative formulation – see expression (2) in Section 2), *prior to signing a contract (the sale)*. As in our sale example, the principal may find it desirable to provide agents with incentives to acquire information or invest prior to signing a contract, because this improves the chance of selecting a more able agent; that is, an agent with a better type (See Compte-Jehiel 2002b). In this context, our analysis suggests that a dynamic screening procedure, that leaves the competing agents some time to acquire information about their types, would outperform static screening procedures.³⁸

Another example along these lines is the case of a sponsor who wishes to induce potential contestants (and possibly the ablest one) to exert high research effort. When research outcomes are not measurable or contractible, one option for the sponsor is to organize a tournament in which the winner gets a fixed prize. As mentioned in the introduction, inducing high research effort is sometimes more economically achieved by reducing the number of contestants (rather than increasing the prize). In such settings, it is important for the sponsor to screen among potential contestants so as to induce the participation of the ablest ones only.³⁹

How should one organize screening? This issue is of primary importance, as failing to screen good contestants may jeopardize the success of the tournament, and Fullerton and McAfee have identified why some screening procedures (that would auction rights to participate in the tournament) could fail to screen properly.⁴⁰

Though screening is often thought of as a pure adverse selection problem, it seems plausible that some contestants may only have a rough idea of how successful their research effort will be, and that by investing a bit *prior to the tournament*, they could have a much better idea of how able they are. Besides, the sponsor could clearly benefit from such investments,

³⁸Cremer and Khalil (1992) consider a principal-agent setup in which the agent can learn his type before signing the contract. Most of their analysis bears on the one-agent case in which no screening is needed (their main insight is that the principal should not induce the agent to learn his type before the contract is signed). In the multi-agent section of their paper, Cremer and Khalil restrict attention to contracts in which the agents do not have incentives to learn their type. However, (unlike in the one-agent case) such contracts need not be optimal.

³⁹When contestants have identical abilities (as in Che and Gale), this can easily be achieved by a random selection for a subset of contestants. However, when they are not ex ante identical, a finer screening device is required.

⁴⁰They suggest to consider all-pay auctions.

since this should help him select the *truly* ablest contestants. Here again, a dynamic screening procedure would outperform static screening procedures.⁴¹

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⁴¹To illustrate, and in order to abstract from the issues raised in Fullerton McAfee, assume that the outcome of research is measurable, and that it can be contracted on. Formally, the outcome of research for contestant i is a random variable x_i , and we assume that the sponsor auctions (through an ascending auction) a contract that pays $P(x)$ for the outcome x . The cost of research is assumed to be identical across contestants, and equal to γ . The bidders differ in how likely they are to produce good research, that is, contestant i 's distribution over research outcome is $f(\cdot | \theta_i)$ where θ_i is interpreted as contestant i 's ability. When all bidders are aware of their ability, the auction selects the bidder for which $E[P | \theta_i]$ is highest. What if some bidders do not know θ_i precisely, but only have a rough idea of it, and if at some cost c , they could get to know θ_i ? This is precisely the setting we have analyzed, with the conclusion that it is generally worthwhile to design a procedure that allows participants to take some time to acquire information during the process.

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