

Bargaining with Reference Dependent Preferences.

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1 Introduction

In real bargaining contexts, psychology does seem to play an important role. The game theoretic approaches to bargaining - which endogenize the bargaining strategies of the parties (see Osborne-Rubinstein 1990 for a review) - largely ignore psychological considerations. Psychology has pointed out a number of deviations from standard behaviors (see Rabin for a recent survey). In particular, there seems to be ample evidence that subjects' preferences are defined relative to reference points (see Tversky and Kahneman) and that these reference points may shift over time. Several authors have investigated the effects of shifting reference points in consumption models (Gilboa Smeidler, Strahilevitz-Loewenstein). The aim of this paper is to analyze the strategic implications of shifting reference points in bargaining games à la Rubinstein.

Our main hypothesis is that a player's reference point may shift from one bargaining phase to the next, as a function of the offers received in the past. Our main finding is that bargaining parties will reach agreements gradually as opposed to immediately as a result of these shifting reference points. The

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intuition is roughly as follows. If party A starts the negotiation by offering an equal split $(1/2, 1/2)$ to party B, there is a risk - if breakdown occurs and a new bargaining phase starts- that party B's reference point switches to a new value that it is greater than it was at the start (while party A's reference point would remain unchanged). More importantly, there is a risk that party B will stop making any serious offer, thus making a breakdown inevitable with the consequences as just described. In such a case, where party B would have a reference point higher than that of party A, the final split would be biased in favor of party B. Of course, anticipating this, party A will not start off with the equal split $(1/2,1/2)$ offer, and bargaining will take several periods.

More precisely, we consider a bargaining game in which at the end of any period there is a risk of breakdown and when a breakdown occurs bargaining starts again with new reference points, which are determined by the most generous offers made so far. (Parties also suffer a cost c whenever a breakdown occurs.) We analyze the equilibria of these games and show that equilibria (which have a recursive structure) necessarily involve some form of gradualism.

Our analysis explains why it may be risky to start off negotiations with generous offers, an advice often found in negotiation books. The risk is that, through the possible change of reference point, this offer will be taken for granted in the future, and eventually whet one's opponent appetite. Hence, our analysis may help understand how gradualism - which is so widespread in real bargaining contexts - may be caused by some psychological traits such as shifting reference points.

Previous game theoretic approaches to delays in bargaining have mostly focused on incomplete information. Most such models have the feature that offers made at the bargaining stage have signalling effects on the private information held by the parties, and thus many equilibrium behaviors are possible. Some recent contributions (Abreu and Gul (2000) and Compte

and Jehiel (2002)) avoid that problem by taking the view that parties may with positive probability be obstinate thereby always insisting on the same share. Then, there is essentially one equilibrium, and whenever the two parties may be obstinate equilibrium behavior takes the form of a war of attrition in which long delays are possible. But, these equilibria do not capture the idea of gradualism in the sense that the equilibrium offers made by the parties remain almost always the same throughout the interaction (the interesting dynamics is about the probability with which parties accept the inflexible demand of the opponent). In a companion paper (Compte and Jehiel 2003 which is an elaboration on Compte and Jehiel 1995) we have shown that some history-dependent outside options could have the effect of inducing gradualism in bargaining. Our finding that changing reference points may induce gradualism has close connections with that insight. But, in our model there are no outside options. Thus another view on our paper is that in bargaining contexts, changing reference points play a role similar to that of history-dependent outside options in standard neoclassical setups.

2 The model

We consider a bargaining game described as follows. There are two parties $i = 1, 2$ who negotiate over the partition of a pie of size 1. Parties move in alternate order. At the start of the interaction, one of the players is drawn at random with probability $1/2$ to make an offer to the other party. The responder may then accept or reject the offer. If he accepts the is the end of the bargaining and the partition offer is implemented. Otherwise, the game moves to the next round where it is the other player who makes the offer. And so on. At the end of each round, there is an exogenous probability β of breakdown. In case of breakdown, a new bargaining phase starts, with again one of the player being drawn at random to make the first offer. Moving from one bargaining phase to the next one is assumed to cost c to each party.

Bargaining phases are labelled $n = 1, \dots$. The various rounds of a bargaining phase are labelled $m = 1, \dots$

The key ingredient of this paper is that preferences are defined relative to reference points and reference points move from one phase to another. Specifically, the utility that player j derives from the partition offer $y = (y_i, y_j)$ when his reference point is r_j is defined by

$$v_j(y, r_j) = y_j - r_j.$$

Remark: Alternatively, the reference point r_j can be interpreted as a minimum aspiration level of the player that defines the level of offer at which the derived utility is nul. We will discuss alternative interpretations of the reference point in Section 4.

Consider a bargaining phase n at which player j 's reference point is equal to r_j . In phase n , player j evaluates an agreement on $y = (y_i, y_j)$ that would take place in round $n' \geq n$ according to $u_j^n(y, r, n')$ where:

$$u_j^n(y, r, n') = v_j(y, r_j) - (n' - n)c.$$

That is, player j evaluates partition offers to occur possibly in later phases according to the reference point of the current phase. Note also that our payoff specification assumes that there is no time discounting. Despite the absence of time discounting, the parties have incentives to reach early agreements as otherwise they run the risk of paying higher costs for switching more often to new bargaining phases. In the classical setup with steady preferences, immediate agreement would obtain as an equilibrium outcome.

To complete the description of the model, we assume that reference points vary from one bargaining phase to another according to how generous the current offers are, given the current evaluation. Formally, let X_j^n denote the largest offer received by party j during phase n . The difference $X_j^n - r_j^n$ is a measure of the offers received by player j in phase n . We

assume that

$$\begin{aligned} r_j^{n+1} - r_j^n &= f(X_j^n - r_j^n) \text{ if } X_j^n \geq r_j^n, \\ &= 0 \text{ otherwise,} \end{aligned}$$

where $f(\cdot)$ is a non decreasing function such that $f(0) = 0$ and $f(x) \leq x$. The function $f(\cdot)$ is meant to capture the extent to which the received offers affect one's own reference point. The conditions imply that $f(x) \geq 0$, and thus the reference points may only increase over time. The condition $f(x) \leq x$ ensures that a player's reference point can never exceed the most generous offer he received in the past.

3 Main insights

Before giving formal results, we wish to explain informally our main insights. For simplicity, in this Section, we restrict our attention to the case in which $f(\cdot)$ is linear in its argument, i.e. $f(x) = \mu x$ with $\mu \in (0, 1]$.

Consider the first phase of bargaining and suppose the most generous offer received in this phase by player $i = 1, 2$ is $X_i^{(1)}$. The bargaining process may accidentally break down, thus inducing a switch to a second bargaining phase. In such a scenario, the phase 2 reference points would be given by

$$r_i = \mu X_i^{(1)}$$

In this second phase, only offers (x_1, x_2) for which $x_i \geq r_i$ for $i = 1, 2$ are individually rational. Thus, the total surplus that players can derive (beyond their minimum aspiration value) is equal to $1 - r_1 - r_2$.

In the symmetric setting that we consider, this surplus $1 - r_1 - r_2$ will end up being shared equally between the two players in expectation (for this conclusion we will have to restrict attention to symmetric equilibria). However, several periods and thus several bargaining phases may be required

before the agreement is reached. We let η denote the expected number of new phases required before an agreement is reached. This expected number η will in equilibrium be a function of the remaining surplus $1 - r_1 - r_2$, and we will consider equilibria such that it is an *increasing* function of this surplus.¹

To summarize, if bargaining accidentally breaks down in phase 1, party i will end up obtaining a share x_i such that

$$x_i - r_i = \frac{1 - r_1 - r_2}{2}$$

and this will require on average η new phases. Thus, from the viewpoint of phase 1, player i 's expected payoff would be equal to

$$r_i + \frac{1 - r_1 - r_2}{2} - \eta c$$

or, equivalently,

$$\frac{1}{2} + \mu \frac{X_i^{(1)} - X_j^{(1)}}{2} - \eta c \tag{1}$$

The interpretation of (1) is as follows. The difference $X_i^{(1)} - X_j^{(1)}$ represents how generous the offers made by player j were in phase 1 relative to those made by i . When this difference is positive, party i is in a favorable position in case the negotiation moves to a second phase.

So far the breakdown issue was treated as exogenous, but players may actually act *strategically* in this respect. By not making any further (serious) offers, players may ensure that no agreement will be found until the current bargaining phase breaks down. A new bargaining phase would then follow with the consequences on expected payoffs as shown above. As a result, players will refrain from making offers much more generous than those they have been receiving, because they fear that the other party might exploit

¹This is a fairly natural assumption, but we were unable to derive it as a necessary property of equilibrium behavior.

them by waiting for a breakdown. As a result, the negotiation process will have to be gradual.

How gradual does the process has to be?

The above observations are suggestive that the negotiation process must be gradual, but how gradual should it be? To address that issue, we first derive an upper bound on the degree by which parties are ready to improve their offer in each round taking as given the breakdown equilibrium outcome (this step follows the logic of Compte and Jehiel 1995). We next endogenize the breakdown outcome and we derive a lower bound on the expected number of phases that is necessary to reach an agreement (this step does not have its counterpart in Compte and Jehiel 1995, 2003).

Specifically, we compare the payoff that player i would obtain by waiting till the process moves to the next phase, with the payoff he would obtain at most if he were to increase the generosity of his offer by an amount of Δ . We let $\Delta^t = X_i^{(1,t)} - X_j^{(1,t)}$ denote the difference between player i and player j 's most generous offers received up to date t . By waiting for a breakdown, player i would obtain at least:

$$\frac{1 + \mu\Delta^t}{2} - \eta c$$

If player i instead increases by Δ the generosity of his offer, player j may next secure (again by waiting for a breakdown) a payoff equal to

$$\frac{1 - \mu\Delta^t + \mu\Delta}{2} - \eta c$$

which implies that in phase 1 player i cannot expect to obtain a share of the pie larger than

$$1 - \frac{1 - \mu\Delta^t + \mu\Delta}{2} + \eta c$$

Hence in equilibrium, only offers for which Δ satisfies

$$\frac{1 + \mu\Delta^t}{2} - \eta c \leq 1 - \frac{1 - \mu\Delta^t + \mu\Delta}{2} + \eta c$$

can be made, which implies:²

$$\begin{aligned}\Delta &\leq \bar{\Delta}, \text{ where} \\ \bar{\Delta} &\equiv 4\eta c/\mu\end{aligned}\tag{2}$$

The scalar $\bar{\Delta}$ thus provides an upperbound on the extra generosity of a new offer made in equilibrium. On the other hand, the value of $\bar{\Delta}$ gives an idea on the number of rounds (and in turn of bargaining phases) required to reach an agreement. Intuitively, the smaller $\bar{\Delta}$, the larger the number of bargaining rounds required to reach an agreement, hence the larger the expected number of phases η . But, by (2) the larger η the larger $\bar{\Delta}$. Trading off these two effects yield the desired quantification of gradualism.

More precisely, to reach a balanced agreement, each party must gradually increase the generosity of his offer until an equal split offer (1/2,1/2) is possible. This requires (up to integer problems) at least $2 \cdot \frac{1/2}{\Delta}$ rounds, and the expected number of breakdowns that will occur before a balanced agreement on a pie of size 1 can be reached is thus no smaller than $\beta/\bar{\Delta}$. We thus have, using Equation (2),

$$\eta \geq 1 + \beta/\bar{\Delta} = 1 + \beta\mu/4\eta c.\tag{3}$$

which implies,

$$\eta \geq \min\left(1, \frac{1}{2}\sqrt{\frac{\beta\mu}{c}}\right).\tag{4}$$

4 Main results

We now move to a more formal analysis of the game. The main technical difficulty that has to be addressed is that we need to find equilibrium behavior in every possible round n , for any possible reference points (r_1, r_2) .

²We use the monotonicity of $\eta(\cdot)$ to derive an upper bound that is independent of the surplus left.

We will bypass these difficulties by exploiting the recursive structure of the game.

4.1 The recursive structure.

How does the subgame starting in round n with reference points (r_1, r_2) (referred to as game I) differ from the game for which the size of the pie is $1 - r_1 - r_2$ and reference points are $(0, 0)$ (referred to as game II)?

These two games are very similar, as the following Lemma shows:

Lemma 1 *Consider any given strategy profile σ of game II, and construct a strategy profile $\tilde{\sigma}$ of game I in which all offers are translated by (r_1, r_2) . That is, if (y_1, y_2) is offered under σ , then $(r_1 + y_1, r_2 + y_2)$ is offered under $\tilde{\sigma}$. To each subgame of game II corresponds a unique subgame of game I (obtained by translation of all past offers). Consider two such subgames. The strategy profile σ yields the same payoffs in the subgame of game II as the strategy profile $\tilde{\sigma}$ in the subgame of game I.*

The intuition is straightforward. Adding r_i to each offer to player i offsets the fact that only offers above r_i are valued. Besides, given our assumption about the evolution of reference points, it is easy to check that variations in reference points from one phase to the next are identical in both games (a formal proof is given in the Appendix).

We now exploit the properties described above to simplify the analysis of the game and restrict our attention to recursive equilibria, as defined below.

Definition *A recursive equilibrium σ defines a subgame perfect equilibrium σ^K for each possible size $K \leq 1$ of the pie, and it has the following property: in any subgame starting in round n with reference points (r_1, r_2) , the continuation strategy $\tilde{\sigma}$ coincides with σ^K with $K = 1 - r_1 - r_2$ up to a translation: whenever player i would propose (x_1, x_2)*

under σ , he proposes $(\tilde{x}_1, \tilde{x}_2)$ under $\tilde{\sigma}$, where \tilde{x}_i is defined by

$$\tilde{x}_i = r_i + x_i$$

Also, since the subgame where player 1 is selected to start is identical to that where player 2 is selected to start up to a change of identity of the players, we shall focus on equilibria that are *symmetric*: if (σ_1, σ_2) is played in the subgame where player 1 is selected to start round 1, then (σ_2, σ_1) is played in the subgame where player 2 is selected to start round 1.

Finally, for any given recursive equilibrium σ , define $\eta(K)$ as the expected number of phases. We will restrict our attention to *monotone* recursive equilibria for which $\eta(K)$ is non-decreasing in K .

4.2 Results.

Our objective is to show that in equilibrium, parties only gradually get to an agreement. More precisely, consider any date t in round n . The current reference point for player i is denoted r_i^n , and the most generous offer received by party i in round n , up to date t , is denoted X_i^t . In what follows, we shall derive an upper bound on the difference $\Delta^t = X_i^{t+1} - X_i^t$.

As a first step towards getting this upperbound, we first derive a lower bound on the payoff that player i may obtain in equilibrium when he ends the current round. Formally, for any K, r_1, r_2 , we define

$$v_i^{K, \text{end}}(r_1, r_2) = r_1 + \frac{K - r_1 - r_2}{2} - \eta(K)c.$$

We have:

Lemma 2 *Assume that the pie has size K . Consider a date t in phase 1. If current most generous offers are (X_1^t, X_2^t) and player i decides that in the current phase, he will make no more generous offers nor accept any offer, then continuation equilibrium payoff for player i is at least equal to $v_i^{K, \text{end}}(r_1, r_2)$ where $r_1 = f(X_1^t)$ and $r_2 = f(X_2^t)$.*

When player i makes no further offers (and reject all offers) in the current phase, he prevents agreement in that phase, and eventually, a new phase starts. In this new phase, reference points move to $r_i \geq f(X_i^t)$ and $r_j = f(X_j^t)$. The proof of the Lemma exploits symmetry to show that in the subgame starting after breakdown, and up to further bargaining costs, players share the surplus $K - r_1 - r_2$ equally. It also exploits recursiveness to show that (up to bargaining costs), each player i gets a share $r_i + (K - r_1 - r_2)/2$. It finally exploits monotony to show that bargaining costs cannot exceed $\eta(K)c$. A formal proof is the Appendix.

We are now ready to derive an upperbound on Δ_t . When a player, say player 1, makes a generous offer in phase 1, he increases the value of r_2 in case the current phase ends. Since $v_2^{K,end}(r_1, r_2)$ increases in its second argument, one implication of the above Lemma is that generous offers increase the other party's option value. For party 1, making a too generous offer may have the effect of increasing so much party 2's option value that party 1 would actually prefer to wait for breakdown.

Formally, let

$$\mu = \min_{x,y>0} \frac{f(x+y) - f(x)}{y}$$

We have:

Proposition 3 *Assume that the pie has size K . Consider a date t in phase 1 and assume current most generous offers are (X_1^t, X_2^t) . In equilibrium, if party 1 makes an offer (y_1, y_2) at t , this offer must satisfy:*

$$y_2^t - X_2^t \leq \frac{4c\eta(K)}{\mu}$$

Proof. First assume that players are still in phase 1. consider phase 1. At date t , party 1 may secure $v_1^{1,end}(f(X_1^t), f(X_2^t))$. If he makes an offer (y_1^t, y_2^t) at t , with $y_2^t > X_2^t$, then $X_2^{t+1} = y_2^t$, and party 2 obtains a payoff at least equal to $v_2^{1,end}(f(X_1^t), f(X_2^{t+1}))$ by ending the current round at $t + 1$,

hence party 1 must be getting an expected share of the pie at most equal to $1 - v_2^{1,end}(f(X_1^t), f(X_2^{t+1}))$, which implies,

$$v_1^{1,end}(f(X_1^t), f(X_2^t)) + v_2^{1,end}(f(X_1^t), f(X_2^{t+1})) \leq 1.$$

Using the definition of $v_i^{1,end}$, we get,

$$v_2^{1,end}(f(X_1^t), f(X_2^{t+1})) - v_2^{1,end}(f(X_1^t), f(X_2^t)) \leq 2c\eta(K)$$

which implies,

$$(f(X_2^{t+1}) - f(X_2^t))/2 \leq 2c\eta(K).$$

The upperbound on $y_2^t - X_2^t$ follows from the definition of μ . ■

Note that thanks to recursiveness and monotony, the same bound applies to phases other than phase 1. Proposition 3 thus implies that the most generous offers may only grow at small pace, whatever bargaining phase they are playing.

We may now prove our main result: we will check that only sufficiently generous offers may be accepted, and obtain a lower bound on the number of periods necessary to reach an agreement as an implication of Proposition 3.

Formally, player 1 may only accept an offer (y_1, y_2) if

$$y_1 \geq v_1^{1,end}(f(\max(X_1^t, y_1)), f(X_2^t)),$$

because otherwise, he would prefer to wait for breakdown. Combining this observation with Proposition 3 we get:

Proposition 4 *Getting to an agreement requires at least τ offers and η^* expected phases, where τ and η^* satisfy the two following inequalities:*

$$\begin{aligned} \tau &\geq \frac{1 - 2c\eta^*}{4c\eta^*} \frac{\mu}{2 - \mu} \\ \eta^* &\geq 1 + \beta\tau \end{aligned}$$

To fix ideas, consider two extreme cases, where β/c is very small, and where β/c is very large. In the first case, the number of expected phases may remain close to 1 (because the probability of breakdown is very small). Nevertheless, the number of offers necessary to reach agreement may be quite large, i.e. comparable to $1/c$. In the second case, the number of expected phases will be large compared to 1, and Proposition 4 implies that it is at least comparable to $\sqrt{\beta/c}$. The number of offers is then comparable to $\sqrt{1/(\beta c)}$.

Proof. Let $X_1 = \max(y_1, X_1^t)$ and $X_2 = X_2^t$. Player 1 may only accept (y_1, y_2) if

$$X_1 \geq \frac{1}{2} + \mu \frac{X_1 - X_2}{2} - \eta^* c. \quad (5)$$

Let

$$\underline{X} = \min_{\substack{s.t.(5) \\ X_1 \geq 0, X_2 \geq 0}} X_1 + X_2$$

By Proposition 3, the generosity of offers may only increase gradually: $X_1^t + X_2^t$ increases by at most $\Delta = 4c\eta^*/\mu$ at every date. It follows that at least \underline{X}/Δ moves are required to get an agreement, that is, $\tau \geq \underline{X}/\Delta$. To compute \underline{X} , observe that $X_1 + X_2$ is minimized when the constraints (5) and $X_2 \geq 0$ are binding, hence

$$\underline{X} = \frac{1 - 2c\eta^*}{2 - \mu}$$

Since in each period, there is a probability β that the current round terminates, we also have: $\eta^* \geq 1 + \beta\tau$, which concludes the proof of Proposition 4 ■

5 Discussion.

In essence, gradualism in our model stems from changes in preferences that arise due to offers received in the past. Though we have emphasized the role of reference points, one could imagine other source of changes in preferences.

We provide here an example, in which changes in preferences would be driven by changes in aspiration levels. (See Yukl (1974) and White and Neale (1994) for experiments concerning the effect of offers on aspiration levels, and the effect of aspirations on negotiation outcomes).

Denote by a_i player i 's aspiration level. Assume that the utility that player i derives from the partition offer $y = (y_i, y_j)$ depends both on the value of y_i and on how this value compares to his aspiration level a_i :

$$v_j(y, a_j) = (1 - \alpha)y_i + \alpha(y_i - a_i).$$

where $\alpha \in (0, 1)$ captures the weight the agent puts on his aspiration level. Note that this formalization of preferences would coincide with that of Section 2, with the reference point being chosen equal to a fraction α of the aspiration level.

Changes in aspiration level induce change in preferences. Various assumptions concerning these changes seem plausible. One may for example assume that a party's aspiration level adjust as a function of the most generous offer received. One may also assume that a party's aspiration level is a function of the most generous offers received by each party, say a function of the payoff that would result from splitting the difference between most generous offers.

In both these extensions, a generous offer made to the other party has the effect of increasing the opponent's aspiration level in case breakdown occurs. Because a player with higher aspirations ends up with a larger share of the pie, players should be cautious and not start with too generous offers: In a symmetric equilibrium, one should expect aspiration levels to remain balanced throughout the bargaining game. A complete analysis of these extensions deserves further research.

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Appendix

Proof of Lemma 1: Consider a history h in game II, leading to a date t_0 in round n_0 .

Denote by $(Y_1^{(1)}, Y_2^{(1)}), \dots, (Y_1^{(n_0-1)}, Y_2^{(n_0-1)})$ the sequence of most generous offers made in the phases preceding phase n_0 . To this history of game II corresponds a history \tilde{h} of game I in which all offers are translated by (r_1, r_2) . At date t_0 of game I, after history \tilde{h} , players are in round $n_0 + n$. Denote by $(\tilde{Y}_1^{(n)}, \tilde{Y}_2^{(n)}), \dots, (\tilde{Y}_1^{(n+n_0-1)}, \tilde{Y}_2^{(n+n_0-1)})$ the sequence of most generous offers made in rounds n to $n + n_0 - 1$. We show below that the strategy profile σ yields the same payoffs in the subgame of game II starting after h as the strategy profile $\tilde{\sigma}$ in the subgame of game I starting after \tilde{h} .

Given h , consider a realization of σ for which (y_1, y_2) is accepted in round $m (\geq n_0)$ at t . To this realization of σ corresponds a realization of $\tilde{\sigma}$ for which, following \tilde{h} , $(r_1 + y_1, r_1 + y_2)$ is accepted in round $m + n$ at t .

Note first that

$$\tilde{Y}_i^{(n+k-1)} = Y_i^{(k)} + r_i.$$

Denote by $r_i^{(k)}$ (respectively $\tilde{r}_i^{(n+k-1)}$) the reference point induced by h in round k (respectively by \tilde{h} in round $n + k - 1$). The law of evolution of reference points implies

$$r_i^{(k)} = r_i^{(k-1)} + \mu(Y_i^{(k)} - r_i^{(k-1)})$$

and

$$\tilde{r}_i^{(n+k-1)} = \tilde{r}_i^{(n+k-2)} + \mu(\tilde{Y}_i^{(n+k-1)} - \tilde{r}_i^{(n+k-2)})$$

Since, for $k = 1$, $r_i^{(k)} = 0$ and $\tilde{r}_i^{(n+k-1)} = r_i$, we obtain by induction on k that the equality

$$\tilde{r}_i^{(n+k-1)} - r_i^{(k)} = r_i \tag{6}$$

holds for all $k = 1, \dots, n_0$.

Now after history h in game *II*, the realization considered yields a payoff equal to $y_i - r_i^{(n_0)} - nc$ to player i , while after history \tilde{h} in game *II*, the realization considered yields a payoff equal to $y_i + r_i - \tilde{r}_i^{(n+n_0-1)} - nc$ to player i . By (6), these two payoffs coincide. ■

Proof of Lemma 2: Consider any given recursive symmetric monotonic equilibrium σ . Given player i 's proposed strategy, the current phase will eventually terminate without an agreement being reached. A new phase will then start. Let r_1 and r_2 be the new reference points in that phase. We have $r_i \geq f(X_i^t)$ and $r_j = f(X_j^t)$.

Consider now the subgame that starts at this new phase, and denote by σ' the equilibrium strategies induced by σ in that subgame. Recall that $\sigma(K)$ is the equilibrium strategy profile for the game in which the size of the pie is K . By recursiveness, the strategy profile σ' coincides with $\sigma(K')$ where $K' = K - r_1 - r_2$, up to a translation of (r_1, r_2) .

Consider now the equilibrium $\sigma(K')$, and denote by ω a possible realization induced by $\sigma(K')$. To each realization ω corresponds a number of phase $n(\omega)$ after which agreement occurs and a final partition $(x_1(\omega), x_2(\omega))$. By symmetry, to each realization ω we may associate another realization ω' which has same probability and for which

$$n(\omega) = n(\omega'), \text{ and } x_1(\omega') = x_2(\omega) = K' - x_1(\omega)$$

It follows that player i 's expected payoff is equal to

$$E_{\sigma(K')} x_1(\omega) - (n(\omega) - 1)c = \frac{K'}{2} - E_{\sigma(K')} (n(\omega) - 1)c = \frac{K'}{2} - (\eta(K') - 1)c,$$

hence, from the perspective of the original subgame, the equilibrium strategy profile σ' generates for player i an expected payoff equal to

$$r_i + \frac{K'}{2} - (\eta(K') - 1)c - c,$$

which, since $\eta(\cdot)$ is non decreasing, concludes the proof. ■

An explicit construction of a symmetric recursive monotonic equilibrium. We examine the case where $f(x) = x$, and provide an explicit construction of a (recursive) equilibrium.

Consider an increasing sequence $\chi = (X^{(1)}, \dots, X^{(m)}, \dots)$, with $X^{(1)} > 0$, and define a strategy profile σ^χ associated to it. Recall that X_i^t denotes the current most generous offer received by player i . We consider a pie of size K and define $X^t = K - X_1^t - X_2^t$. X^t measures the distance between most generous offers.

Let m be such that $X^{(m)} < K \leq X^{(m+1)}$. If player i is drawn to start, then player i starts by offering $K - X^{(m)}$. At any later date t , and so long as breakdown did not occur:

1. If $X^t \in (X^{(m-2k-1)}, X^{(m-2k)}]$ then: if it is player i 's turn to move, he sticks to his current most generous offer X_i^t , while if is player j 's turn to move, she makes a more generous offer $X_i^t + (X_t - X^{(m-2k-1)})$.
2. If $X^t \in (X^{(m-2k)}, X^{(m-2k+1)}]$ then: if it is player j 's turn to move, she sticks to his current most generous offer X_j^t , while if is player i 's turn to move, he makes a more generous offer $X_j^t + (X_t - X^{(m-2k)})$.

If breakdown occurs after date t offer, reference points move to $r_1 = X_1^t$ and $r_2 = X_2^t$, and continuation strategies are determined by the usual translation, using the strategy profile $\sigma^\chi(X^t)$.

The path of offers induced by the proposed strategy profile is such that the distance between most generous offers gradually reduce:

$$X^1 = K, X^2 = X^{(m)}, \dots, X^t = X^{(m-t+2)}, \dots, X^{m+2} = 0.$$

Note that by construction, this same path over X^t is generated, whether breakdown occurs or not.

Our objective is to derive a sequence χ for which σ^χ is a monotone recursive symmetric equilibrium.

Assume $X = X^{(m)}$ and player i is supposed to move and make a more generous offer. We let $\bar{V}^{(m)}$ denote the expected gain in excess of X_j obtained by j under σ^X , and let $\underline{V}^{(m)}$ denote the expected gain in excess to X_i obtained by i under σ^X . By construction, for a fixed $X = K - X_1 - X_2$, these expected gains are independent of (X_1, X_2) .

We have the following relationships:

$$\begin{aligned}\underline{V}^{(m+1)} &= (1 - \beta)\bar{V}^{(m)} + \beta\left[\frac{\bar{V}^{(m)} + \underline{V}^{(m)}}{2} - c\right], \text{ and} \\ \bar{V}^{(m+1)} &= (X^{(m+1)} - X^{(m)}) + (1 - \beta)\underline{V}^{(m)} + \beta\left[\frac{\bar{V}^{(m)} + \underline{V}^{(m)}}{2} - c\right]\end{aligned}$$

which implies:

$$X^{(m+1)} - X^{(m)} = (\bar{V}^{(m+1)} - \underline{V}^{(m+1)}) + (1 - \beta)(\bar{V}^{(m)} - \underline{V}^{(m)})$$

To check incentives, we consider the most unfavorable case for say player i , that where he is supposed to make a more generous offer at $X = X^{(m)}$. If player i conforms, he obtains $\underline{V}^{(m)}$ (in addition to X_i), while if he decides never to make a more generous offer, breakdown will eventually occur. This costs c , but he will then get either $\bar{V}^{(m)}$ or $\underline{V}^{(m)}$ (in addition to X_i), depending on the identity of the player drawn at the start of the new phase. So we must have:

$$\underline{V}^{(m)} \geq \frac{\bar{V}^{(m)} + \underline{V}^{(m)}}{2} - c$$

or equivalently,

$$\bar{V}^{(m)} - \underline{V}^{(m)} \leq 2c \tag{7}$$

We choose the sequence χ so that (7) is binding, which gives $X^{(1)} = 2c$ and for any $m \geq 1$,

$$X^{(m+1)} - X^{(m)} = 2c(2 - \beta)$$

For this choice of χ , σ^X is a monotone recursive symmetric equilibrium.³

³ σ^X has to be amended slightly once $X \leq X^{(0)}$ with $0 = (1 - \beta)X^{(0)} + \beta(X^{(0)}/2 - c)$: in that region, both players are willing to offer what the other party demands immediately.

The number of rounds in equilibrium is approximately $1/(2c(2 - \beta))$, and the expected number of phase is at least $\beta/(2c(2 - \beta))$.