Cautiousness

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Abstract

A standard approach in modeling uncertainty in economic models is to assume that there is a probability distribution over the states of nature and that the decision maker receives a signal that is correlated with the state. In the vast majority of papers in this literature, it is assumed that the decision maker knows or behaves as if he knew the joint distribution over states and signals. We argue that this assumption is unrealistic, and perhaps more importantly, that this unrealistic assumption can lead directly to unrealistic conclusions. We suggest an alternative approach that is less demanding on the decision maker’s knowledge, and illustrate the approach in several simple examples.
1 Introduction

Agents often lack detailed knowledge about the consequences of their actions. The traditional route to modelling lack of knowledge takes whatever is not known as a random variable and assumes that the agent faces uncertainty about the realization of that random variable.

For many problems, the consequence is not a random variable and the agent is not facing objective uncertainty. The uncertainty may reflect the agent’s ignorance rather than the realization of a random variable. The decision theory perspective is that if the agent satisfies Savage (1954)’s axiom, he behaves as though the consequence was a random variable, that is, as though he had a subjective probabilistic belief over possible consequences and maximized expected utility given that belief.

This "as if" perspective has been extraordinarily useful in shaping our models, and although Savage (1954) is not necessarily meant to be a description of the way people make decisions, nor a suggestion that agents actually form beliefs, it is standard to view expected utility maximization as a description of decision making.

As analysts, we nevertheless face a difficulty. Much of economics is concerned with understanding regularities in an economic agent’s behavior. Regularities presumably arise because of similarities in the circumstances driving behavior or between the situations faced. Understanding regularities would thus seem to require not only a descriptive theory of behavior but also an understanding of the links the agent makes between the situations that he faces.1

In most game theory applications, this difficulty is addressed by putting structure on the beliefs that an agent holds across the situations that he faces. Consider the following example.

Tom participates in an auction in which \( n \) other bidders participate. In that particular auction situation, valuations consist of a vector \( v = (v_1, \ldots, v_n) \). Tom knows \( v_1 \) but he ignores \( (v_2, \ldots, v_n) \). A typical auction model assumes that Tom is facing (or behaves as if he was facing) a distribution over auction situations, that is, a distribution over valuation vectors. As an analyst evaluating welfare properties of different auction formats, or as an econometrician identifying valuations from bids, we treat valuation vectors as drawn from an objective distribution, and we assume that Tom behaves optimally as if he had detailed knowledge of that objective distribution. This assumption provides us with a link between the various auction situations that Tom is supposed to face, as each valuation \( v_1 \) comes with a posterior belief over others’ valuations.

Said differently, a new situation is not, strictly speaking, the realization of an objective random variable. For lack of a better model however, we typically treat it as though it was the realization of an objective random variable, related to the distribution over past situations. Standard models then adopt an omniscient

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1 The observation that expected utility theory lacks a description of the process by which beliefs are formed across problems has been the basis for cased-based decision theory (Gilboa and Smeidler, 1995).
perspective, inherited from Harsanyi (1967/68): the agent behaves optimally as though he knew with precision that objective distribution.

This paper starts with a similar premise: agents strive to find out optimal behavior in the face of some given distribution over situations, as if that distribution was an objective one. Our perspective however is to model decision making when detailed knowledge of this objective distribution is lacking.

In a nutshell, the standard/omniscient perspective is tantamount to providing the agent with the ability to correctly evaluate and compare all possible choice rules that map data to decisions. The path proposed limits and suggests plausible ways to limit the choice rules that the agent compares. By limiting the choice rules that are considered, the agent behaves as if he had less knowledge of the underlying distribution.

Our motivation stems from various considerations. The first one is plausibility. Consider the following example. Tom is asked about the number of coins in a jar (the true number/state is $s$), and some number $z$ comes to his mind ($z$ is a point estimate). A standard Bayesian model would treat $z$ as a signal of the true state $s$ and assume a joint distribution $\omega$ over $(s, z)$, reflecting the situations of this kind that Tom faces. This modelling strategy implies that (i) each realization $z$ comes with a precise probabilistic belief about the underlying state $s$; and (ii) his belief is objectively correct, in the sense that it coincides with the Bayesian posterior $\omega(\cdot | z)$.

Both assumptions seem unrealistic. Tom is probably aware that he cannot get it right. Yet he probably cannot tell whether he is over or under-estimating $s$, and his perception of the magnitude of the errors he is making is probably rough. In any event, precise knowledge of the objective distribution $\omega$ seems out of reach.

Lack of realism is not our main concern however. Models may shape our intuitions in valuable ways even if they lack realism (Rubinstein 1991). Concerns arise when unrealistic assumptions lead to unrealistic conclusions.

Standard models sometimes yield unrealistic conclusions because behavior may end up being driven, not by the uncertainty about the underlying state, but by the (assumed) certainty about the prior distribution (over states and signals/data). For example, the assumption makes any comparative statics with respect to quality of information delicate, because an agent getting noisier estimates about the state will tend to put more weight on the prior. We may thus end up with a notion of "informational rents" and the "insight" that rents are easier to extract from poorly informed individuals. But this is an (unfortunate) artifact of the model.

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2 That is, situations where Tom must find some number $s$ and comes up with a point estimate $z$.

3 More generally, the beliefs that we hold often seem to be crude ones, and we often find ourselves unable to form precise probability estimates. In the words of Savage (1954, p58):

"The postulate of personal probability imply that I can determine, to any degree of accuracy whatsoever, the probability (for me) that the next president will be a Democrat. Now it is manifest that I cannot determine that number with great accuracy, but only roughly."
Conclusions also seem unreasonable when the equilibrium construction hinges on the precise knowledge of prior distributions. Belief free equilibria in repeated games with imperfect private monitoring are a good example of this.

Bandit problems also illustrate this problem. That an individual would modify his behavior based on past signals, or that he would sometimes experiment, seems reasonable. That he would finely tune his experimentation strategy to his priors over possible stochastic processes that generate the data seems both unnecessarily complex and unreasonable. We would not want dynamic models involving learning to deliver insights that hinge on learning being optimal, and probably be happy with models that involve lesser sophistication.

Related literature.

Earlier attempts to weaken the assumption that agents would know (or behave as if they knew) the underlying distribution over states and data/signals have essentially followed two directions.

One direction is that taken by the robustness literature (which can be seen as a response to Wilson’s critique). That literature takes the standard Bayesian route that treats anything that is not perfectly known (here, the prior distribution) as a random variable, but assuming a known prior over priors (Bergemann and Morris, 2005 and Sims, 2003).

Another direction is the one taken by bounded rationality models. The path followed consists of maintaining the assumption that agents choose the alternative that maximizes expected utility, but to distort the way alternatives are evaluated. This can done by adding errors to valuations of alternatives (random stimuli/utility models, Block and Marschak (1960)), possibly putting structure on the sources of errors (procedural models, Osborne and Rubinstein (1998)), or by implicitly defining a process that generates subjective beliefs (different from objective ones). This is the case for non-partitional models (Geanakoplos (1989)), case based beliefs (Gilboa and Schmeidler (1995)) and analogy based beliefs (Jehiel (2005)).

Finally, there has been objections to the assumption that agents would form precise probabilistic beliefs and then maximize expected utility as if they were perfectly confident of their beliefs. This assumption has its roots in Savage (1954) and some have suggested alternatives to Savage: Bewley (1986), Schmeidler (1986) and Gilboa and Schmeidler (1989) all capture the idea that the agent may not be fully confident about his prior.4

2 Decision making under objective uncertainty

The model we present is a standard model of decision making. We emphasize the objective nature of the uncertainty that the agent faces, but as explained in introduction, this assumption is implicit in most game theory applications.

Formally, we define $A$ as the set of alternatives, and $S$ as the set of states. A state $s$ summarizes everything that the agent needs to know to properly evaluate

\footnote{Nevertheless, the agent who would take seriously the idea that he ought to come up with a set of priors would have to be confident in the exact way he lacks confidence.}
which alternative is best. *Preferences over alternatives* are then characterized by a utility function \( u(a, s) \) that specifies a welfare level for each \( a \in A \) and \( s \in S \).

The agent is ignorant of the underlying state \( s \) but he has some imperfect knowledge of it. We let \( z \) denote the data/signal that he gets. We shall refer to the pair \((z, s)\) as the current *situation* that the agent faces. We have in mind that the agent’s decision are based on the data/signal \( z \).

The agent faces objective uncertainty. Specifically, the situation \((z, s)\) belongs to a range of situations \( D \) that the agent faces, and we let \( \omega \in \Delta(D) \) denote the distribution over situations that the agent faces.

### 2.1 The standard approach.

Rational decision making assumes that for any that for any signal \( z \) that he might receive, the agent takes the alternative that maximizes his expected welfare. Denoting by \( r^*(z) \) that optimal decision, we have:

\[
r^*(z) \equiv \arg \max_a E_\omega[u(a, s) \mid z].
\]

Alternatively, one may define the set \( \mathcal{R} \) consisting of all possible decision rules \( r() \) that map \( Z \) into \( A \). For each rule \( r \in \mathcal{R} \), one can define the expected utility (or performance)

\[
v_\omega(r) = E_\omega u(r(z), s).
\]

The optimal decision rule \( r^* \) solves:

\[
r^* = \arg \max_{r \in \mathcal{R}} v_\omega(r).
\]

**Example 1 An estimation problem**

An agent has to determine a number that is complicated to evaluate (a quantity of oil in an oil field, a number of coins in a jar, etc.). The agent makes a report, and he is penalized as a function of the distance between the report that he makes and the true number (e.g., quadratic preferences).

Formally, we choose \( u(a, s) = -(s - a)^2 \) to describe preferences, and assume that

\[
z = s + \varepsilon
\]

where \( s \) and \( \varepsilon \) are drawn from independent normal distributions centered on \( s_0 \) and 0 respectively:

\[
s = s_0 + \eta
\]

with \( \eta \sim N(0, \sigma_\eta^2) \) and \( \varepsilon \sim N(0, \sigma_\varepsilon^2) \).

The optimal report then consists of choosing

\[
r^*(z) = E_\omega[s \mid z] = z - \rho(z - s_0) \text{ where } \rho = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2}.
\]
In this example, the parameter \( z \) can be interpreted as a noisy estimate of the true state, and the optimal rule above can be interpreted as that the estimate should be taken with caution and regressed to the mean (\( s_0 \)).

2.2 Implicit informational assumptions

Strictly speaking, the basic model outlined above does not require that the agent knows the distribution \( \omega \). One could take the view that the agent has learned the optimal decision rule \( r^* \). Nevertheless, whatever view one advocates, assuming that the agent follows \( r^* \) is tantamount to the agent knowing in details the distribution \( \omega \). In the estimation problem above for example, where preferences are assumed to be quadratic, it requires the agent to behave as if he knew the conditional expectations \( E_{\omega}[s \mid z] \).

In practice, it is generally assumed that the agent knows \( \omega \). He can then form a (correct) belief \( \beta_z \in \Delta(S) \) about the true state for each signal \( z \) that he may receive:

\[
\beta_z = \omega(\cdot \mid z),
\]

and next use that belief to derive the optimal rule, that is:

\[
\begin{align*}
    r^*(z) &= a^*[\beta_z] \text{ where for any } \beta \in \Delta(S), \\
    a^*[\beta] &= \arg \max_a E_{\beta}u(a, s).
\end{align*}
\]

The distinction between signals and belief is thus immaterial, and one generally identifies signals and beliefs:

\[
z \equiv \beta_z.
\]

2.3 Issues

At least descriptively, the standard model outlined above looks implausible. We had aimed for a decision model where the agent lacks detailed knowledge of the underlying state \( s \). We end up with a model where the agent knows perfectly (or behaves as if he knew perfectly) how the state and his signal about the state are jointly distributed.

Plausibility is not the main issue we wish to raise. A model may be unrealistic and yet deliver useful insights. We wish to point out that sometimes, insights may be driven, not by the uncertainty about the underlying state, but by the certainty the agent implicitly has about the distribution \( \omega \). This concern is what motivates us to propose an informationally less demanding model (see next subsection). We provide two illustrations below.

Regression to the mean

In the estimation problem above (Illustration 1), optimal behavior calls for regressing to the mean. From an agent’s perspective, there is probably a rough sense in which if an estimate looks extreme, then one should be cautious in taking it at face value. Choosing a more conservative figure that would look closer to "normal" might then seem appropriate. In many applications however,
there doesn’t exist a "normal" estimate. If one has to estimate a quantity of oil in an oil field, there is nothing to make most estimates seem abnormal or extreme. If I am asked to make a point estimate for a number of coins in a jar, there is nothing that would make me think the estimate is extreme. That estimate just happens to be mine. Of course, if confronted with other people’s estimates and seeing that mine lies far above all others, some reconsideration of my own estimate will probably seem advisable.

In other words, regressing to the mean may make sense when an appropriate reference point is available, but not otherwise. And even if one is available, the agent’s perception is likely to fall into one of few categories: "more or less normal", "somewhat high", "somewhat low", "extremely high" or "extremely low", without his being able to say more about precisely how extreme an estimate seems to be.

Comparative statics

Consider a population of identical agents, each facing the same estimation problem as that described in Example 1. It may be useful to think of agents all facing the same jar filled with coins, and being asked the number of coins in the jar. We assume that by varying the length of time an agent can spend studying the jar, the precision of their estimate varies. We model this by assuming that the noise term \( \varepsilon \) has higher variance when the time spent is smaller.

The prediction of this model is that as the variance of the noise term increases, the agents’ reports should put more weight on \( s_0 \), lowering the variance of the estimates. For any two agents with estimates \( z_1 \) and \( z_2 \) respectively, and letting \( r_i = r^*(z_i) \), we have:

\[
E[r_1 - r_2]^2 = (1 - \rho)^2 E[z_1 - z_2]^2.
\]

This implies that the variance of the reports vanishes as the estimates become noisier:\(^5\)

\[
\lim_{\sigma_\varepsilon \to 0} E[r_1 - r_2]^2 \leq 0
\]

In other words, as agents have less time to make their estimates, their opinions converge to one another (in the sense of vanishing variance). This is of course an (unfortunate) artifact of the model and of the implicit informational assumption that they all know \( \omega \) (or behave as if they knew \( \omega \)).

2.4 An alternative approach

The standard approach embodies strong informational assumptions because there are implicitly no limits on the agent’s ability to compare alternative rules. We propose a theory in which weaker informational assumptions are made by limiting the agent’s ability to evaluate or compare alternatives. In that theory, behavior is driven by correct welfare considerations, but only to a limited extent.

\(^5\) This is because \( 2(\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2})^2 \sigma_\eta^2 \) tends to 0 as \( \sigma_\varepsilon \) increases.
Formally, the primitives of our model consist of (i) $D$, the range of situations over which the model applies; (ii) $\omega \in \Delta(D)$, the distribution over situations in $D$ that the agent faces, and (iii) $R$, a set of rules that apply to situations in $D$, where each rule maps the agent’s data/signal to a decision. We let $r^*_\omega = \arg \max_{r \in R} v_\omega(r)$. Our main assumption is:

**A1**: The set of considered rules $R$ consists of a limited set of rules, i.e. $R \subseteq \overline{R}$. Although the agent does not know $\omega$, he identifies (or learns) which rule $r^*_\omega$ is optimal in $R$, and he follows it.

The standard approach corresponds to the case where no restrictions are imposed on $R$, i.e., $R = \overline{R}$. A1 thus corresponds to a weakening of the standard assumption.

A1 requires that the agent identifies which rule is optimal despite not knowing $\omega$. We are agnostic about how the agent does it, though learning is a natural candidate. We emphasize that how the agent has come to know which rule is optimal is outside our model.

A1 limits the agent’s ability to compare rules. However, A1 assumes a perfect ability to discriminate across rules in $R$. This assumption could be weakened, assuming that agents can only imperfectly evaluate which rule is best. The qualitative features of behavior that we care about are robust to the agent not knowing exactly which of the two rules is optimal.

Being able to identify which rule is optimal in $R$ is precisely what we interpret as an implicit informational assumption. This assumption implicitly assumes that if a rule is added to the set the agent compares, the agent knows more.

### 2.5 Discussion.

**Information.** From an (omniscient) outsider’s perspective, A1 is tantamount to the agent behaving as if he had information on the distribution $\omega$. With no restrictions on $R$, the agent behaves as if he knew $\omega$ and then followed expected utility maximization. With restrictions, the assumption is weakened. For example, if $R$ consists of two rules, say

$$R = \{r_0, r_1\}$$

then one can define a partition of $\Delta(D)$ into two subsets $\mathcal{D}_{r_0}$ and $\mathcal{D}_{r_1}$ depending on which rule is better. When, say $r^*_\omega = r_0$, the agent understands that the optimal rule is $r_0$ and he follows it. So he behaves as if he knew that $\omega \in \mathcal{D}_{r_0}$. We do not have in mind however that the agent can formulate with precision

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6 Denoting by $r^*$ the rule that the agent perceives as best, and denoting by $\mu_\omega(r)$ the probability that $r$ is perceived as best under $\omega$, we could assume:

**A2**: $\mu_\omega(r) = \phi_r(\{v_\omega(r)\}) \in R$ where for any vector $v = \{v^m\} \in R$, $\phi(v) = \{\phi_r(v)\} \in R \in \Delta(R)$. For example, the function $\phi$ could be a logit function ($\phi_r(v) = \exp \beta v^r / \sum \exp \beta v^m$).

7 $\mathcal{D}_{r_0} = \{\omega \mid v_\omega(r_0) = \max_{r \in R} v_\omega(r)\}$

8 and without being able to exploit this information any further. We shall come back to that issue in Section 7.
that $\omega$ lies in subset $D_{r_0}$, only the omniscient outsider can. Our perspective is that although there is an objective process $\omega$, the agent might not be able to even conceive of what states are, nor understand what probabilities could mean, and yet be able to take decisions, trying to make out the best of his data. If he could compare all possible ways to use data, he would behave as if he knew $\omega$. Under A1, we examine circumstances where he is limited in his ability to compare alternative ways to use data.

Rules. We view rules as plausible processes by which an individual uses the data that he gets. Rules are mappings from data $z$ to decisions, and it is conceivable that an agent would form belief $\beta(z)$ based on his data $z$, and then apply subjective expected utility maximization. However, we see no reason to restrict attention to, or to necessarily include such rules as plausible rules. In the applications we have in mind, rules reflect some aspect of behavior (caution or various degrees of caution for example), possibly contingent on some crude belief state or mental state, or crude categories, that can be expressed in common language. Estimates are reliable (or not), an option is complex or simple to evaluate, a theory is likely (or not likely, or somewhat likely) to be true, I may be upset or pleased with the relationship. We emphasize the use of common language, because eventually the description of our insights relies on common language.

Set of rules. The set of rules to be considered is the analyst’s choice, in the same way that $\omega$ is in the standard approach. We do not suggest that there is a unique set of rules that the analyst should consider. Different sets (say $R$ or $R'$) could in principle lead to different insights about behavior. The set of rules the analyst chooses will be governed by the questions of interest in a given problem.\footnote{This is analogous to practice in the standard approach. When analyzing auctions, one could argue that for any auction of interest, there are both private value and common value components. Nevertheless, it is common to focus attention on one or the other components, with different insights obtained for the two.}

Restrictions could capture limits on what the agent can learn, but this is not our only motivation. The set of rules can be viewed as an instrument available to the analyst to vary the extent to which the agent’s optimal behavior takes into account the details of the underlying distribution $\omega$. This modeling strategy effectively limits agents’ ability to exploit in arbitrary or special ways some information ($\omega$) that agents would plausibly not use in the first place. It may thus be seen as a robustness test for insights derived with standard models.

Finally, by considering all possible rules, the standard approach attempts to endogenize all aspects of behavior at once. Our perspective is that many aspects of behavior are probably determined by considerations that lie outside the environment under scrutiny. By treating some aspects of behavior as exogenous, the analyst focuses on those aspects that he wishes to address.
We return to the estimation problem. We first define a more general class of preferences that allows for asymmetric costs of overshooting or undershooting the report. We next suggest plausible rules and apply our approach.

We consider preferences characterized by a utility function \( u(a, s, \beta) \), where \( \beta \geq 0 \) parameterizes the additional loss from overshooting the target. Specifically, we assume that \( u(a, s, \beta) \) depends only on the difference \( d = a - s \). We let

\[
\begin{align*}
    u(a, s, \beta) &= L_\beta(a - s) \\
    L_\beta(d) &= -d^2 - \beta \max(d, 0).
\end{align*}
\]

The quadratic case thus corresponds to the case where \( \beta = 0 \). Note that the state that fully describes preferences now consists of the pair \((s, \beta)\).

As before, \( z \) denotes the data or signal that the agent gets, \((z, s, \beta)\) refers to the situation that the agent faces, and \( \omega \in \Delta(S \times Z) \) captures the uncertainty that the agent faces.\(^{10}\)

**Plausible rules.** One rule that the agent could follow would be to take his estimate at face value:

\[ r_0(z) \equiv z. \]

We shall refer to it as the naive rule, as this would be the optimal rule if the agent was making no mistakes in his estimates. Other rules might have him exercise caution, and pick an action that distorts, upward or downward, his initial estimate. We define rule \( r_\gamma \) as:

\[ r_\gamma(z) \equiv z - \gamma. \]

We fix \( \gamma_0 \) and analyze below the case (case 1) where the agent only compares three rules:

\[ R_1 = \{r_0, r_{\gamma_0}, r_{-\gamma_0}\}. \]

We shall also discuss the case (case 2) where the agent compares many rules:

\[ R_2 = \{r_\gamma\}_{\gamma \in \mathbb{R}}. \]

**Analysis.** When the agent uses rule \( r_\gamma \), each realization \( \varepsilon \) induces a difference \( d = x - \gamma - s = \varepsilon - \gamma \). Denoting by \( f \) the distribution over errors, the expected performance associated with rule \( r_\gamma \) can be written as:

\[
v_{\omega, \beta}(r_\gamma) = E_\varepsilon L_\beta(\varepsilon - \gamma) = -[\sigma^2 + \gamma^2 + \beta \int_{\varepsilon > \gamma} (\varepsilon - \gamma)f(\varepsilon)d\varepsilon]
\]

which implies:

\[
v_{\omega, \beta}(r_\gamma) - v_{\omega, \beta}(r_0) = -\gamma^2 + \beta\left(\int_0^\gamma \varepsilon f(\varepsilon)d\varepsilon + \gamma \Pr(\varepsilon > \gamma)\right).\]

\(^{10}\)For simplicity we assume that the preference parameter \( \beta \) is fixed across situations.
**Proposition 1**: Consider case 1. If $\beta < 2\gamma_0$, then performance is maximum for the naive rule $r_0$. When $\beta > 2\gamma_0$, then the cautious rule $r_\gamma$ may become optimal if the noise has sufficiently high variance.

**Proof**: \(\int_0^\gamma \varepsilon f(\varepsilon) d\varepsilon \leq \gamma \Pr(\varepsilon \in (0, \gamma))\), so the difference $v_\omega(r_\gamma) - v_\omega(r_0)$ is below $-\gamma^2 + \gamma \beta / 2$, which implies the first statement. Now if the noise has large enough variance, $\Pr(\varepsilon > \gamma)$ is close to $1/2$, which implies the second statement. \(\square\)

The following figure plots the respective regions where $r_0$ or $r_\gamma$ is optimal when $\gamma_0$ is set equal to 1. Qualitatively, this figure illustrates the combinations of asymmetric loss functions ($\beta$) and noisiness of estimates ($\sigma_\varepsilon$) that call for cautious behavior.

![Graph showing regions where r0 or r_\gamma are optimal](image)

Adding more rules would allow the agent to more finely adjust the cautiousness level $\gamma$ to the particular distribution $\omega$ (characterized here by the pair $(\sigma_\varepsilon, \beta)$) that the agent faces. As the number of rules considered increases however, this fine tuning implicitly assumes more precise information about $\omega$. At the limit where all rules $r_\gamma$ are considered (case 2), the optimal rule is obtained by differentiating (1).

**Proposition 2**: Under case 2, optimal shading $\gamma^*$ solves:

\[
\gamma^* = \frac{\beta}{2} \Pr(\varepsilon > \gamma^*).
\]

Proposition 2 implies that optimal shading is stronger when $\varepsilon$ is more dispersed or when the cost parameter $\beta$ is larger.

**Discussion**.

*Tracking relevant aspects of $\omega$.* A central assumption of our approach is that the agent does not know precisely the specific distribution $\omega$ he is facing. $A1$ however is an assumption that the agent behaves as if he understood some aspects of $\omega$ that are relevant to his decision problem.
This is done in a crude way in case 1, but it nevertheless permits us to partition the parameter space \((\beta, \sigma, \varepsilon)\) and understand under which circumstances the agent exerts (or does not exert) caution. Case 2 gives a more complete picture, as the magnitude of the caution he exerts is fit to the dispersion of his errors (but this implicitly requires a fine knowledge of this dispersion).

**What’s relevant?** There is an (implicit) relationship between the set of rules considered and the aspects of \(\omega\) that are relevant. For example, the rules \(r_\gamma\) do not attempt to adjust the cautiousness level as a function of the estimate \(z\). As a consequence, the distribution over \(s\) is irrelevant; only the distribution over errors is relevant.

**Other classes of rules.** Depending on the data or signals that the agent receives, other classes of rules may be seen as plausible. For example, assume that in addition to \(z\), the agent receives a noisy estimate of \(s_0\), say \(y = s_0 + \varepsilon_0\). By observing \(z - y\), the agent may decide whether \(z\) is surprisingly high, and behave normally or cautiously depending on whether he perceives that \(z\) is normal or high. Formally

\[
r_{\gamma,h}(z,y) = \begin{cases} 
  z & \text{if } z < y + h \\
  z - \gamma & \text{if } x > y + h 
\end{cases}
\]

The rule \(r_{\gamma,h}\) thus includes the notion or perception that \(z\) is abnormally "high" or "normal". Compared to case 1, including \(r_{\gamma_0,h_0}\) in the set of plausible rules allows us to derive determine whether (i.e. for which \(\omega\)) exploiting the perception that \(z\) is "high" or "normal" is useful to the agent or not, and thus derive whether and when the qualifications "high" and "normal" are relevant (when applied to a particular parameter \(z\)).

### 4 Choice problems between two alternatives

A rule can then be interpreted as a *heuristic* that applies to a particular class of situations that the agent faces. With this interpretation in mind, our approach assumes that an agent has various heuristics available, and that he is able to determine which heuristic works best for him on average over the situations in \(D\) that he faces.

We apply this idea to a simple class of choice problems with two alternatives, one of which is easier to evaluate than the other. We also extend our analysis to the case in which each alternative has two consequences, one of which is easier to evaluate than the other. We endow agents with two heuristics, a naive heuristic, and a cautious one, and derive circumstances under which the cautious heuristic is better.

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11 Recall that \(s = s_0 + \eta\) and \(z = s + \varepsilon\).

12 This exploitation may turn out to be beneficial when \(z - y\) is sufficiently correlated with \(s - s_0\). To see why, consider the case where \(R_3 = \{r_0, r_{\gamma_0}, r_{\gamma_0,h_0}\}\), one can find which rule is best as a function of the parameters. When \(\sigma_{\varepsilon_0}\) or \(\sigma_\eta\) become large, \(r_{\gamma_0,h_0}\) corresponds to a random choice between \(r_0\) and \(r_{\gamma_0}\), and it is therefore dominated. If neither \(\sigma_\varepsilon\) nor \(\sigma_\eta\) is too high, then for intermediate values of \(\beta\) and high enough \(\sigma_\varepsilon\), rule \(r_{\gamma_0,h_0}\) becomes optimal.
This application will enable us (in Section 5) to draw a link between several much studied phenomena: ambiguity aversion, status quo bias, procrastination, temptation, and the winner’s curse. All these phenomena will be understood through a single lens: cautious behavior as a response to noisy evaluations.

4.1 A basic choice problem

We consider a class of choice problems with two alternatives, one of which, labelled \( a \), can be easily evaluated by the agent, while the other, labelled \( b \), is more difficult to evaluate. To each alternative \( k \in \{a, b\} \) we associate a state \( s_k \) with the understanding that taking alternative \( k \) yields utility \( u(s_k) \) to the agent.\(^{13}\) If the agent knew the state \( s = (s_a, s_b) \), the optimal decision would consist of choosing the alternative \( k \in \{a, b\} \) for which \( s_k \) is largest.

The agent forms a possibly noisy estimate of \( s_a \) and \( s_b \), denoted \( x_a \) and \( x_b \) respectively.\(^{14}\) We assume first that \( x_a = s_a \) and \( x_b = s_b + \varepsilon \).

That is, the agent learns \( s_a \) perfectly but gets a noisy signal of \( s_b \). We let \( z = (x_a, x_b) \) and denote by \( \omega \) the distribution over situations \( (z, s) \) that the agent faces.

**Plausible rules.**

One naive rule would be that the agent takes his estimates at face value, and chooses the alternative \( k \) for which \( x_k \) is largest. For any \( z = (x_a, x_b) \),

\[
r_0(z) \equiv \arg \max_k x_k.
\]

A more cautious rule would be to first distort his estimate \( x_b \) by \( \gamma \):

\[
r_\gamma(x) \equiv a \text{ if } x_a > x_b - \gamma \\
\equiv b \text{ otherwise}.
\]

The performance of rule \( r_\gamma \) is given by the expected utility he obtains under \( \omega \) when he follows \( r_\gamma \). We denote it \( v(r_\gamma) \):

\[
v(r_\gamma) = E_\omega u(r_\gamma(x)).
\]

We examine the case where the set of plausible rules is

\[
R_1 = \{r_0, r_{\gamma_0}\}
\]

for some \( \gamma_0 > 0 \), assumed to be small.

**Analysis.**

\(^{13}\)If \( k \) is lottery, \( s_k \) would thus correspond to the certainty equivalent associated with \( k \).

\(^{14}\)An alternative assumption would be to assume that he forms an estimate of the difference \( d = s_a - s_b \), as it may sometimes be easier to evaluate differences than each alternative separately.
Intuitively, when \( \sigma \) is a better alternative. For large enough noise, caution is called for too. A shall refer to it as alternative. But it becomes a problem otherwise, and caution becomes desirable events where because it permits to overcome the selection bias.

We have:

\[
\Delta = E[e^{\lambda \theta} - 1 | -\theta + \varepsilon \in (0, \gamma_0)]
\]

We have:

**Proposition 3:** Let \( \overline{x} = \lambda + \frac{2 \theta_0}{\sigma^2} \). If \( \overline{x} > 0 \), then for \( \sigma_\varepsilon^2 \) large enough (i.e. \( \sigma_\varepsilon^2 \geq 2\gamma_0/\overline{x} \)), performance is maximum for the cautious rule \( r_{\gamma_0} \). And for \( \sigma_\varepsilon^2 \) small enough, performance is maximum for the naive rule.

There are two cases worth emphasizing.

**case 1:** \( \theta_0 = 0 \) and \( \lambda > 0 \). On average alternatives \( a \) and \( b \) are equally attractive. Because the agent is risk averse, high enough noise in evaluations of alternative \( b \) leads to caution.

**case 2:** \( \theta_0 > 0 \) and \( \lambda = 0 \). The agent is risk neutral, but on average \( a \) is a better alternative. For large enough noise, caution is called for too. Intuitively, when \( \sigma_\varepsilon \) becomes extremely large, both rules generate a random choice (essentially driven by the error term). The cautious choice \( a \) being better on average, \( r_{\gamma_0} \) is a better rule.

Proposition 3 illustrates two different motives for caution. One motive is a combination of risk aversion and noisy evaluations. The other motive is that which ever rule one uses, the agent is subject to a selection bias: \( b \) is favored in events where \( \varepsilon \) is positive. This is not a problem when on average \( b \) is a better alternative. But it becomes a problem otherwise, and caution becomes desirable because it permits to overcome the selection bias.

**Proof:** Define \( \Delta(x) = E[e^{\lambda \theta} - 1 | -\theta + \varepsilon = x] \) and let \( \rho = \frac{\sigma_\varepsilon^2}{\sigma^2 + \sigma_\varepsilon^2} \). Conditional on \( -\theta + \varepsilon = x \), \( \theta \) is normally distributed, with mean \( \mu(x) = (1 - \rho)\theta_0 - \rho x \) and variance \( (1 - \rho)\sigma^2 \). This implies \( \Delta(x) = \exp(\lambda \mu(x) + \frac{\lambda^2}{2}(1 - \rho)\sigma^2) - 1 \). Denote by \( x^* \) the solution to \( \Delta(x) = 0 \). We have \( x^* = \frac{\tau_0^2}{2} \), and for any \( x \in (0, x^*) \), \( \Delta(x) > 0 \). So if \( x^* > \lambda_0 \), that is if \( \sigma_\varepsilon^2 \geq 2\gamma_0/\overline{x} \), then \( \Delta \geq \Delta(\lambda_0) > 0 \).

\(^{15}\)Indeed, \( x^* = \frac{1 - \rho(\theta_0 + \lambda \sigma^2/2)}{\rho^2} = \frac{\sigma_\varepsilon^2}{\sigma^2}(\lambda \sigma^2 + 2\theta_0) = \frac{\tau_0^2}{2} \)
4.2 Cautious behavior.

Proposition 3 illustrates that when an agent is facing a choice between two alternatives, he may prefer to be cautious and opt for a rule that favors the easier to evaluate alternative. Case 1 and 2 above provide two different reasons for being cautious.

Caution arises because, for the class of situations considered, the agent correctly perceives that one alternative is more difficult to evaluate than the other alternative, and then labels alternatives accordingly (as $b$ and $a$ respectively). If he did not and if, say the alternatives had equal chances of being labelled $a$ and $b$, or $b$ and $a$, then the naive rule would be best.\footnote{More generally, if the perception of the relative difficulty in evaluating $b$ is uncorrelated with the mistakes he makes in evaluating $a$ and $b$, then the naive rule is best.}

This illustrates two channels by which cognitive abilities matter: through the accuracy of evaluations (hence the correlation between $z$ and $s$), and through the perceived asymmetry in the alternatives considered.

From the perspective of experimental economics, which generally attempts to identify biases in decision making or cognitive limitations, the comments above suggest that observed behavior may reflect an agent’s optimal response to his (somewhat accurate) perception of his own cognitive limitations: that he finds an alternative more difficult to evaluate may be a useful signal of his own evaluation errors, and he may find it optimal to use that clue to shy away from that alternative.

We review below various phenomena that may be interpreted or explained through this lens.

Ambiguity aversion. Caution builds on a perceived asymmetry between the alternatives considered, and on the particular way situations are classified. Instances where this classification seems likely to occur are when one of the alternatives involves more abstract reasoning or more complex computations, or when one of the alternatives is described in vaguer terms. When this classification indeed occurs, agents may tend to be cautious and shy away from complex or vague alternatives. Staying away from vague or complex alternatives looks like a preference for precise lotteries (ambiguity aversion), but from our perspective, this behavior may just stem from standard risk aversion combined with the fact that in general vague outcomes are more prone to estimation errors or more difficult to evaluate than precise lotteries.\footnote{The latter view is also consistent with the experimental results reported in Fox and Tversky (1995). They consider the Ellsberg set up: Alternative 1 is the precise alternative where the agent gets $100 if he draws the color of his choice from an urn composed of 50 red and 50 black balls; Alternative 2 is the vague alternative where the agent gets 100$ if he draws the color of his choice from an urn composed of an unknown number of red and black balls. Fox and Tversky elicit the individuals’ willingness to pay for each alternative. They find that these willingnesses to pay differ substantially across alternatives only if the individuals are confronted with both alternatives simultaneously. Our interpretation is that it is only when confronted with both alternatives simultaneously that the agent will perceive alternative 2 as more vague than alternative one, hence that more caution is called for.} \footnote{This is also consistent with recent experiments reporting that “ambiguity aversion” is correlated with the complexity of the lottery that the agent faces (Halevy (2007)).}
Status quo bias, confirmatory bias, ex post rationalization. Another instance where one alternative would seem easier to evaluate is when one alternative is the status quo. Agents may thus be cautious towards the novel alternative. Caution gives rise to behavior that looks like a status quo bias or a confirmatory bias. But the behavior is actually an agent’s optimal response given the categorization of choice problems that he makes. Similarly, in repeated choice problems between two types of decisions, say a and b, we tend to have more precise information about the consequences of the decisions that we take than over those that we do not take. Once decisions of a particular type (say type a) have been taken, cautiousness consists of favoring the type of decisions we have become familiar with. We often refer to ex post rationalization as a bias. But from our perspective, it may just be an appropriate response to noisy evaluations of decisions that we are not familiar to.

Winner’s curse. We can apply the basic model to a choice between "not buying" (option a) and "buying" (option b), with the idea that the agent only gets a noisy estimate of the gain from buying. Case 2 shows that if on average "always buying" is a bad option, then noisy estimates call for cautious behavior, that is, using the rule $r_\gamma$ that favors the "not buying" alternative. This cautious behavior looks like a response to the winner’s curse, but it is just an appropriate response to noisy estimation of valuation when on average "always buying" is a bad option. With this interpretation, winner’s curse is a selection bias, and it is stronger when competing with many others (because the option "buying always" worsens).

Procrastination and temptation

Finally, elaborating on the simple choice problem above, cautiousness could also explain behavior that looks like procrastination or temptation.19 Cautiousness calls for shading the estimates associated with consequences that are more difficult to evaluate (for risk averse individuals). As in general, future consequences are more difficult to evaluate, cautiousness calls for shading estimates of future prospects. This cautious rule may thus generate behavior that looks like temptation (and procrastination) but it is just an appropriate response to noisy evaluations of future consequences.

4.3 Heuristics and biases

One interpretation of optimal rules in our model is that they are good heuristics: they work well on average across the problems that the individual faces. In this sense, avoiding vague or complex alternatives, or favoring alternatives that we understand better (i.e., the status quo), or shading estimates of future prospects are good heuristics, all derived from the same motive: caution.

19We have in mind an extension of our basic model to deal with situations in which each alternative generates two types of consequences, one easy to evaluate, the other more difficult, having in mind for example that consequences in the future are more difficult to evaluate than consequences in the present. Formally, each alternative $k$ is characterized by $(s_0^k, s_1^k)$ and noisy estimates $(z_0^k, z_1^k)$. A cautious rule that deflates the weight put on future consequences $(z_1^k)$ may outperform a naive that takes estimates at face value.
From an experimenter’s perspective, some choices may look suboptimal or subject to biases. However, these choices may simply stem from the agent’s using heuristics based on broad clues such as complexity or vagueness. From the agent’s perspective, using such heuristics may be valuable, because the clues are correlated with the estimation errors that the agent makes in evaluating alternatives. Naturally, if he was certain that he wasn’t making estimation errors for the problem he currently faces, then caution would not be called for. The difficulty, however, lies not in his using a particular heuristic, but in his inability to condition more finely on the type of problem faced.

This view of heuristics as rules that are good on average across problems is consistent with Tversky and Kahneman (1974)’s view. Our model can be interpreted as one where heuristics compete.

5 Choice theory and welfare

Choices reveal not only a person’s relative preferences over outcomes but also his perception of the uncertainty he faces. Choice theory aims at separating perceptions of uncertainty from preferences. It postulates axioms where that distinction can be made unambiguously. The theory makes no claim that agents would necessarily obey the axioms. However, it generally argues that a rational agent would want to satisfy the axioms, presumably out of a concern for coherence across decisions. The theory then provides a way to achieve such coherence: If a decision maker wanted to satisfy the axioms, then one way to do that would be to attach (personal) probabilities to events and maximize (subjective) expected utility.

Now would a decision maker fare well if indeed his preferences satisfied the axioms? Would he fare well if indeed he attached probabilities to events and maximize expected utility? What if he has poor ability to form beliefs, or if his beliefs are mistaken?

The questions may be puzzling, and to some, reflect a misunderstanding of Savage: choice theory takes preferences as input, and preferences are supposed to reveal what is best for the agent; so there cannot be any welfare issues associated with asking that preferences satisfy axioms. Also, beliefs cannot be mistaken, because they just reflect or represent preferences.

Our perspective is different. Our view is that choices indeed reveal both a person’s preference over outcomes and his perception of the uncertainty faced, but perceptions might not be reliable. In other words, stated preferences over acts reveal comparisons based on perceptions of these acts (and not on the acts as viewed by an outsider). We see no ground for a normative view based on perceptions of acts.

Tversky and Kahneman describes heuristics that people employ in assessing probabilities or predicting values (for example, that of judging frequency by availability; that is, the ease with which similar occurrences are brought to one’s mind). They argue that these heuristics are useful, but that they lead to estimation errors for some types of problems.
Correlatively, we take subjective expected utility maximization as one special rule that one could use, the performance of which ought to be assessed and compared to other plausible rules.

Formally, subjective expected utility maximization takes as input a belief $\beta \in \Delta(S)$, and possibly the agent’s perception of his own preferences $\tilde{u}$, and computes, for each perception $z = (\beta, \tilde{u})$, the "optimal" decision:

$$r^{SEU}(z) = \arg \max_a E_{\beta} \tilde{u}(a, s).$$

Our view is that the perception $z$ is a signal, and $r^{SEU}$ is just one particular rule, one that takes beliefs and perceived preferences at face value and mechanically maximizes subjective expected utility. Evaluating that rule requires defining the underlying (objective) process that generates $s$ and $z$, that is, $\omega \in \Delta(S \times Z)$. One can evaluate the actual performance of that rule by computing:

$$v_\omega(r^{SEU}) = E_\omega u(r^{SEU}(z), s).$$

There is of course a special case where $r^{SEU}$ is a good rule: when perceptions are correct, that is, when $\tilde{u} = u$ and when $\beta$ coincides with the true posterior $\omega(\cdot | z)$. As soon as perceptions are not correct however, other rules may perform better: Rules that treat beliefs with caution, or rules that rather than using finely described probability distributions over states use cruder perceptions (possibly because these cruder perceptions may be less noisy). Thus if our aim is in understanding the behavior of agents who attempt to use rules that perform well, there is no reason to restrict attention to agents who form probabilistic beliefs; even if we restrict attention to such agents, there is no reason to expect them to take these beliefs at face value.\(^{21}\)

### 6 Related literature

Most decision and game theory applications assume that agents know (or behave as if they knew) the joint distribution $\omega$ over states and perceptions/signals. We review different strands of literature that attempt to depart from that assumption.

#### 6.1 Wilson’s critique and the robustness literature

We often refer to Wilson (1987) to acknowledge our unease with common knowledge assumptions in game theoretic interactions, in particular when they involve "one agent’s probability assessment about another’s preferences or information".

\(^{21}\)The same critique applies to the various weakenings of the Savage axioms that have been proposed, including those that lead to representations of uncertainty in terms of probability sets. Choices are then represented as functions of these probability sets. Once viewed as advice or a decision making process that agents might consider following, these belief sets ought to be viewed as signals, and the agent should aim to use rules that best utilize these signals.
That uneasiness should not be restricted to games; it ought to arise even in decision problems, when one assumes that an agent knows (or acts as if he knew) the joint distribution over states and signals.

The uneasiness stems from two reasons: (i) lack of realism of the assumption, and more importantly, (ii) lack of realism of the consequences of the assumption, with behavior adjusting very finely to details of the environment. The drastic simplification that we propose – looking at few alternative rules – partitions the set of environments \( \omega \) into few subsets, and small changes in the environment \( \omega \) "rarely" induce changes in the rule used (and consequently, changes in behavior).

Following Wilson’s critique, the literature on robust mechanism design and on robust equilibrium analysis (in particular the work by Bergemann and Morris (2012)) attempts to weaken the common knowledge assumption that we usually make when analyzing games of incomplete information. The route follows Harsanyi’s suggestion, and it consists of working with a richer type space. Specifically, in the face of possibly imperfect knowledge of the joint distribution \( \omega \) over states and signals, the traditional route consists of adding one extra layer of uncertainty: one perturbs the model by assuming some randomness over the actual distribution \( \omega \) and by adding signals correlated with the realized \( \omega \). One thus obtains an enriched information structure, characterized by a new joint distribution, say \( \tilde{\omega} \), over states and signals, and it is this enriched or perturbed game that is analyzed, under the presumption that this more sophisticated signal structure would be commonly known, with the hope that behavior would not be too sensitive to these perturbations.22

As an internal test to the standard approach (which builds on a strong common knowledge assumption), the robustness test above is clearly a legitimate one, as we would not want equilibrium predictions to depend finely on knowledge that seems out of reach (not only to an outside observer, but even to players themselves).23 The route we propose is not an internal test to the standard approach, but an alternative way to deal with player’s lack of detailed knowledge about the distribution \( \omega \) they happen to face.

### 6.2 Bounded rationality models

Bounded rationality models generally maintain the assumption that agents choose the alternative that maximizes expected utility, but they distort the way alternatives are evaluated. We have reviewed various ways by which this has been done. We discuss here the simplest model, that is, Thurstone (1927) random stimuli model, which directly adds errors to valuations of alternatives.

The model can be described as follows. Think of an individual trying to

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22 Note that strictly speaking, enriching the type space does not weaken the common knowledge assumption. It replaces it with another that might be viewed as even less plausible.

23 Bergemann and Morris (2011, page 4) argue: "the absence of the observability of the belief environment constitutes a separate reason to be skeptical towards an analysis which relies on very specific and detailed assumptions about the belief environment".

We share that view. However we view that it applies not only to outsiders observing the interaction, but also to players themselves.
assess which of two colors $k=1,2$ is brighter. Call $R_k$ the (true) brightness level and call $S_k$ the random stimulus received by the agent, assuming that

$$S_k = R_k + \sigma_k \varepsilon_k.$$  

In assessing whether $R_1 > R_2$, the individual is assumed to compare the stimuli and report that $R_1 > R_2$ when $S_1 > S_2$. The noisy perception may thus lead to mistakes in evaluating whether $R_1$ lies above or below $R_2$.

Going back to our choice problem between two alternatives $k=1,2$, and interpreting $s_k$ as some physical property of alternative $k$ that the agent cares about, our model may be viewed as a random stimuli model in which the individual has a noisy perception of $s_k$:

$$z_k = s_k + \sigma_k \varepsilon_k.$$  

The true welfare (or utility) associated with alternative $k$ is $u_k = u(s_k)$, but the agent observes a noisy signal of that utility, say $\hat{u}_k = u(z_k)$. One rule consists of maximizing perceived utility (thus taking at face value the signals $\hat{u}_k$).

However, as with maximization of subjective utility, there is no reason that maximizing perceived utility is a good rule for welfare. Using our terminology, maximization of perceived utility corresponds to the naive rule $r_0$, but other rules such as the cautious rule $r_\gamma$ might improve welfare.

Intuitively, returning to the color example, there are two types of mistake one can make: saying that $R_1 > R_2$ when $R_2 > R_1$, and saying that $R_2 > R_1$ when $R_1 > R_2$. If only one type of mistake is costly, then clearly a cautious rule will perform better. Another example, in line with the winner’s curse type of example, is as follows: if option 1 is better on average, and if perceptions regarding option 2 are noisier, then one ought to be cautious when $z_2 > z_1$.

Note that all the random utility models following Block and Marschak (1960) and McFadden (1973), or the applications of that model to games (i.e., the quantal response models following McKelvey and Palfrey (1995)) have followed the same path, namely, assuming that agents maximize $\hat{u}_k$. For the econometrician who interprets $\hat{u}_k$ as the true utility to the agent, the assumption that the agent maximizes $\hat{u}_k$ is clearly legitimate. But if one truly interprets $\hat{u}_k$ as a perceived utility, then the joint distribution over true and perceived utility matters, and the rule that maximizes perceived utility has no reason to be welfare maximizing, unless specific assumptions are made relative to that joint distribution.

### 6.3 Robust decision theory

As mentioned in introduction, there has been objections to the assumption that agents would form precise probabilistic beliefs and then maximize expected

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24 In both Block and Marschak (1960) and McKelvey and Palfrey (1995) for example, imperfections in perceptions motivate stochastic choices.
utility as if they were perfectly confident of their beliefs. We discuss here the robust decision model of Hansen and Sargent (2001), who model situations in which the agent does not have full confidence in his belief. Formulated within our framework, the agent receives data in the form of a belief \( \beta \in \Delta(S) \), that is \( z \equiv \beta \in Z \equiv \Delta(S) \). For an agent who would take his belief at face value, an optimal rule consists of maximizing (subjective) expected utility. For an agent who does not entirely trust his belief, a plausible rule is the one suggested by Hansen and Sargent:

\[
 r^\theta(z) \equiv \arg \max_a \min_{z'} E_{\omega} u(a, s) + \theta d(z', z),
\]

where \( \theta \) is some positive parameter and \( d(\cdot, \cdot) \) is a distance between beliefs. For any \( \theta < \infty \), rule \( r^\theta \) can be interpreted as a caution rule, with \( 1/\theta \) measuring the degree of caution. At the limit where \( \theta \) gets arbitrarily large, \( r^\theta \) coincides with \( r^{SEU} \).

The rules \( r^\theta \) are plausible, yet, as for subjective expected utility maximization, in the absence of an assumption that link signals and welfare, that is, an assumption regarding the (objective) distribution \( \omega \in \Delta(S \times Z) \) over states and data, one cannot evaluate the performance of these rules, and compare them to other rules. Once this is done, one may indeed define:

\[
 v_\omega(r^\theta) = E_{\omega} u(r^\theta(z), s),
\]

enabling us to make comparisons between rules.

When beliefs are noisy,\(^{25}\) then, among rules \( r^\theta \), one cautious rule \( r^{\theta_0} \) may turn out to be optimal. We emphasize ‘among rules \( r^\theta \)’ because in the absence of a restriction on the set of rules considered, the optimal rule would be:

\[
 r^*(z) \equiv r^{\infty}(\omega(\cdot | z))
\]

In other words, an agent may find attractive to follow \( r^{\theta_0} \) rather than \( r^{SEU} \). But this is the case only because beliefs are noisy and because there is an implicit restriction on the set of rules that the agent compares. As in our basic model, cautiousness can be seen as an appropriate response to noisy perceptions when the set of rules considered is constrained.

7 Further comments and developments

Limited information. The agent does not know with precision the process \( \omega \) that generates data, but being able to identify that \( "r_{k_0} \) is best in \( R^n \) is equivalent to him being partially informed of \( \omega \). This notion of partial information is quite different from the standard one: to model imperfect information on \( \omega \), a standard model would presume that the process \( \omega \) is drawn from a distribution

\(^{25}\)That is, when \( z \) does not always coincide with \( \omega(\cdot | z) \). If \( z = \omega(\cdot | z) \) beliefs are correct. This is for example the case considered in Barillas and Sargent (2009). Note however that when beliefs are correct, and using a caution rule \( r^\theta \) with \( \theta < \infty \) is clearly not called for.
over processes (call \( \phi \) that distribution), and "\( r_{k_0} \) is best in \( R \)" would then be viewed as a signal about the actual process \( \omega \). Based on that signal and the presumption that the agent knows \( \phi \), the agent would then derive the optimal decision rule \( r \).  

This does not mean that the information "\( r_{k_0} \) is best in \( R \)" cannot be viewed as a signal under our approach. It can, but then it should be included in the description of the data that the agent obtains. Data would then consist of a pair \( z^e \equiv (z, k_0) \), where \( k_0 \) stands for the signal "\( r_{k_0} \) is best in \( R \)", and a rule would then map \( z^e \) to decisions. The modeler’s task would then be to define the subset of rules \( R_e \) that the agent can plausibly compare, and determine what additional insights one would derive from that more elaborate data generating process.  

**Limited skill or limited awareness.** Our agent is aware that his estimate \( z \) is not correct, and a rule defines a particular way to exploit the estimate. Because he only considers a restricted set of rules, he has a limited ability to exploit that estimate (compared to an omniscient agent who would know the process \( \omega \) that generates \( z \)). This limitation may be interpreted as a limit on skill: an agent who uses a more limited set of rules \( R' \subset R \) has fewer abilities to exploit \( z \) than one using \( R \). This limitation may also be interpreted as stemming from limited awareness: an agent may not consider rules because he is not aware that they might be useful.

To illustrate with a famous example, consider a situation in which more or less able detectives must decide, based on a series of indices, whether they ought to continue investigations thinking that (a) intrusion on the premise has occurred, (b) no intrusion has occurred, or (c) it is uncertain whether (1) or (2) is true or not. The underlying state \( s \in \{I,N\} \) indicates whether intrusion has occurred or not, the data \( z = (z^b, \ldots) \) consists of a sequence of indices, among which \( z^b \in \{0,1\} \) indicates whether dog barking has been reported or not. A rule maps a sequence of indices to a decision \( d \in \{a, b, c\} \). Detectives do not know the joint distribution \( \omega \) over state and data, but they are able to compare a limited set rules. Differences in abilities (or awareness) across detectives can be modelled as differences in the set of rules that detectives consider. A detective who considers all possible mappings from data to decisions will figure that reports of dog barking indicates intrusion, and he will also possibly deduce – depending on the probability that barking is reported when it occurs, that no report indicates no intrusion. A detective who considers fewer rules may miss the latter connection.

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26 Rational inattention models (see Sims (2003)) adopt that perspective. These models constitute an attempt to model agents who are imperfectly informed about the data generating process, but they keep the standard assumption that the actual data generating process is drawn from a distribution that the agent knows.

27 In comparison, the standard approach presumes that the agent can find the optimal rule across all possible rules \( r^e \). In the standard model, modelling partial or limited information on \( \omega \) thus requires that the agent distinguishes between rules belonging to a set larger than if he knew \( \omega \) with precision.

28 This is to the extent that there are no fake reports or that one does not confuse real dog barking with dreams of dog barking.
Further developments.

References.


Bergemann, D and S. Morris (2011), Robust Predictions in Games with Incomplete Information, Discussion paper n°1821, Cowles Foundation


