On the virtues of the ascending price auction: New insights in the private value setting*

Olivier Compte† and Philippe Jehiel‡

First version: September 2000

This version: December 2000

Abstract

This paper shows that in a private value setting in which one bidder may at any time refine her assessment of her valuation at some positive cost, the ascending price auction induces higher expected welfare than the sealed-bid second price auction does. When the number of bidders is above a threshold, it generates higher revenue as well. In the ascending price auction, a key feature of equilibrium behavior is that as long as there are more than two bidders left, the bidder who may refine her information (and has not done so yet) stays above her expected current valuation. This is because she has the option to acquire information when there are two bidders left, and drop out at no cost if her realized valuation turns out to be low. In contrast, in the sealed-bid format, there is no such option and that bidder will bid her expected valuation.

*We thank seminar participants at CORE for useful comments.

†C.E.R.A.S.-E.N.P.C., C.N.R.S. (URA 2036), 28 rue des Saints-Pères 75007 Paris France. e-mail: compte@enpc.fr

‡C.E.R.A.S.-E.N.P.C., C.N.R.S. (URA 2036), 28 rue des Saints-Pères 75007 Paris France. e-mail: jehiel@enpc.fr and University College London, e-mail: p.jehiel@ucl.ac.uk.

Key words: auctions, private value, information acquisition, option value.

1 Introduction

Consider a single object auction. Each bidder has some private information about how much he values the object. Bidders' valuations are not affected by the information held by other bidders. That is, we are in the private value setting.

The ascending price auction and the sealed-bid second price (or Vickrey auction) both yield the same outcome in this setting. In both formats, bidders have a (weakly) dominant strategy: drop out when the price reaches the valuation in the ascending format, and bid the valuation in the Vickrey format. They are thus equivalent auctions. Note that this true whether or not bidders are ex ante symmetric.

Consider now a slight modification of the above setup. We still consider the private value setting; that is, the private information held by a bidder does not affect the valuation of other bidders. But, we now assume that one bidder, say bidder 1, may acquire a better information about her valuation at some cost c.

We show that the ascending price auction and the sealed-bid second price auction are no longer equivalent formats despite the fact that we are considering the private value setting. The *expected welfare* generated in the ascending format is higher than that generated in the sealed-bid format. And when the number of bidders is above a threshold, the *expected revenue* generated is also higher in the ascending format.

The main reason for this result is as follows: the sealed-bid format forces bidder 1 to decide whether or not to refine her assessment of her valuation without having any information on how much the other bidders value the object. In contrast, in the ascending format, bidder 1 has the option to wait and stay in until there are only two bidders left to acquire information (and possibly learn that she values the object much more than the remaining bidder does). As a result, bidder 1 decides on better grounds whether it is worth acquiring further information on the valuation. Thus, the ascending format permits a better information acquisition strategy, which results in higher expected welfare.

When the number of bidders is too large, it is not optimal for bidder 1 to acquire information in the sealed-bid format, because the chance of getting the object is very small. Bidder 1 therefore bids her initial expected valuation. In contrast, in the ascending format, bidder 1 can wait until there are only two bidders left to acquire information. So in the ascending format bidder 1's information acquisition strategy is independent of the total number of bidders, and (to the extent that acquisition costs are not too large) bidder 1 acquires information with positive probability, independently of the number of bidders. Besides, when information acquisition costs are not too large, it is worthwhile for bidder 1 to acquire information even if the current price exceeds her initial expected valuation. So because bidder 1 waits until there are only two bidders left to acquire information, it may well be that bidder 1 stays in well above her initial expected valuation. And when the number of bidders is large, it is actually most likely that in events where she acquires information, bidder 1 stays in well above her initial expected value.[1] As a result, in events where she acquires information, either she learns that her valuation is small and drops out immediately, but this does not adversely affect revenues compared to the second price auction (because the current price is most likely to lie above bidder 1's initial expected valuation); or she

[1] This is because as the number of bidders get larger, the price at which only two bidders remain active tends to increase.

learns that her valuation is higher than the current price, in which case the effect on revenues is positive.

Related literature: Our paper is related to two strands of literature in auction theory, i.e. the comparison of auction formats (and more precisely here the comparison of the second price and ascending price auction formats), and the analysis of information acquisition in auctions. To the best of our knowledge, this paper is the first attempt to analyze the issue of information acquisition in the ascending auction format.

The non-equivalence of the second price auction and the ascending price auction has been noted in the affiliated value paradigm (see Milgrom-Weber 1982). There the two formats differ because the information conveyed about others' signals are not the same in the two formats and therefore the assessment of the valuation is not the same. Milgrom-Weber (1982) consider a symmetric setup and show how, in the affiliated value paradigm, the ascending format may generate higher expected revenue. Our paper can be viewed as providing a new explanation as to why the ascending format generates higher expected revenue.

Maskin (1992) also considers an interdependent value setup and shows in the two-bidder case that the ascending price auction generates an efficient outcome even when bidders are not ex ante symmetric, as long as bidders have one-dimensional signals and a single crossing condition holds.[2] But, with two bidders, the ascending format and the sealed-bid format yield the same outcome, so they cannot be compared.[3]

[2] See Dasgupta-Maskin 1999, Ausubel 1999, Perry-Reny 2000 for extensions of this kind of results and Jehiel-Moldovanu 2000 for limits of it in the multidimensional private information case.

[3] See Krishna 2000 for an investigation of when the ascending price auction generates an efficient outcome in this kind of contexts and Compte-Jehiel 2001 for illustrations of the efficiency differences between the second price and ascending price auctions in

Other comparisons between auction formats have been made under the assumption that bidders are risk averse. Yet, the second price format and the ascending format are still equivalent in the private value setting even if bidders are risk averse. In the context of auctions with negative externalities (see Jehiel-Moldovanu 1996), Das Varma (1999) has shown that the ascending format could (under some conditions) generate higher revenues than that generated by the sealed-bid (first-price) auction format, because in the ascending format a bidder can stay longer to be able to combat a harmful competitor if that is the remaining bidder.

The literature on information acquisition in auctions is restricted to sealed-bid types of auction mechanisms. In a private value model, Hausch and Li (1991) show that first price and second price auctions are equivalent in a symmetric setting (see also Tan 1992). Stegeman (1996) shows that second price auction induces an ex ante efficient information acquisition in the single unit independent private values case (see also Bergemann and Valimaki 2000). However, as our paper shows, the ascending price auction may induce an even greater level of expected welfare in this case.

Models of information acquisition in interdependent value contexts (in static mechanisms) include Matthews (1977), (1984) who analyzes in a pure common values context whether the value of the winning bid converges to the true value of the object as the number of bidders gets large,[4] Persico (1999) who compares information acquisition incentives in the first price and second price auction in the affiliated value setting and Bergemann and Valimaki (2000) who investigate, in a general interdependent value context, the impact of ex post efficiency on the ex ante incentive of information acquisiton (however, even if ex ante and ex post efficiency is achieved in Bergemann-

interdependent valuation contexts with possibly multi-dimensional private information.

[4] See also Hausch and Li 1993 for an analysis of information acquisition in common value settings.

Valimaki's sense this need not imply that the most efficient mechanism is obtained, see the efficiency analysis in Section 3).

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 provides the analysis of the ascending price auction and the second price auction as well as revenue and efficiency comparisons between the two formats. Some discussion follows in Section 4.

2 The model

There is one object for sale. We consider n potential risk-neutral bidders $i \in N = \{1, ..., n\}$. When a bidder does not get the object, he gets a payoff normalized to zero.

Each bidder $i = 1, ..., n$ has a valuation θ_i for the object. The valuations θ_i , $i = 1, ...n$ are realizations of the random variables $\tilde{\theta}_i$, $i = 1, ...n$, which are assumed to be independently distributed from each other. Each random variable $\tilde{\theta}_i$ has a density denoted by $g_i(\cdot)$, defined over $\left[\underline{\theta}, \overline{\theta}\right]$. We will assume that $g_i(\theta) > 0$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

We assume that each bidder $i = 2, ..., n$ knows his own valuation, whereas bidder 1 is only imperfectly informed about θ_1. More precisely, we assume that bidder 1 observes the realization α of a signal that is imperfectly correlated with her valuation θ_1, and independent of other bidders' valuations. We denote by $f(\cdot, \cdot)$ the density over (θ_1, α). We assume that f is defined over $\left[\underline{\theta}, \overline{\theta}\right] \times [\underline{\alpha}, \overline{\alpha}]$ and that $f(\theta, \alpha) > 0$ for all $(\theta, \alpha) \in \left[\underline{\theta}, \overline{\theta}\right] \times [\underline{\alpha}, \overline{\alpha}]$.

Also, for simplicity and in order to highlight comparative statics with respect to the number n of bidders, we assume that the random variables $\tilde{\theta}_i$, $i = 2, ...n$ are drawn from the same distribution. Accordingly, we denote by $g(\cdot)$ the common density $g_i(\cdot)$ of every bidder $i = 2, ...n$.

At any point in time, bidder 1 may decide to learn the realization of θ_1. This costs her $c > 0$.

The information structure is assumed to be common knowledge among all bidders.

Auction format: We will mostly consider the ascending price auction. At some point we will make some comparisons with the sealed-bid second price auction.

The ascending price auction is defined as follows.[5] The price starts at a low level, say 0, at which each bidder is present. The price gradually increases. Each bidder may decide to quit at every moment. Bidder 1 may also decide to learn θ_1 at every moment. When a bidder quits, this is commonly observed by every bidder.[6] The auction stops when there is only one bidder left. The object is allocated to that bidder at the current price.

A strategy for bidder $i = 2, ..n$ specifies for each current price p, private information (valuation θ_i) and public information (who left and at what price) whether or not to drop out.[7]

A strategy for bidder 1 specifies bidder 1's behavior depending on whether or not bidder 1 has acquired information about θ_1. If bidder 1 has learned θ_1, her strategy is defined in the same way as for bidders $2, ...n$. If bidder 1 has not yet learned θ_1, her strategy specifies for each current price p, private information (α) and public information (who left and at what price) whether or not to drop out and whether or not to acquire information (about θ_1).

[5] We present here the continuous time/price version of the ascending price auction. This raises some technical difficulties regarding the definition of equilibria in undominated strategies. The equilibria we will refer to are the limits as $\varepsilon > 0$ tends to 0 of the equilibria in undominated strategies of the corresponding game in which time is discrete and after each round the price increases by the increment ε.

[6] It is immaterial whether or not bidders $i = 2, ...n$ observe whether or not bidder 1 learns θ_1. However, it is clearly more realistic to assume that they do not.

[7] In case all the remaining bidders quit at the same date, one of them is selected at random with equal probability to get the object. He then pays the current price.

The sealed-bid second price auction is defined as follows. Each bidder i simultaneously sends a bid b_i to the seller. The bidder with maximal bid, i.e. $i_0 = \arg\max_i b_i$ gets the good and pays the second highest bid, i.e. $\max_{i \neq i_0} b_i$ to the seller.[8]

In the sealed-bid format, bidder i ($i = 2, ...n$)'s strategy consists in submitting a bid b_i as a function of θ_i. Bidder 1's strategy specifies whether or not to acquire information as a function of α, and given the information at the bidding stage, which bid b_1 to submit.

3 On the virtue of the ascending price auction

3.1 The ascending price auction

Consider the ascending price auction. We first observe that bidders $i = 2, ...n$ have a dominant strategy: drop out at their valuation θ_i.

The key issue is whether and when bidder 1 decides to refine her assessment of her valuation. We need some preliminary definitions. We first define for each $\alpha \in [\underline{\alpha}, \overline{\alpha}]$:

$$V(\alpha) \equiv E(\tilde{\theta}_1 \mid \alpha) \tag{1}$$

to be the expected valuation of bidder 1 given the realization α. This is bidder 1's expected valuation at the start of the game.

We next define for each $(p, \alpha) \in \left[\underline{\theta}, \overline{\theta}\right] \times [\underline{\alpha}, \overline{\alpha}]$:

$$H(p, \alpha) \equiv E(\max(\tilde{\theta}_1, \tilde{\theta}_2) - \tilde{\theta}_2 \mid \alpha \text{ and } \tilde{\theta}_2 \geq p) - c.$$

and

$$K(p, \alpha) = E(\max(V(\alpha), \tilde{\theta}_2) - \tilde{\theta}_2 \mid \tilde{\theta}_2 \geq p).$$

[8] If there are several bidders with maximal bids, one of them is selected at random with equal probability to get the good, and pays that bid to the seller.

The value $H(p,\alpha)$ (respectively $K(p,\alpha)$) corresponds to bidder 1's expected payoff when the current price is equal to p, only bidder 2 remains active (and will remain active up to his valuation), and bidder 1 decides to acquire information (respectively decides not to acquire information ever, and drop out at $\min\{p, V(\alpha)\}$). For each $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, we define:

$$p^*(\alpha) \equiv \sup_p \left\{ p \in \left[\underline{\theta}, \overline{\theta}\right] \mid H(p,\alpha) \geq K(p,\alpha) \text{ or } p = \underline{\theta} \right\}. \tag{2}$$

We also define for each $(p,\alpha) \in \left[\underline{\theta}, \overline{\theta}\right] \times [\underline{\alpha}, \overline{\alpha}]$:

$$G(p,\alpha) \equiv E(\max(p, \theta_1) - \theta_1 \mid \alpha),$$

and we let

$$p^{**}(\alpha) \equiv \inf_p \left\{ p \in \left[\underline{\theta}, \overline{\theta}\right] \mid G(p,\alpha) - c \geq 0 \text{ or } p = \overline{\theta} \right\} \tag{3}$$

Finally, we denote by $N(p)$ the total number of remaining bidders when the current price is equal to p. Equilibrium behavior is characterized as follows:

Theorem 1 *Consider the ascending price auction. The strategies defined below constitute the unique perfect Bayesian equilibrium in undominated strategies. Bidder $i = 2, ...n$ with valuation θ_i drops out when the price reaches θ_i. Assume the current price is p and that bidder 1 has not acquired information about θ_1.*

If $N(p) = 3$, bidder 1 does not acquire information; she stays in if $p < \max\{p^*(\alpha), V(\alpha)\}$, and she drops out otherwise.

If $N(p) = 2$, we distinguish two cases:

(i) $p^*(\alpha) \leq p^{**}(\alpha)$: then bidder 1 with initial private information α never acquires information and drops out when the price reaches $V(\alpha)$.

(ii) $p^{**}(\alpha) < p^*(\alpha)$: then bidder 1 acquires information only if $p \in [p^{**}(\alpha), p^*(\alpha))$, she drops out at $\min(p, V(\alpha))$ if $p \geq p^*(\alpha)$, and she stays in and acquires

information at $p^{**}(\alpha)$ if $p < p^{**}(\alpha)$.
Finally, when bidder 1 has acquired information about θ_1 and the current price is p, bidder 1 drops out immediately whenever $\theta_1 < p$ and stays in till the price reaches θ_1 otherwise.

The notable feature of the equilibrium is that $p^*(\alpha)$ may lie well above $V(\alpha)$ and therefore, in many cases bidder 1 stays in much above her expected initial valuation, even though she has not yet acquired information about θ_1.

The reason why she does so is that as long as there are more than two bidders left, she still has the option of acquiring information about θ_1 when there are two bidders left. Moreover when there are two bidders left, and bidder 1 learns that her valuation θ_1 is low (typically below the current price), she does not suffer because she can still drop out before the remaining bidder does.

An Example.

To illustrate the behavior of bidder 1 implied by Theorem 1, we provide a simple example:

Example 1 *Suppose θ_i, $i = 2, ...n$ is uniformly distributed on $\left[\underline{\theta}, \overline{\theta}\right]$. Conditional on α, θ_1 is assumed to take value $\overline{\theta}$ with probability α and $\underline{\theta}$ with probability $1-\alpha$ so that $V(\alpha) = \alpha\overline{\theta} + (1-\alpha)\underline{\theta}$. The variable α is assumed to be uniformly distributed on $[\underline{\alpha}, \overline{\alpha}]$ where $0 < \underline{\alpha} < \overline{\alpha} < 1$ and c is assumed to be sufficiently small (i.e., $\underline{\theta} + \frac{c}{1-\alpha} < V(\alpha) < \overline{\theta} - \frac{2c}{\alpha}$ for all $\alpha \in [\underline{\alpha}, \overline{\alpha}]$).*

Under the assumptions above, we have $G(p, \alpha) = (1-\alpha)(p - \bar{\theta})$, which implies

$$p^{**}(\alpha) = \underline{\theta} + \frac{c}{1-\alpha}.$$

We also have $H(p, \alpha) = \alpha \left[\overline{\theta} - \frac{\overline{\theta}+p}{2}\right] - c$, which implies that $H(V(\alpha), \alpha) > 0$.

So $p^*(\alpha)$ solves $H(p,\alpha) = 0$, that is,

$$p^*(\alpha) = \overline{\theta} - \frac{2c}{\alpha}.$$

which implies that for all $\alpha \in [\underline{\alpha}, \overline{\alpha}]$:

$$p^{**}(\alpha) < V(\alpha) < p^*(\alpha).$$

By Theorem 1, the equilibrium behavior of bidder 1 with initial private information α is thus determined as follows:

(i) As long there are no less than three bidders left, $N \geq 3$, bidder 1 does not acquire information on θ_1, and she stays in till the price reaches $p^*(\alpha)$ at which price she drops out.

(ii) As soon as there are two bidders left $N = 2$ (and thus the current price p must lie below $p^*(\alpha)$ given the above behavior), bidder 1 acquires information on θ_1 if $p > p^{**}(\alpha)$ and stays in without acquiring information (at p) if $p < p^{**}(\alpha)$; she drops out if she learns that $\theta_1 = \underline{\theta}$. She stays in till the remaining other bidder drops out if $\theta_1 = \overline{\theta}$.

The argument.

Before starting the proof of Theorem 1, we gather preliminary observations in the following Lemma.

Lemma 1 *For any $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, we have:*
(i) $H(.,\alpha)$ is decreasing (in p), $p^*(\alpha) < \bar{\theta}$, and $H(p^*(\alpha),\alpha) \geq 0$.
(ii) $G(.,\alpha)$ is increasing (in p).
(iii) $H(p,\alpha) \leq G(p,\alpha) - c + V(\alpha) - p$
(iv) If $p^{**}(\alpha) < (\leq) p^*(\alpha)$, then $\quad p^{**}(\alpha) < (\leq) V(\alpha)$

Proof. Define

$$\phi(p,\alpha) = E[\max(\theta_1, p) - p \mid \alpha] = \int_{\theta_1 \geq p} (\theta_1 - p) f(\theta_1 \mid \alpha) d\theta_1.$$

We have

$$\frac{\partial \phi}{\partial p}(p, \alpha) = -\Pr\{\theta_1 \geq p \mid \alpha\} \tag{4}$$

By definition of H, we have:

$$H(p, \alpha) = \frac{\int_{\theta_2 \geq p} \phi(\theta_2, \alpha) g(\theta_2) d\theta_2}{\int_{\theta_2 \geq p} g(\theta_2) d\theta_2} - c$$

Since $\frac{\partial \phi}{\partial p} < 0$ when $p \in (\underline{\theta}, \bar{\theta})$, we obtain, for any $p < \bar{\theta}$,

$$H(p, \alpha) < \phi(p, \alpha) - c. \tag{5}$$

Besides, it is easy to check that

$$\frac{\partial H}{\partial p}(p, \alpha) = \frac{g(p)}{\int_{\theta_2 \geq p} g(\theta_2) d\theta_2} [H(p, \alpha) - \phi(p, \alpha) + c].$$

which implies that $H(., \alpha)$ is decreasing. Besides $H(\bar{\theta}, \alpha) = -c < 0$ and $K(p, \alpha) \geq 0$ for all $p \in [\underline{\theta}, \bar{\theta}]$. Hence by definition of $p^*(\alpha)$, we have $p^*(\alpha) < \bar{\theta}$ and $H(p^*(\alpha), \alpha) \geq 0$, which concludes (i).

Now by definition of G, we have

$$G(p, \alpha) = p - V(\alpha) + \phi(p, \alpha),$$

which implies (ii) (from Equation (4)), and (iii) (thanks to inequality (5)). Then (iv) follows because if $p^{**}(\alpha) < p^*(\alpha)$, then

$$H(p^*(\alpha), \alpha) < H(p^{**}(\alpha), \alpha) < G(p^{**}(\alpha), \alpha) - c + V(\alpha) - p^{**}(\alpha)$$

and because $H(p^*(\alpha), \alpha) \geq 0$ and $G(p^{**}(\alpha), \alpha) = c$. ■

We are now ready to start the proof of Theorem 1, which is made in several steps articulated as 7 lemmas. The first two lemmas are straightforward and do not require any proof.

Lemma 2 *Bidder $i = 2, ...n$ has a (weakly) dominant strategy, i.e., drop out when the price reaches his valuation θ_i.*

Lemma 3 *Suppose the current price is p, bidder 1 is still in and she has learned the realization of θ_1. Then bidder 1 has a (weakly) dominant strategy: she drops out immediately if $\theta_1 < p$, and she stays in till the price reaches θ_1 otherwise.*

In what follows, we assume that bidders behave according to Lemma 2 and 3, and we derive the optimal behavior of bidder 1 in the event where she has not yet acquired information.

Lemma 4 (key step) *Suppose bidder 1 with private information α has not learned θ_1, the current price is $p < \max(p^*(\alpha), V(\alpha))$ and there are N bidders left where $N > 2$. Then it is not optimal for bidder 1 to drop out (at p).*

Proof. Clearly if $p < V(\alpha)$, bidder 1 does not drop out, since she strictly prefers staying till the price reaches $V(\alpha)$ and then dropping out (without ever acquiring information on θ_1). (The strict preference derives from the behavior of bidders $i = 2, ...n$, and the assumption that $g(\cdot)$ is strictly positive on $\left[\underline{\theta}, \overline{\theta}\right]$.)

Suppose $p^*(\alpha) > V(\alpha)$, let the current price p be such that $V(\alpha) < p < p^*(\alpha)$, and assume there are $N > 2$ bidders left. Assume (contrary to the claim of the Lemma) that bidder 1 drops out at p.

Then we claim that she has a strictly better strategy: consider ε small enough so that $p + \varepsilon < p^*(\alpha)$, and assume that bidder 1 waits till the price reaches $p+\varepsilon$ or some price $p' \leq p+\varepsilon$ at which $N(p') = 2$.[9] Then drop out if $N(p+\varepsilon) \geq 3$, or acquire information at p' and behave as specified in Lemma 2.

[9] The event under which the number of remaining bidders jump directly to 1 has 0 probability (because g has no mass point). [In the discrete jump game that approximates the continuous time game, the probability of this event gets arbitrarily small as price increments tend to 0.]

When bidder 1 follows the above strategy (instead of dropping out) and when the other bidders conform to the strategies specified in Lemma 2, the event where $N(p') = 2$ with $p' \in (p, p+\varepsilon]$ has positive probability (because g is positive). In any such event, bidder 1 acquires information at p' and since $N(p') = 2$, she obtains an expected payoff equal to $H(p', \alpha)$. This payoff is at least equal to $H(p+\varepsilon, \alpha)$ because $H(\cdot, \alpha)$ is decreasing, and $H(p+\varepsilon, \alpha)$ is positive because $p+\varepsilon < p^*(\alpha)$, because $H(p^*(\alpha), \alpha) \geq 0$, and because $H(\cdot, \alpha)$ is decreasing (see Lemma 1). So this strategy is preferred to dropping out. ■

Lemma 5 *Suppose there are* N, $N > 2$, *bidders left. Then it is not optimal for bidder* 1 *to acquire information (That is, she acquires information only when there are two bidders left.).*

Proof. It will be convenient to denote by $\theta^{(2)}$ the second largest valuation among bidders other than bidder 1, and by $\boldsymbol{\theta}^{(2)}$ the corresponding random variable.

Suppose there are $N > 2$ bidders left and suppose bidder 1 chooses to acquire information on θ_1 and the price is $p \leq \max(p^*(\alpha), V(\alpha))$. (If the price were larger, bidder 1 would have dropped earlier by the previous Lemma.)

We show that it is a strictly better strategy for bidder 1 (i) to drop out at price $\max(p^*(\alpha), V(\alpha))$ if there are still three or more bidders left, and (ii) otherwise to acquire information as soon as there are two bidders left.

Under the latter event (that is if $\theta^{(2)} \leq \max(p^*(\alpha), V(\alpha))$), the two strategies yield the same payoff. Under the former event (which has positive probability because $p^*(\alpha) < \overline{\theta}$ (by Lemma 1 (i) and because $V(\alpha) < \overline{\theta}$), bidder 1 obtains 0 when she drops out, and she obtains an expected payoff equal to

$$E\left[H(\boldsymbol{\theta}^{(2)}, \alpha) \,\middle|\, \boldsymbol{\theta}^{(2)} \geq \max(p^*(\alpha), V(\alpha))\right]$$

when she acquire information at p. And this payoff is negative because for any $p \geq (>) \max(p^*(\alpha), V(\alpha))$, $K(p,\alpha) = 0$ and $H(p,\alpha) \leq (<) K(p,\alpha)$ (by definition of $p^*(\alpha)$), and because the event $\theta^{(2)} > p^*(\alpha)$ has positive probability. ■

Lemma 6 *Suppose there are $N > 2$ bidders left and the current price is $p \geq \max(p^*(\alpha), V(\alpha))$. Then it is (strictly) optimal for bidder 1 to drop out.*

Proof. Suppose that the current price is $p \geq \max(p^*(\alpha), V(\alpha))$ and that bidder 1 does not drop out. Then either she will never acquire (in any subgame) the information on θ_1 in which case she would certainly be better off by dropping out at p (since $p \geq V(\alpha)$). Or she will acquire information at some price $p' > p$. But the best she can hope to get in expectation by acquiring information at price p' is $H(p', \alpha)$ (if there are more than two bidders left at the price p' at which bidder 1 acquires information, then bidder 1 gets even less). This expected payoff however is negative because (by definition of $p^*(\alpha)$) $H(p', \alpha) < K(p', \alpha)$, and because for any $p' \geq V(\alpha)$, $K(p', \alpha) = 0$. ■

Lemma 7 *Whatever the number of bidders left, it is not optimal for bidder 1 with private information α to drop out at price $p < V(\alpha)$.*

Proof. Dropping out at $p < V(\alpha)$ with no information acquisition is dominated by waiting till $V(\alpha)$ without acquiring any information ever. ■

Lemma 8 *When there are $N = 2$ bidders left, then:*
(a) if $p > p^*(\alpha)$ or $p < p^{**}(\alpha)$, it is not optimal for bidder 1 with private information α to acquire information.
(b) Otherwise, it is optimal for bidder 1 to acquire information.

Proof. Acquiring information at p yields $H(p, \alpha)$. If $p > p^*(\alpha)$, $H(p, \alpha) < K(p, \alpha)$, hence acquiring information is worse than dropping out at $\min(p, V(\alpha))$.

We show next that if $p < p^{**}(\alpha)$, bidder 1 would rather acquire information at $p + \varepsilon$ than at p.

When bidder 1 acquires information at $p + \varepsilon$ (rather than p), bidder 1's expected payoff is unchanged when the other bidder (say 2)'s valuation is above $p + \varepsilon$. In the event where $\{\theta_2 \in (p, p + \varepsilon)\}$, bidder 1 obtains the object at a price at most equal to $p + \varepsilon$. Hence her expected payoff, conditional on this event, is at least equal to

$$V(\alpha) - p - \varepsilon.$$

When she acquires at p, then conditional on that same event, she obtains an expected payoff at most equal to

$$E(\max(p, \tilde{\theta}_1) - p - c \mid \alpha) = G(p, \alpha) - p + V(\alpha) - c.$$

When $p < p^{**}(\alpha)$, $G(p, \alpha) - c < 0$, and since ε can be chosen very small, bidder 1 strictly prefers to acquire information at $p + \varepsilon$.

By a similar argument, one shows that if it is optimal for bidder 1 to acquire information at some $p' > p^{**}(\alpha)$, then it is optimal for bidder 1 to acquire information at any $p \in [p^{**}(\alpha), p']$.

To conclude, we show that if $p = p^*(\alpha)$, it is optimal for bidder 1 to acquire information. If bidder 1 does not acquire information at p, then (by (a)) it will never be optimal for bidder 1 to acquire information. So it is optimal for bidder 1 to drop out at $\min\{p, V(\alpha)\}$, which gives her an expected payoff equal to $K(p, \alpha)$. Since by definition of $p^*(\alpha)$, $K(p, \alpha) = H(p, \alpha)$, it is also optimal for bidder 1 to acquire information. ■

3.2 Comparison with the second price auction

In what follows we make efficiency and revenue comparisons between the ascending price and the second price auction. We start by characterizing equilibrium behavior in the second price auction.

It will be convenient to denote by $\theta^{(1)}$ (respectively $\theta^{(2)}$) the largest (respectively second largest) valuation among bidders other than bidder 1, and by $\tilde{\theta}^{(1)}$ and $\tilde{\theta}^{(2)}$ the corresponding random variables. Let α be bidder 1's initial private information. Bidder 1's expected payoff if she acquires information on θ_1 is:

$$E(\max(\tilde{\theta}_1, \tilde{\theta}^{(1)}) - \tilde{\theta}^{(1)} \mid \alpha) - c \tag{6}$$

If she does not acquire information, her expected payoff is:

$$E(\max(V(\alpha), \tilde{\theta}^{(1)}) - \tilde{\theta}^{(1)})) \tag{7}$$

Bidder 1 with type α acquires information on θ_1 whenever (6) is larger than (7), and she does not otherwise.

Revenue.

Our main result here is to show that when the number of bidders is above a threshold, the ascending price auction generates more revenues than the second price auction.

The first point to be noted is that the decision whether or not to acquire information on θ_1 depends on how likely bidder 1 believes ex ante that θ_1 will be larger than $\theta^{(1)}$. When there are sufficiently many bidders n, this probability is sufficiently low (whatever α and $c > 0$), and therefore bidder 1 does not acquire information in the sealed-bid format (because the cost c is borne whatever the realization).

In contrast, in the ascending price auction format, the total number n of bidders is irrelevant for bidder 1's decision whether or not to acquire information on θ_1 (see the expressions for $p^*(\alpha)$ and $p^{**}(\alpha)$). This is because bidder 1 can always (costlessly) wait till there are two bidders left, and then decide whether or not to acquire the information. As a result, for c not too large, bidder 1 will sometimes acquire information, even when the number of bidders is large.

Now assume that the number of bidders is large so that bidder 1 does not acquire information in the sealed-bid format. In any event where bidder 1 does not acquire information in the ascending price auction, allocation and revenues are independent of the auction format. We will show next that under the event where bidder 1 acquires information in the ascending price auction, and if the number of bidders is large, revenues are lower in the second price auction.

In what follows, we choose c small enough so that

$$p^*(\alpha) > V(\alpha)$$

for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$.[10] Let $R(x, y, z) = \min(\max(x, y), z)$, and let A_α denote the event where bidder 1 has private information α and acquires information in the ascending price auction:

$$A_\alpha = \{\alpha, \theta^{(1)} \geq p^{**}(\alpha), \theta^{(2)} \leq p^*(\alpha)\}$$

It will also be convenient to consider the following two events:

$$B_\alpha = \{\theta^{(2)} \geq V(\alpha)\} \text{ and } C_{\alpha,a} = \{\theta_1 \geq p^*(\alpha) + a, \theta^{(1)} \geq p^*(\alpha) + a\}$$

Note that under $A_\alpha \cap C_{\alpha,a}$, bidder 1 acquires information and the revenue is at least equal to $p^*(\alpha) + a$, hence it exceeds $\theta^{(2)}$ by at least a. In what follows we choose $a > 0$ such that $p^*(\alpha) + a < \bar{\theta}$.[11]

In the second price auction, under the event A_α, expected revenue is equal to

$$R_\alpha^{\text{second}} \equiv E\left[R(V(\alpha), \theta^{(1)}, \theta^{(2)}) \middle| A_\alpha\right].$$

In the ascending price auction, under the event A_α, expected revenue is equal to

$$R_\alpha^{\text{ascending}} \equiv E\left[R(\theta_1, \theta^{(1)}, \theta^{(2)}) \middle| A_\alpha\right].$$

[10] Note that this condition implies that $p^{**}(\alpha) < V(\alpha) < p^*(\alpha)$, by Lemma 1.
[11] This is possible thanks to Lemma 1.

Under A_α, the event B_α has a probability close to 1 when n is large. In what follows, we fix $\varepsilon > 0$ and choose n large enough so that $\Pr\{B_\alpha | A_\alpha\} \geq 1-\varepsilon$. Since $R(V(\alpha), \theta^{(1)}, \theta^{(2)}) = \theta^{(2)}$ under B_α, and $R(V(\alpha), \theta^{(1)}, \theta^{(2)}) = V(\alpha)$ under the complement event, we obtain:

$$R_\alpha^{\text{second}} = \Pr\{B_\alpha | A_\alpha\} E\left[\theta^{(2)} \middle| B_\alpha, A_\alpha\right] + (1-\Pr\{B_\alpha | A_\alpha\} V(\alpha) \leq E\left[\theta^{(2)} \middle| B_\alpha, A_\alpha\right]$$

We also obtain:

$$R_\alpha^{\text{ascending}} \geq (1-\varepsilon) E\left[R(\theta_1, \theta^{(1)}, \theta^{(2)}) \middle| B_\alpha, A_\alpha\right] + \varepsilon \underline{\theta}$$

Under the event $C_{\alpha,a}$, $R(\theta_1, \theta^{(1)}, \theta^{(2)})$ exceeds $\theta^{(2)}$ by at least a. And since $R(x, \theta^{(1)}, \theta^{(2)})$ is no smaller than $\theta^{(2)}$ for all x, we get:

$$R_\alpha^{\text{ascending}} \geq \Pr\{C_{\alpha,a} | B_\alpha, A_\alpha\} a + E\left[\theta^{(2)} \middle| B_\alpha, A_\alpha\right] - \varepsilon(\bar{\theta} - \underline{\theta})$$

Since a is fixed, since the term $\Pr\{C_{\alpha,a} | B_\alpha, A_\alpha\}$ does not vanish with n (it actually increases with n), and since ε can be chosen arbitrarily small, the revenue $R_\alpha^{\text{ascending}}$ is strictly larger than R_α^{second}. Thus we have proved:

Proposition 1 *Choose c small enough so that $p^*(\alpha) > V(\alpha)$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. Then there exists $\overline{n}$ such that if the number of bidders n is larger than $\overline{n}$, the revenue from the ascending price auction exceeds that obtained from the second price auction.*

Efficiency.

We have made revenue comparisons between the ascending and the sealed-bid formats. Another question of interest is how the two formats compare on efficiency grounds. In what follows, welfare is measured by the valuation of the winner minus the information acquisition cost if bidder 1 has acquired information.

A very nice implication of the private value setting is that the private incentives of bidder 1 coincide with the social incentives both in the second price sealed-bid auction and in the ascending price auction. However, the decision is not made with the same information in the two formats. Since there is more information available to bidder 1 (about θ_i, $i = 2, ...n$) in the ascending price format when she has to make her decision (she knows that $\theta^{(1)}$ lies above the current price p and that $\theta^{(2)}$ lies below the current price p), it follows that the expected welfare is higher in the ascending format than it is in the sealed-bid format (A formal proof is given next).

Proposition 2 *Expected welfare is higher in the ascending price auction than it is in the sealed-bid second price auction.*

Proof. First note that in equilibrium, when she wins the object, bidder 1 pays $\theta^{(1)}$ for the object, whether she has acquired information or not, and whether the format is the ascending price or the second price auction. When bidder 1 has acquired information, bidder 1's gain (under the realizations $\alpha, \theta^{(1)}$) is equal to

$$E[\max(\theta_1, \theta^{(1)}) - \theta^{(1)} - c \mid \alpha]$$

and when bidder 1 has not acquired information, bidder 1's expected gain (under the realizations $\alpha, \theta^{(1)}$) is equal to

$$E[\max(V(\alpha), \theta^{(1)}) - \theta^{(1)} \mid \alpha].$$

It follows that, for any realization α, and for which ever format (ascending or second price), the induced expected welfare W_α^{format} and bidder 1's expected payoff G_α^{format} satisfy

$$G_\alpha^{format} = W_\alpha^{format} - E[\theta^{(1)}] \tag{8}$$

Since in the ascending price auction, bidder 1 has the option to either acquire information immediately or to never acquire information, and since this option precisely corresponds to that available in the second price auction, we have

$$G_{\alpha}^{\text{second price}} \leq G_{\alpha}^{\text{ascending}},$$

which, given (8), concludes the proof. ■

It is interesting to assess Proposition 2 in the light of the literature on information acquisition in mechanism design (see Stegeman 1996 and Bergemann-Valimaki 2000). The view there is that in a private value setting (like the one considered here), the Vickrey auction (or second price auction here) guarantees both ex ante (at the information acquisition stage) and ex post (given the information of agents) efficiency (see Theorem 1 in Bergemann-Valimaki). However, in our setting the Vickrey auction is not the best mechanism from the viewpoint of efficiency: the ascending price auction induces an expected welfare strictly higher than that induced by the (static) Vickrey auction.

To summarize, in a private value setting, the Vickey auction is optimal in the class of direct truthful mechanisms in which bidders get no information about the private information held by other bidders. But mechanisms like the ascending price auction - because they allow for a better information transmission about the information held by others - appears to outperform the Vickrey mechanism, at least when only one bidder may acquire information. The analysis of the optimal unconstrained mechanism (in which any kind of information transmission is allowed) is left for future research.

4 Discussion

We have assumed so far that (i) when bidder 1 decides to acquire information, she instantaneously learns her valuation and (ii) only bidder 1 is imperfectly informed about his valuation. These assumptions may seem unrealistic and the purpose of this Section is to discuss the case where it takes time for bidder 1 to acquire information about her valuation, as well as a case where several bidders may refine their valuation.

Delayed information acquisition.

We start with the case where it takes time for bidder 1 to acquire information and for the sake of illustration, we will assume that when bidder 1 decides to acquire information at t, she learns her valuation at $t + T_0$.

In the ascending price auction, bidder 1 still has the option of waiting till there is one other bidder left and then acquiring information abour her valuation. However, this is no longer such a great option (at least when T_0 is not too small) because bidder 1 faces the additional risk of learning her valuation too late to avoid buying the good at a price above her valuation.

A slight modification of the ascending format will permit though to obtain conclusions similar to the ones obtained previously, thus showing the superiority of dynamic mechanisms over static ones even in this modified formulation of information acquisition.

Specifically, consider the following modification of the ascending price auction. The price starts at a low level, say 0, at which each bidder is present. The price gradually increases (say by Δ per unit of time). Each bidder may decide to quit at every moment. Bidder 1 may also decide to acquire information at every moment. When a bidder quits, this is commonly observed by every bidder. As soon as there are 2 bidders left, the auction stops for T units of time, and then resumes. The auction stops for good when there is only one bidder left. The object is allocated to that bidder at the current price.

If one choose $T \geq T_0$, then the modified ascending auction in effect allows bidder 1 to acquire information and learn her type as soon as there are two bidders left. In the modified ascending auction, the strategies described in Theorem 1 are still in equilibrium, and they yield the same outcome as the one that would prevail in the ascending price auction with immediate learning. Welfare and revenue comparisons with the second price auction are therefore unchanged.

Our analysis thus provides a rationale for designing auctions with multiple stages.

Several poorly informed bidders

The case where several bidders are poorly informed and may devote resources to acquire information requires extensive further research. A key feature of our analysis is that the poorly informed bidder remains active above her expected valuation. We wish to point out here that this conclusion will carry over to the more general case where several bidders may decide to acquire information.

For the sake of illustration, assume that there are now two poorly informed bidders, say bidder 1 and 2, that they are ex ante symmetric and that the signals initially received by bidders 1 and 2, e.g. α_1 and α_2, are uninformative. In what follows we let V denote bidder 1 and 2's common expected valuation. We have shown that if bidder 2 were informed, it would be optimal for bidder 1 to wait until there are only two bidders left to acquire information, and otherwise to drop out at some price p^*.

When bidder 2 is poorly informed however, it may no longer be a good strategy to wait until p^* to drop out: if the poorly informed bidder 2 follows that same strategy, both bidders 1 and 2 may end up being the two remaining bidders and learning that their valuation is low, hence wanting to drop out at the same price, and thereby getting the object at a loss with substantial probability.

Nevertheless, as long as the current price p does not exceed V by a too large amount, this loss will be small, and if the information acquisition cost is not too large, poorly informed bidders will still derive a positive expected profit from staying in above V: staying in until the price reaches p^* will no longer be optimal, but staying in until the price recahes some intermediate level $\hat{p} \in (V, p^*)$ will.

References

[1] Ausubel, L. (1999): "A generalized Vickrey auction," mimeo.

[2] Bergemann, D. and J. Valimaki (2000): "Information acquisition and efficient mechanism design," mimeo.

[3] Compte, O. and P. Jehiel (2001): "On the value of competition in procurement auctions," forthcoming *Econometrica*.

[4] Dasgupta, P and E. Maskin (2000): "Efficient auctions," *Quarterly Journal of Economics*

[5] Das Varma (1999): "Standard auctions with identity dependent externalities," mimeo Duke University.

[6] Hausch, D.B. and L. Li (1991): "Private values auctions with endogeneous information: revenue equivalence and non-equivalence," mimeo Wisconsin University.

[7] Hausch, D.B. and L. Li (1993): "A common value model with endogeneous entry and information acquisition," *Economic Theory* **3**: 315-334.

[8] Jehiel, P. and B. Moldovanu (1996): "Strategic nonparticipation," *Rand* Journal of Economics, **27**, 84-98.

[9] Jehiel, P. and B. Moldovanu (2000): "Efficient design with interdependent valuations," forthcoming *Econometrica*.

[10] Krishna, V. (2000): "Asymmetric English auctions," mimeo.

[11] Maskin, E. (1992): "Auctions and privatizations," in *Privatization* edited by H. Siebert, 115-136.

[12] Matthews, S. (1977): "Information acquisition in competitive bidding process," mimeo Caltech.

[13] Matthews, S. (1984): "Information acquisition in discriminatory auctions," in *Bayesian Models in Economic Theory*, ed. M. Boyer and R. E. Khilstrom, Amsterdam, North Holland.

[14] Milgrom, P. and R. Weber (1982): "A theory of auctions and competitive bidding," *Econometrica*, **50**, 1089-1122.

[15] Persico, N. (2000): "Information acquisition in auctions," *Econometrica.*

[16] Perry, M. and P.J. Reny (2000): "An ex-post efficient auction," mimeo.

[17] Stegeman, M. (1996): "Participation costs and efficient auctions," *Journal of Economic Theory* **71**: 228-259.

[18] Tan, G. (1992): "Entry and R and D in procurement contracting," *Journal of Economic Theory* **58**: 41-60.

[19] Vickrey, W. (1961): "Counterspeculation, auctions and competitive sealed tenders," *Journal of Finance*, **16**, 8-37.