Voluntary Contributions to a Joint Project: Revisiting Admati and Perry 's Contribution Game[¤]

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Abstract

We revisit Admati and Perry (1991) 's model of voluntary contributions to a joint project. Their main result that equilibrium contributions are small appears not to be robust to the introduction of asymmetries and/or to the introduction of partial refunds in case the project is not completed.

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1 Introduction

This paper analyzes a problem of voluntary contributions to a joint project between several agents. A commonly observed pattern is that the completion of the project requires several steps in which agents make small partial contributions at a time. An important insight reported in Admati and Perry (1991) is that whenever contributions are sunk, the equilibrium pattern of contributions is one in which each agent makes small contributions at a time. In a subscription game in which contributions are not sunk but are conditional on the completion of the project, they observe that in equilibrium the project is completed in two (big) steps. They conclude that the sunk character of contributions may explain why the completion of a joint project is made step by step.

We consider two variants of Admati and Perry's contribution model: First, we consider a variant in which agents are asymmetric and do not value the project equally. Second, we consider a variant in which the contributions made by agents are partially refunded in case the project is not completed - formally, each agent gets back a given fraction of the total contribution he made if the project is not completed: this variant can be viewed as a combination of the subscription game and the contribution game considered in Admati and Perry.

In both variants, we show that whenever the project is socially desirable, the project is completed, and it is completed in at most three steps and at most two strictly positive contributions (the …rst contribution may sometimes be zero depending on the identity of the …rst mover). Thus equilibrium contributions are not small in either of these two variants. We conclude that the sunk character of contributions does not per se provide a satisfactory explanation of the commonly observed contribution pattern in which agents alternate in making small steps towards the completion of the transaction.

In the rest of the paper, we analyze the two extensions mentioned above: In Section 2 we consider the case of asymmetric agents; In Section 3 we consider the variant in which players get back a fraction of their contributions when the project is not completed. Finally in Section 4 we briefly consider the related literature.

2 The Asymmetric Case

There are two players, $i = 1, 2$. Each player i values the immediate completion of the project according to V_i. In contrast with Admati and Perry (1991), we do not require that $V_1 = V_2$ so that we allow for asymmetries between the players. To fix notation, we will assume that $V_1 \geq V_2$. The cost of completing the project is K. Whether and when the project is completed depends on the contributions made by the players over time, as discussed below.

Players take turns in making contributions, starting with player 1 in period 1. The project is completed as soon as the cumulative contribution made by both players reaches the total cost K. Throughout the paper, we assume that neither player can afford to complete the project alone, that is, $V_i < K$ for $i = 1; 2$.

Let c_i^t be the amount of player i's contribution in period t. A history at time τ is a sequence of contributions $(\{c_1^t\}_{t=1}^{\tau-1}; \{c_2^t\}_{t=1}^{\tau-1})$ made by the agents prior to time τ. If it is not player i's turn to move in period t then $c_i^t = 0$. A strategy for player i specifies the size of the player's contribution for each history after which it is player i's turn to move.

Both players are impatient, and discount both contributions and benefits using a discount factor $\delta < 1$.[1] One possible interpretation of δ is that at the end of each period, the game terminates with probability $1 - \delta$, in which event the contributions made up to that date are lost. Let T be the first time at which the cumulative contribution reaches K. If the project is not completed then $T = \infty$. An outcome of the game is defined by a triple $(T; \{c_1^t\}_{t=1}^T; \{c_2^t\}_{t=1}^T)$. For $i = 1; 2$, player i's payoff is given by

$$U_i(T; \{c_1^t\}_{t=1}^T; \{c_2^t\}_{t=1}^T) = \delta^{T-1} V_i - \sum_{1 \leq t \leq T} \delta^{t-1} c_i^t.$$

We look for subgame perfect equilibria of the game.

Remark: The above setup assumes that the cost of contributing c_i is exactly c_i. In their contribution game, Admati and Perry consider more general contribution costs $W(c_i)$ assumed to be convex (strict convexity is required in their setup to get the uniqueness of Subgame Perfect Nash Equilibrium outcome).[2] When the cost function is sufficiently convex, it is not surprising that the completion of the project requires several steps because even absent of strategic considerations, breaking the total contribution into several pieces is more efficient. The Admati-Perry result that equilibrium contributions are small is thus more striking when costs are not too convex because then some strategic consideration must be at work. In order to focus on the strategic motives, we have chosen to present our results in the linear cost case. However our main result that in the asymmetric case ($V_1 \neq V_2$)

[1] We could allow for asymmetric impatiences together with asymmetric valuations. This would lead to similar insights as the one in Proposition 1 (but it would make the notation heavier).

[2] For the linear case, Marx and Matthews 2000 point out the existence of equilibria not considered in Admati and Perry (1991).

the completion is made in two big consecutive steps would still hold even if more general cost functions were considered (see remark 2 after proposition 1).

To present our result, it will be convenient to let X_i^t denote the total contribution made by player i up to (and including) period $t - 1$, and by X^t the contribution still required at (the start of) period t to complete the project. We have:

$$X_i^t = \sum_{1 \le s \le t-1} c_i^s, \text{ and}$$
$$X^t = K - X_1^t - X_2^t.$$

Proposition 1 Assume that $V_1 > V_2$, and $\delta V_2 + V_1 - K > 0$. The contribution game has a unique subgame perfect equilibrium outcome. In equilibrium, the project is completed in three periods and two (strictly positive) contributions. On the equilibrium path, contributions are as follows: player 1 contributes nothing; player 2 contributes $K - V_1$; player 1 contributes V_1.

Remark 1: If $V_2 > V_1$, and $\delta V_1 + V_2 - K > 0$, on the equilibrium path the project is completed in two periods: player 1 contributes $K - V_2$ in period 1, and then player 2 contributes V_2 in period 2. (When $V_1 = V_2$, Marx and Matthews (2000, Proposition C2) observe that such a path can emerge as a Subgame Perfect Nash Equilibrium path.)

Remark 2: Suppose the cost of contributing c_i is $W(c_i) = w + \overline{w}(c_i)$ where $w \ge 0$ and $\overline{w}(\cdot)$ is a strictly convex function. Suppose that $V_1 > V_2$ and let $C_1 = W^{-1}(V_1)$. Suppose also that $\delta V_2 - W(K - C_1) > 0$. Then to the extent that the function $\overline{w}(\cdot)$ is not too convex,[3] the contribution

[3] It is sufficient that

$$\min_{c_0 + c_1' = c_1} \frac{1}{\delta} W(c_0) + W(c_1') > W(c_1) \text{ for all } c_1 \le C_1;$$

game has a unique Subgame Perfect Equilibrium path: player 1 contributes nothing in the first period, player 2 contributes $K - C_1$ in the second period and then player 1 contributes C_1.

Proposition 1 shows that when players are asymmetric, i.e. $V_1 \neq V_2$, and the project is socially desirable, i.e. $V_1 + V_2 - K > 0$, the project is completed in equilibrium for δ sufficiently close to 1. Besides, there are at most two strictly positive contributions in equilibrium, and the player who values the project most makes the last contribution; that contribution is equal to his valuation for the project so that his equilibrium payoff is nul. The intuition is that the player who values the project most is unable to commit not to contribute the rest whenever the amount left to be contributed is inferior to his valuation; since a player cannot in equilibrium contribute more than his valuation, the result of Proposition 1 follows.

This result shows that with asymmetries, there cannot exist an equilibrium where players contribute step by step (which contrasts with the result obtained by Admati and Perry in the symmetric case). The introduction of asymmetries shows that the sunk character of contributions does not per se induce the players to contribute step by step.

Proof of Proposition 1:

It will be convenient to denote by $(i; X)$ any subgame where it is player i's turn to move and there remains X to contribute to complete the project. We prove the following claim.

Claim A: In any subgame $(1; X)$, player 1 contributes the rest if $X \leq V_1$, and he contributes nothing if $V_1 < X$. In any subgame $(2; X)$, player 2

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which holds for any fixed δ if $\overline{w}(.)$ is not too convex. Also, when $w > 0$ and $\overline{w}(.)$ is not too convex (that is, when $w + \overline{w}(c_1) \leq n \left[w + \overline{w}(\frac{c_1}{n}) \right]$ for all n and $c_1 \leq C_1$), our claim holds for any δ, $0 < \delta < 1$:

contributes the rest if $X < (1-\delta)V_2$, he contributes nothing if $(1-\delta)V_2 < X \le V_1$, and he contributes $X - V_1$ if $V_1 < X \le K$.

We first show that claim A holds for all $X \le X_0$ if $X_0 < (1-\delta)V_1$. Then we will prove that it holds for all $X \le K$ by induction on the value of X_0.

Step 1: Choose X_0 such that $(1-\delta)V_2 < X_0 < (1-\delta)V_1$. Claim A holds for all $X \le X_0$.

If $X < (1-\delta)V_i$, then, if it is player i's turn to move, it is a strictly dominant strategy for that player to contribute the rest (because $V_i - X > \delta V_i$).

If $X \in ((1-\delta)V_2; (1-\delta)V_1)$, then, given that player 1 contributes the rest when it is his turn to move, it is strictly optimal for player 2 to contribute nothing. This is so because then he gets δV_2, while he only gets $V_2 - X < \delta V_2$ if he contributes the rest, or $\delta V_2 - c_2 < \delta V_2$ if he contributes $c_2 \in (0; X)$

Step 2: Choose any X_0 such that $(1-\delta)V_2 < X_0 < V_1$. Assume claim A holds for all $X \le X_0$. Then it also holds for all $X \le X_0' = X_0 + \varepsilon$, with $\varepsilon = (1-\delta)(V_1 - X_0)$.

point a. Consider the subgame $(2; X)$. We show that when $X \ge X_0$, player 2's equilibrium contribution is at most equal to $X - X_0$.

If player 2 contributes $X - X_0$, player 2 obtains $\delta V_2 - (X - X_0)$ (because the subsequent subgame is $(1; X_0)$ and the induction hypothesis applies to that subgame). Clearly, player 2 obtains strictly less by making a larger contribution (yet smaller than X). By contributing the rest, player 2 would obtain $V_2 - X$. However, since $X_0 > (1-\delta)V_2$, we have:

$$\delta V_2 - (X - X_0) > V_2 - X.$$

point b. Consider the subgame $(1; X)$, with $X > X_0$. If in equilibrium, player 1 contributes $c_1 \ge X - X_0$, then player 1 is strictly better off contributing the rest (that is, $c_1 = X$). (In the subgame following c_1, the induction hypothesis applies.)

Let $X' = X - c_1$. In case $X' > (1 - \delta)V_2$, player 2 contributes nothing, and player 1 next contributes the rest; In case $X' \leq (1 - \delta)V_2$, player 1 gets at most $\delta V_1 - [X - (1 - \delta)V_2] < V_1 - X$. So in both cases, player 1 is strictly better off contributing the rest.

point c. Consider the subgame $(1; X)$, with $X > X_0$. In equilibrium, the total contribution made by player 1 in this subgame is at least equal to X_0.

In equilibrium, either player 2 is the first player driving the total contribution above $K - X_0$; then player 1 contributes X_0 in the subsequent subgame (by point a and the induction hypothesis); or player 1 is the first player driving the total contribution above $K - X_0$; then it must be that player 1 contributes at least X_0 (because by point b, player 1 contributes the rest). In both cases, the total contribution made by player 1 is at least equal to X_0.

point d. Consider the subgame $(1; X)$, with $X_0 < X < X_0 + \varepsilon$. It is strictly optimal for player 1 to contribute the rest.

If player 1 chooses a contribution $c_1 < X$, total completion requires at least two periods and player 1's total contribution is at least equal to X_0 (by point c). Since player 1 is better off when his contributions are cast at the last date where he contributes, he obtains a payoff at most equal to

$$\delta[V_1 - X_0]$$

By contributing X immediately, he would obtain $V_1 - X$, which is strictly larger than $\delta[V_1 - X_0]$ (since $X \leq X_0 + \varepsilon$ and by definition of ε).

Finally, given that it is optimal for player 1 to contribute the rest, it is optimal for player 2 to contribute nothing.

Step 3: Assume claim A holds for all $X < V_1$. Then it also holds for all $X \leq K$.

Consider a subgame $(2; X)$, with $X \geq V_1$. By contributing $X - V_1 + \varepsilon$, player 2 secures $\delta V_2 - K + V_1 - \varepsilon$. This is so because after player 2's contribution there remains only $V_1 - \varepsilon$ to contribute. Since ε can be chosen arbitrarily small, player 2 's equilibrium value in the subgame $(2; X)$ is at least equal to $v_2(X) = \delta V_2 - X + V_1$. Also, contributing strictly more than $X - V_1$ gives player 2 a payoff strictly smaller than $v_2(X)$, so making such a contribution cannot be optimal.

Consider a subgame $(1; X)$, with $X > V_1$. If player 1 makes a strictly positive contribution, then either the project is not completed and his payoff is negative, or the project is completed and his total contribution is strictly larger than V_1 (because player 2 never contributes more than $X' - V_1$ when there remains X' to contribute). It is therefore optimal for player 1 to make no contribution.

Given that player 1 never contributes when $X > V_1$, it is optimal for player 2 to contribute $X - V_1$.

Finally, combining Steps 1 and 2 implies that claim A holds for all $X < V_1$. Applying step 3 permits to conclude. ■

3 The partial refund case

In this Section, we interpret $1 - \delta$ as a probability of breakdown. We assume that in case of breakdown, each player gets back a fraction α of his total contribution.[4]

[4] In a more general setup, Bagnoli and Lipman (1989) have noticed that the total refund case may help select core outcomes if attention is restricted to equilibria in undominated

Formally, consider any date t where player i moves and assume that after player i's contribution, the cumulative contribution made does not reach K (in case the total contribution reaches K, the project is completed). With probability δ, the game moves to date $t+1$. With probability $1-\delta$, the game terminates, and each player i gets a refund $\alpha \sum_{1 \le s \le t} c_i^s$. Let τ be the date at which the game terminates. An outcome of the game is defined by a triple $(\tau; \{c_1^t\}_{t=1}^{\tau}; \{c_2^t\}_{t=1}^{\tau})$. For $i = 1; 2$, player i's payoff is given by

$$\begin{aligned} u_i(\tau; \{c_1^t\}_{t=1}^{\tau}; \{c_2^t\}_{t=1}^{\tau}) &= V_i - \sum_{1 \le t \le \tau} c_i^t \text{ if } \sum_{1 \le t \le \tau} \left(c_1^t + c_2^t \right) \ge K \\ &= (-1+\alpha) \sum_{1 \le t \le \tau} c_i^t \text{ otherwise} \end{aligned}$$

Remark 1: When $\alpha = 0$ the game coincides with that analyzed in subsection 2.1, where contributions are sunk. To see why, consider a date T and a sequence of contributions $\{c_1^t\}_{t=1}^{T}; \{c_2^t\}_{t=1}^{T}$ for which $\sum_{1 \le t \le \tau} c_1^t + c_2^t \ge K$. Player i's expected payoff is then equal to

$$\delta^{T-1}[V_i - \sum_{1 \le t \le T} c_i^t] + (1-\delta) \sum_{1 \le \tau < T} \left[\delta^{\tau-1}(-1+\alpha) \sum_{1 \le t \le \tau} c_i^t \right]$$

which reduces to

$$\delta^{T-1} V_i - \sum_{1 \le t \le T} \delta^{t-1} c_i^t$$

when $\alpha = 0$.

Remark 2: When $\alpha = 1$, contributions are fully refundable. This game is formally equivalent to the subscription game analyzed in Admati and Perry - Section 5.

Remarks 1 and 2 above show that the game of this subsection is a combination of the subscription game and the contribution game considered in Admati and Perry. We have the following Proposition:

strategies.

Proposition 2 Assume that $\alpha > 0$ and $\delta(V_1 + V_2) > K$. Then the game has a unique subgame perfect equilibrium path. If $V_1 = V_2$, then the project is completed in two periods and two contributions. Otherwise, the project is completed in at most three periods and two (strictly positive) contributions.

Proposition 2 shows that the project is completed when it is socially desirable, i.e. $V_1 + V_2 > K$, and δ is sufficiently close to 1. It also shows that there are at most two strictly positive contributions in equilibrium and therefore this variant too cannot explain that players make small contributions at a time.

It should be noted that Proposition 2 holds true irrespective of whether or not players are symmetric and irrespective of the refund share α, as long as $\alpha > 0$. Thus, even if parties are symmetric, and even if contributions are almost sunk (i.e. α close to 0), the players will not in equilibrium contribute step by step. Yet, the extent to which contributions are sunk (i.e. the magnitude of $1 - \alpha$) may affect the outcome (to the detriment of the player who values most the project), and possibly the order in which players contribute to the project.

Proof of Proposition 2:

In what follows, we assume that $V_1 \geq V_2$. We denote by $(i; X_1; X_2)$ a subgame where it is player i's turn to move and where the current total contribution made by player k is X_k for $k = 1; 2$.

Step 0: Consider a subgame $(i; X_1; X_2)$. Player i contributes the rest $K - X_i - X_j$ whenever

$$V_i - K + X_1 + X_2 > \delta V_i + (1 - \delta)\alpha X_i. \quad (1)$$

Indeed, under that condition, contributing the rest is a dominant strategy for player i, since by not contributing the rest he gets at most $\delta V_i + (1 - \delta)\alpha X_i$

- which corresponds to i's expected payoff when j contributes the rest in the next period.

We now define X_1^* and X_2^* to be the solutions to the following system of equations:[5]

$$V_i - K + X_1^* + X_2^* = \delta V_i + (1 - \delta)\alpha X_i^*, \ i = 1; 2: \tag{2}$$

We first establish:

Step 1: Consider any subgame $(i; X_1; X_2)$ and let $X = K - X_1 - X_2$. Assume $X_j > X_j^*$ (with $j \neq i$) and $0 < X < V_i - \alpha X_i$. Then in equilibrium, player i completes the project immediately by contributing X.

The proof of this step is analogous to that of steps 1 and 2 in the proof of Proposition 1. Let $X^* = K - X_1^* - X_2^*$. The claim of step 1 clearly holds for $X \leq X^*$.[6] We show now that if it holds for all X such that $X^* \leq X \leq X_0$, then it also holds for all X such that $X^* \leq X \leq X_0 + \varepsilon$, where $\varepsilon = \frac{1-\delta}{2}[V_i - \alpha X_i - X_0]$.

Assume that the claim of step 1 holds for all X such that $X^* \leq X \leq X_0$. Then in any subgame $(j; X_1; X_2)$ with $X_j > X_j^*$ and $X^* \leq X \leq X_0$, it is strictly optimal for player j to contribute nothing. This is because player i completes the project next period and because player j obtains strictly less

[5] Note that we allow X_i^* to be negative: When $V_1 = V_2$, we have $X_1^* = X_2^* > 0$, however, if V_1 is much larger than V_2 and $\alpha < 1$, X_2^* may be negative.

[6] Indeed, either $X_i < X_i^*$, and (since $X \leq X^*$) the following equality holds:

$$V_i - X \geq V_i - X^* = \delta V_i + (1 - \delta)\alpha X_i^* > \delta V_i + (1 - \delta)\alpha X_i;$$

or $X_i \geq X_i^*$, and (since $X_j > X_j^*$) the following inequality holds:

$$V_i - X > V_i - X^* + X_i - X_i^* \geq \delta V_i + (1 - \delta)\alpha X_i + (1 - (1 - \delta)\alpha)(X_i - X_i^*)$$

In both cases, it is strictly optimal for player i to contribute the rest.

by completing it now, since

$$V_j - X \le V_j - X^* = \delta V_j + (1-\delta)\alpha X_j^* < \delta V_j + (1-\delta)\alpha X_j:$$

Similarly, in any subgame $(j; X_1; X_2)$ with $X_j > X_j^*$ and $X \ge X_0$, it cannot be optimal for player j to contribute strictly more $X - X_0$.

Thus (as in point b and c of the proof of Proposition 1) in any subgame equilibrium path starting from $(i; X_1; X_2)$ with $X_j > X_j^*$ and $X \ge X_0$, and for which the project is eventually completed, the total contribution of player i must be at least equal to X_0 (in that subgame).

By contributing the rest, player i would obtain $V_i - X$. If player i does not immediately contribute the rest, total completion requires at least two rounds of contribution, while the total contribution of player i (in that subgame) is at least equal to X_0. Since player i is better off when his contributions are cast at the last date where he contributes, and since he had already contributed X_i, his continuation payoff (computed from the subgame $(i; X_1; X_2)$) is at most equal to

$$\delta[V_i - X_0] + (1-\delta)\alpha X_i$$

which is strictly smaller than $V_i - X$ when $X \le X_0 + \varepsilon$, given our definition of ε.

We now consider subgames $(i; X_1; X_2)$ with $X_j < X_j^*$ and $X_i \le X_i^*$. We distinguish two cases.

case a) $X_2^* \ge K - V_1$:

Note first that Equations (2) imply $X_1^* - X_2^* = (V_1 - V_2)/\alpha$. So we also have $X_1^* \ge K - V_2$ (remember that $V_1 \ge V_2$).

Step 2a: Consider the subgame $(i; X_1; X_2)$. We show that player i's continuation equilibrium value is at least equal to $v_i(X_i)$ defined by

$$v_i(X_i) \equiv V_i - K + X_j^* + X_i;$$

and that player i's equilibrium contribution is at most equal to $X_i^* - X_i$

If player i contributes $X_i^* - X_i + \varepsilon$, then the subsequent subgame is $(j; X_j; X_i^* + \varepsilon)$, and there remains $K - X_i^* - X_j - \varepsilon < V_j - \alpha X_j$ to contribute. The conditions of Step 1 are thus satisfied, and player j contributes the rest. Since we may choose ε arbitrarily small, player i's continuation equilibrium value in the subgame considered is at least equal to

$$\delta V_i - [X_i^* - X_i] + (1 - \delta)\alpha X_i^*;$$

which is equal to $v_i(X_i)$ (by Equation (2)).

If player i contributes the rest (that is, $K - X_1 - X_2$), he gets a payoff equal to $V_i - K + X_1 + X_2$; which is thus strictly smaller than $v_i(X_i)$ whenever $X_j < X_j^*$.

Now observe that if player i contributes strictly more than $X_i^* - X_i$ without completing the project, he gets strictly less than $v_i(X_i)$. Observe also that if player i contributes the rest (that is, $K - X_1 - X_2$), he gets a payoff equal to $V_i - K + X_1 + X_2$; which is thus strictly smaller than $v_i(X_i)$ whenever $X_j < X_j^*$. So it cannot be optimal for player i to contribute strictly more than $X_i^* - X_i$.

Step 3a: Consider a subgame $(i; X_1; X_2)$, (with $X_1 < X_1^*$ and $X_2 < X_2^*$). We show that it is optimal for player i to contribute $X_i^* - X_i$.

Consider a subgame perfect Nash equilibrium path starting from a subgame $(i; X_1; X_2)$ and leading to the completion of the project after T contributions. If player i makes the last contribution, player i's total contribution must be at least equal to $K - X_j^*$. [This is so because whenever $X_j \le X_j^*$ contributing strictly more than $X_j^* - X_j$ for j is not optimal (by step 2a).] Since player i is better off when his contributions are cast at the last date, and since he had already contributed X_i, his expected gain (under the realized sequence of contributions considered) is at most equal to

$$\delta^{T-1}(V_i - K + X_j^* + X_i) + (1 - \delta^{T-1})\alpha X_i \le X_i + \delta^{T-1}(V_i - K + X_j^*)$$

If player j makes the last contribution, it must be that $X_i^T \ge X_i^*$. (Otherwise, player j would not make the last contribution, by step 2a). Since, as above, player i is better off when his contributions are cast at the last date where he contributes, his expected gain is at most equal to

$$\delta^{T-2}v_i(X_i) + (1 - \delta^{T-2})\alpha X_i \le X_i + \delta^{T-2}(V_i - K + X_j^*).$$

Since player i obtains a payoff at least equal to $X_i + V_i - K + X_j^*$ in equilibrium, and since $T \ge 2$ (because in equilibrium, player i contributes at most $X_i^* - X_i$), we must have $T = 2$ in equilibrium. Besides, if player 1 contributes strictly less than $X_i^* - X_i$, player j does not complete the project in the next round, and the number of contributions necessary to complete the project would be equal to 3. So in equilibrium, player i contributes $X_i^* - X_i$, and player j then contributes the rest.

case b). $X_2^* < K - V_1$.

Note first that this condition requires $\alpha < 1$.[7] We let $\Delta = K - V_1 - X_2^*$

[7] To see why, assume $\alpha = 1$ and $X_2^* < K - V_1$. Then $X_1^* - X_2^* = V_1 - V_2$, so $X_1^* < K - V_1$, hence

$$X_1^* + X_2^* < 2K - V_1 - V_2: \tag{3}$$

However, adding equations Equations (2) gives

$$(1 + \delta)(X_1^* + X_2^*) = 2K - V_1 - V_2 + \delta(V_1 + V_2);$$

and consider any subgame $(2; X_1; X_2)$, with $X_2 < \hat{X}_2$, where

$$\hat{X}_2 = \max\{X_2^*; K - V_1 - (1-\alpha)X_1\}.$$

If player 2 contributes $\hat{X}_2 - X_2 + \varepsilon$, the subsequent subgame is $(1; X_1; \hat{X}_2 + \varepsilon)$, and the conditions of Step 1 are satisfied (because $\hat{X}_2 > X_2^*$ and because $K - X_1 - \hat{X}_2 - \varepsilon \leq V_1 - \alpha X_1 - \varepsilon$). So player 1 contributes the rest. Player 1's total contribution is thus at least equal to $X_1 + K - X_1 - \hat{X}_2 - \varepsilon = K - \hat{X}_2 - \varepsilon$. Either $X_2^* \leq K - V_1 - (1-\alpha)X_1$, and then

$$K - \hat{X}_2 - \varepsilon = V_1 + (1-\alpha)X_1 - \varepsilon$$

or $X_2^* > K - V_1 - (1-\alpha)X_1$; in which case

$$K - \hat{X}_2 - \varepsilon = K - X_2^* - \varepsilon = V_1 + \Delta - \varepsilon.$$

Since $\alpha < 1$, and $\Delta > 0$, and since ε may be chosen arbitrarily small, it cannot be optimal for player 1 to contribute a strictly positive amount in any subgame $(1; X_1; X_2)$ for which $X_2 < \hat{X}_2$ (because his total contribution would exceed V_1). In particular, player 1 contributes nothing in the first period.

Consider now the subgame $(2; 0; 0)$. Then $\hat{X}_2 = K - V_1$. Let

$$v_2 = \delta V_2 - (K - V_1) + (1-\delta)\alpha(K - V_1);$$

and note that $v_2 > 0$. When player 2 contributes $K - V_1 + \varepsilon$, player 1 contributes the rest, hence player 2 obtains a payoff equal to $v_2 - \varepsilon$. Since ε may be chosen arbitrarily small, player 2's equilibrium payoff in that subgame is at least equal to v_2. Thus, contributing strictly more than $K - V_1$ cannot be optimal for player 2.

which, combined with (3) yields $V_1 + V_2 < K$, contradicting the assumption that the project is socially desirable.

Since player 1 contributes nothing until $X_2 \geq K - V_1$; it cannot be optimal for player 2 to contribute strictly less than $K - V_1$. It follows that in equilibrium, player 2 contributes $K - V_1$ (and then player 1 contributes V_1). ■

4 Related literature and conclusion

We have already discussed the relationship between our paper and that of Admati and Perry (1991). We now discuss the relationship with Marx and Matthews (2000), Lockwood and Thomas (1999) and our companion paper Compte and Jehiel (2000).

Marx and Matthews (2000) propose another variant on Admati and Perry's contribution game, which challenges the inefficiency result obtained in Admati and Perry: They consider a contribution game where players may make contributions simultaneously. (See also Lockwood and Thomas 1999 for an application to investment games, and Gale (2000) for an analysis of a more general framework). Their main result is that the simultaneity in players' contributions may allow players to sustain efficient outcomes. The simultaneity plays a crucial role in their analysis in that it allows to sustain the no contribution outcome if a player deviates from a prescribed strategy. Note however that the no contribution outcome is credible only in symmetric cases.

Our results show that extending Admati and Perry's game to the case of asymmetric players and to the case of partial refunds restores efficiency.

Although our result has implications concerning the efficiency of the outcome, we wish to emphasize that our main interest is on identifying what can induce the equilibrium contributions to be made step by step. In this respect, our main ...nding is that the sunk character of contributions cannot explain why agents make gradual contributions step by step before completing a joint project whenever there are asymmetries and/or the contributions are partly refunded in case the project is not completed.

In contrast, in Compte-Jehiel 2000a, we show that whenever agents have the option of implementing a partial project, equilibrium contributions must be made step by step whether or not agents are symmetric and whether or not contributions are sunk. The logic of this result is that each party fears that a large contribution will be exploited by the other party who might just opt for a partial implementation of the project. The motive for making partial contributions turns out to be analogous to the motive for making partial concessions in a bargaining context with history dependent outside options (Compte-Jehiel 1995). This analogy is further explored in Compte-Jehiel 2000b.

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