Auctions and Information Acquisition: Sealed-bid or Dynamic Formats?*

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Abstract

The value of an asset is generally not known *a priori*, and it requires costly investments to be discovered. In such contexts with endogenous information acquisition, which selling procedure generates more revenues? We show that dynamic formats such as ascending price or multi-stage auctions perform better than their static counterpart. This is because dynamic formats allow bidders to observe the number of competitors left throughout the selling procedure. Thus, even if competition appears strong ex ante, it may turn out to be weak along the dynamic format, thereby making the option to acquire information valuable. This very possibility also induces the bidders to stay longer in the auction, just to learn about the state of competition. Both effects boost revenues, and our analysis provides a rationale for using dynamic formats rather than sealed-bid ones.

Key words: auctions, private value, information acquisition, option value.

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1 Introduction

Assessing the value of an asset for sale is a costly activity. When a firm is being sold, each individual buyer has to figure out the best use of the assets, which business unit to keep or resell, which site or production line to close. The resources spent can be very large when there is no obvious way for the buyer to combine the asset for sale with the assets that he already owns. Similarly, when acquiring a license for digital television, entrants have to figure out the type of program they will have a comparative advantage on, as well as the advertisement revenues they can expect from the type of program they wish to broadcast. Incumbents may also want to assess the economies of scale that can be derived from the new acquisition. All such activities are aimed at refining the assessment of the valuation of the license, and they are costly.

From the seller's perspective, if the assets are auctioned to a set of potential buyers, the better informed the bidders are the higher the revenues, at least when the number of competitors is not too small.[1] However, when information is costly to acquire, a potential buyer may worry about the possibility that he spends many resources, and yet ends up not winning the asset. Providing the bidders with incentives to acquire information is thus key for the seller.

One commonly used format for selling assets is the sealed-bid auction, in which the winner is selected in a single round. Other formats (which are frequently used when the asset is complex) are multi-stage auctions and ascending price auctions, in which the number of potential buyers is gradually reduced.[2] In the sealed-bid format, information acquisition may only take place prior to the auction. In dynamic formats, information acquisition may take place not only prior to the auction, but also in the course of the auction.

Our objective is to compare dynamic and static auction formats in settings in which some bidders initially know their valuations while others have the option to acquire further information on their valuation at some cost.[3]

[1] The reason is that a bidder with a (mediocre) expected valuation may realize, once informed, that his valuation is quite high, hence the increase in revenue. Of course, he may also realize that his valuation is quite low, but if competition is strong, this effect on revenues is small.

[2] See Ye (2000) for an account of how widespread the practice of multi-stage auctions are.

[3] An alternative interpretation is that agents have heterogeneous information acquisition costs.

The main insight of this paper is that dynamic auction procedures are likely to generate more information acquisition and higher revenues than their static counterparts.[4] More precisely, we highlight a significant benefit induced by formats in which bidders gradually get to know the number of (serious) competitors they are facing, which in turn allow them to better adjust their information acquisition strategy.

To get some intuition for our insight, observe that in the sealed-bid static format, bidders do not acquire information on their valuations whenever there are sufficiently many competitors. The point is that the risk of ending up not buying the good (because it turns out that someone else has a higher value) is then so large that bidders prefer not to waste their money (or time) on getting such precise information. In contrast, in the ascending price auction format, bidders get to obtain a better estimate of their chance of winning just by observing the number of bidders left. In particular, even if competition appears strong ex ante, it may turn out to be weak (if many bidders drop out), and information acquisition may then become a valuable option. This has two effects: first, it generates more information acquisition (hence more revenues - at least when the number of bidders is not too small). Second, it may induce bidders to wait and remain active in the auction, just to learn more about the state of competition. The latter effect also raises the price paid by winners, hence revenues, as compared with the price paid in the sealed-bid format.

It should be emphasized that the reason why dynamic formats generate more revenues here is completely different from the classic reason of affiliated values (Milgrom-Weber 1982) in which ascending formats allow the bidders to learn about the information held by others. Here, the valuations of bidders are not influenced by other bidders' information (we consider a private-value setting), and yet dynamic auction formats generate higher revenues (by modifying bidders' information acquisition strategy on their own valuations).

Our paper can thus be viewed as providing a (new) rationale for using dynamic auction formats. But, note that our discussion has highlighted the role of providing bidders with some estimate of the level of competition (through the number of competitors left), and not all dynamic formats have the property of conveying such an estimate. Dynamic formats that do not have this property are less desirable from the perspective of this paper. For

[4] Though we focus here on revenues, efficiency may also be higher in the ascending format (see Compte-Jehiel 2000).

the sake of illustration, consider the one-object ascending price auction with secret drop-out in which bidders observe the current level of price, but not how many competitors are left. This is the auction format studied in an independent work by Rezende (2005). As it turns out (see Section 5), in our model in which the information acquisition cost is assumed to be bounded away from 0, bidders do not to acquire information on their valuations when there are sufficiently many competitors. So the ascending price auction with secret drop-out is equivalent to the sealed-bid auction, and it is dominated by the ascending price auction (in which bidders observe the number of bidders left) when there are sufficiently many bidders (see Section for further discussion on Rezende's paper).

Related literature:

Our paper is related to various strands of literature in auction theory: the comparison of auction formats (and more precisely here the comparison of the second price and ascending price auction formats), the analysis of information acquisition in auctions and the literature on entry in auctions.

Concerning the comparison between auction formats, we mentioned earlier the work by Milgrom-Weber (1982), who showed that, in affiliated value settings, the ascending and sealed-bid formats differ because the information on others' signals conveyed in equilibrium differ, hence the bidders' assessment of their valuation differ too. In the context of auctions with negative externalities (see Jehiel-Moldovanu 1996), Das Varma (1999) has shown that the ascending format could (under some conditions) generate higher revenues than the sealed-bid (second-price) auction format (in the ascending format a bidder may be willing to stay longer, so as to combat a harmful competitor if he happens to be the remaining bidder).[5]

Concerning information acquisition in auctions, the literature has (in contrast to our work) focused on sealed-bid types of auction mechanisms, and it has essentially examined efficiency issues.[6]

[5] In a companion paper (Compte-Jehiel 2004), we also make a comparison between sealed bid and ascending price auctions, in a setting where information acquisition is exogenous: bidders get more precise signal as time elapses. In contrast, here, information acquisition is endogenous: bidders have to decide whether (and when) to acquire information.

[6] One exception is an independent contribution by Rezende (2005), that examines the ascending price auction with secret drop out. Building on Compte and Jehiel (2000), Rasmussen (2001) considers a two-

In a private value model, Hausch and Li (1991) show that first price and second price auctions are equivalent in a symmetric setting (see also Tan 1992).[7] Stegeman (1996) shows that second price auction induces an ex ante efficient information acquisition in the single unit independent private values case (see also Bergemann and Valimaki 2000). However, in Compte and Jehiel (2000), it is shown that the ascending price auction may induce an even greater level of expected welfare.

Models of information acquisition in interdependent value contexts (in static mechanisms) include Milgrom (1981) who studies second-price auctions, Matthews (1977), (1984) who studies first-price auctions and analyzes in a pure common value context whether the value of the winning bid converges to the true value of the object as the number of bidders gets large,[8] Persico (1999) who compares incentives for information acquisition in the first price and second price auctions in the affiliated value setting, and Bergemann and Valimaki (2000) who investigate, in a general interdependent value context, the impact of ex post efficiency on the ex ante incentives for information acquisition.

Our paper is also related to the literature on endogenous entry in auctions, which includes McAfee and McMillan (1987), Harstad (1990) and Levin and Smith (1994).[9] In these models, each bidder makes an entry decision prior to the auction, at a stage where bidders do not know their valuation. The decision to enter allows the bidder to both participate to the

bidder dynamic auction with deadline (motivated by internet auctions) in which one bidder can refine her valuation at a cost. He observes that the informed bidder may wait till close to the deadline to make the uninformed bidder believe that he can win the object without having to refine her valuation.

[7] Engelbrecht-Wiggans (2001) also compares first price and second price auctions, but he examines the case where bidders acquire information on the number of competitors (rather than on their own valuation). These two formats are then not equivalent, since knowing the number of bidders is valuable in the first price auction only.

[8] See also Hausch and Li (1993) for an analysis of information acquisition in common value settings.

[9] These papers analyze the effect of entry fees or reserve prices on the seller's revenue. McAfee-McMillan (1987) show that in constrast with the case where the number of participants is given exogenously, the optimal reserve price may be zero (this insight is related to that of Bulow-Klemperer (1996) about the positive role of competition in symmetric setups). Levin-Smith (1994) (see also Harstad 1990) further analyze this issue by considering (symmetric) equilibria with possibly stochastic participation. They find that restricting the number of participants to equate the socially optimal number of bidders eliminates the coordination problem that would arise otherwise (if the number of bidders is larger than the socially optimal one, stochastic participation cannot be avoided and may result in no participation).

auction and learn her valuation. These models thus combine the idea of participation costs *and* the idea of information acquisition. This should be contrasted with our model in which there is no participation cost but only a cost to acquire information on the valuation.

Finally, our work is also related to the literature on research contests (Fullerton and McAfee (1998), and more recently Che and Gale (2001)). The main virtue of the ascending price auction identified in this paper is that it increases the incentives to acquire information as, for some realizations of signals, it allows the bidders to realize that competition is less tough than it would have seemed from an ex ante viewpoint. Likewise, Fullerton and McAfee (1998) and Che and Gale (2001) identify conditions under which it is a good idea from an efficiency viewpoint to reduce the number of contestants to just a few (in fact two) in an attempt to increase contestants' incentives to exert effort in the contest.[10]

The rest of the paper is organized as follows. Section 2 describes the basic model. Section 3 develops the basic analysis that paves the way to the revenue comparison performed in Section 4. Further discussion of our model appears in Section 5.

2 The Model

There is one object for sale, worth 0 to the seller and n potential risk-neutral buyers indexed by $i \in N = \{1, ..., n\}$. Each bidder $i = 1, ..., n$ has a valuation θ_i for the object. The valuations θ_i for the various bidders i are assumed to be drawn from independent and identical distributions, with support on a finite number of values $\theta^k, k \in \{1, .., K\}$, with $0 < \theta^1 < \cdots < \theta^k < \cdots < \theta^K = \bar{\theta}$. We denote by $f(\theta^k)$ the corresponding probability that the valuation is θ^k.[11]

[10] There are obviously many differences between an auction setup in which the value of winning is determined by the valuation and a contest in which the prize is common to all contestants, but the two setups share a common feature: less fierce competition increases the incentive to acquire information in our setup or to make effort in the contest application.

[11] The assumption that the distribution is discrete is made so as to guarantee the existence of the equilibrium. Beyond existence issues our analysis does not rely on this assumption. We discuss in Section 5 the continuum type case.

On the interpretative side, one economic motivation for the discreteness of the distribution is that there are only K possible signals that a bidder may observe concerning his valuation, and each θ^k corresponds to an expected valuation given signal k.

In our model, there will be two types of buyers: the *informed buyers*, who know the realization of their own valuation; and the uninformed buyers, who may get informed about their own valuation at some cost c. One possible interpretation is that for some bidders, information acquisition is costless, hence they become informed. We assume that each bidder's informational state (whether he is informed or not) is private information. We denote by n_1 the number of informed bidders, and by n_2 the number of uninformed buyers. We assume that bidders' informational states are drawn from a distribution with full support on (n_1, n_2), $n_1 + n_2 = n$.

For the sake of comparative statics on the number of bidders, we will also consider the *symmetric case* where initially, each bidder i is informed with some probability $q \in (0,1)$ and uninformed with probability $1-q$, and where informational states are drawn from independent distributions.

When bidder i is uninformed, his expected valuation from acquiring the object is:[12]

$$v \equiv \sum_{k=1}^{K} \theta^k f(\theta^k). \tag{1}$$

When he acquires information, bidder i learns the realization θ_i. Other bidders however do not observe that realization; they do not observe either whether information acquisition occurred. Finally, it will be convenient to denote by $\theta^{(1)}$, $\theta^{(2)}$..., $\theta^{(j)}$ the highest valuation, the second highest valuation, and the j^{th} highest valuation, respectively among the (initially) informed bidders. We define similarly $\theta_u^{(1)}$, $\theta_u^{(2)}$..., $\theta_u^{(j)}$ for the (initially) uninformed bidders where $\theta_u^{(j)}$ is obtained using the valuations these bidders would observe if they were to acquire information.[13]

The information structure is assumed to be common knowledge among all bidders.

Remark: The interpretation behind the definition of v (expression (1)) is that in case he wins, bidder i will learn the realization θ_i at no cost. Hence the only motive for spending resources on acquiring information is in checking that it is worthwhile to acquire the object. In some contexts (for example when bidders compete to acquire a firm), the resources that a

[12] In Compte and Jehiel (2000), we analyze the more general case in which an uninformed bidder i gets an imperfect signal about θ_i prior to the auction. No new insight is gained by doing so however.

[13] By modeling the situation as an initial draw by nature determining the valuations of each bidder, $\theta_u^{(j)}$ can be defined irrespective of bidders' information acquisition strategy.

bidder spends on acquiring information are best thought of as an investment that will have to be made anyway, in case that bidder wins.[14] To cover such applications, one would need to modify the expected valuation from acquiring the asset into

$$\sum_{k=1}^{K} \theta^k f(\theta^k) - c. \tag{2}$$

Our analysis would extend in a straightforward way to this alternative formulation (by equating v with the expression above); and our main result that the ascending format generates more revenues would even be stronger in that case. We will however stick to the previous formulation in which v is as shown in (1).

Auction formats:

Throughout the analysis, we will be mostly interested in the comparison between static and dynamic auction formats. We will compare the sealed bid second-price auction and the ascending price auction. The key difference we will be exploiting is that in the static format, information acquisition may only take place prior to the auction, while in the dynamic format, it may also take place in the course of the auction when additional information is available.

The *sealed bid second-price auction.* Each bidder submits a bid. The object is allocated to the bidder whose bid is highest at a price equal to the second highest bid.[15] Each bidder decides prior to the auction whether or not to acquire information.

The *ascending price auction.* Dates are discrete: $t = 0, 1...$ At date $t = 0$, the price starts at 0. At any date $t \geq 0$, each bidder is given in turn the option to drop out of the auction. The order in which bidders are given that option is drawn randomly from a uniform distribution over ranks, and kept secret. As soon as a bidder drops out, the process stops, the auction resumes at date $t+1$, and the current price is unchanged. When no bidder drops out, the auction resumes at date $t + 1$, and the current price increases by a fixed increment Δ (where Δ should be thought of as small).[16]

[14] Such investments for example include the resources spent to assess the synergies with the assets already owned.

[15] In case of ties, each one of the bidders with highest bid gets the object with equal probability.

[16] This assumption can be viewed as a way to break ties when more than one bidder is willing to drop out at the same price. It also ensures that in each step of the auction, the number of active bidders cannot drop

Bidders are assumed to observe the number of bidders still active in the auction. This number will be denoted by m. The auction ends when there is only one bidder left. The object is allocated to that bidder at the current price. Besides observing the price p, the number of active bidder m, and whether they are given the option to drop out, they do not observe anything else.

Information acquisition. In the second price auction, each bidder decides prior to the auction whether or not to acquire information. In the ascending price auction, each uninformed bidder may decide to learn his valuation not only before the auction starts (before $t = 0$), but also during the auction (prior to each date $t = 1, ...$). Learning one's own valuation costs c. The length of time required to acquire information is assumed to be no larger than the lapse of time between two dates. That is, if a bidder decides to learn his valuation prior to date t, we assume that this information is available at date t (see subsection 5.1 for on elaboration on this). The decision to acquire information is not observable to other bidders. To simplify exposition, when a bidder acquires information right before date t, we will write that he acquires information at t^-. For convenience, when the price that will prevail at t is equal to p and the bidder acquires information at t^-, we write that he acquires information at p^-.[17]

Preliminaries.

The above descriptions of the rules of the auctions, the information acquisition technology, the information and payoff structures define games for the sealed bid second-price auction and the ascending price auction. In the sealed-bid auction, a bidder's strategy is described

by more than one. Note that bidders who did not get the option to drop out are not disadvantaged however, because when the auction resumes, the price has not changed and they may still be given the option to drop out at that same price.

An alternative assumption would be that the auction stops when all still active bidders want to exit simultaneously, and that each one of these bidders has an equal chance of being awarded the object. In the absence of an option to acquire information, the two formulations would be equivalent. In the presence of an option to acquire information, our formulation gives bidders who were not given an option to drop out an extra chance of acquiring information.

[17] Note that since the price does not always rise, the latter notation may introduce some ambiguity about the date before which information acquisition occurs. In these cases, specifying the number of active bidders will leave out any such ambiguity.

by a probability of information acquisition (when uninformed) and a bid contingent on the available information to him (i.e. whether or not he is informed, and the realization of his valuation when informed).

In the ascending price auction, the behavioral strategy of bidder i is defined as a function of the (private) history h_i observed by bidder i up to the current date t where h_i stands for the sequence of prices p_s, the number m_s of active bidders for each earlier date $s < t$ and whether or not bidder i was given an option to drop out at s and whether or not bidder i acquired information at s^- (when still uninformed).[18] The behavioral strategy of bidder i specifies for each h_i a probability of information acquisition at t^- (when still uninformed) and a probability of dropping out if given the option to at t (whether or not he is informed).

Throughout the paper, we investigate properties of trembling-hand perfect equilibria with uncorrelated trembles at the various information sets (Selten 1975).[19] We start by stating a few preliminary results, and by introducing further notations. The main challenge will be to evaluate the information acquisition strategy of the uninformed bidders in the ascending auction (i.e. if and when bidders acquire information), as once a bidder is informed, his strategy is rather standard (see below).

Existence.

First, we observe that in each format, an equilibrium exists. This is immediate in the static auction, since bidders have a dominant strategy (concerning bidding), i.e. bidding θ_i if informed, v if uninformed, and information acquisition is a $0-1$ decision. In the dynamic auction, once the price rises above $\bar{\theta}$, it is a dominant strategy for any still active bidder to drop out. So there is only a finite number of histories after which behavior needs to be determined. Since there are only a finite number of types and the action space of every bidder is finite after every history, the existence of a trembling-hand equilibrium follows from Kreps and Wilson (1982).

Informed bidders' strategies.

The behavior of informed bidders is fairly standard in the static sealed-bid second-price auction: each informed bidder submits a bid equal to his valuation. In the dynamic auction,

[18] The last two elements of h_i are private information to bidder i.

[19] Technically, this amounts to looking at the trembling-and perfect equilibria in the agent form representation of the game. (see, for example, the definition in Fudenberg and Tirole 1991)

and because price increments are discrete, dominance arguments do not pin down the strategy of the informed bidders. Yet, standard dominance arguments deliver:

Proposition 1 *In any equilibrium of the dynamic format, an informed bidder with valuation θ drops out at some price p no larger than $\theta + \Delta$ and no smaller than $\theta - \Delta$.*

As the price increment gets small, the behavior of informed bidders is almost determined by Proposition 1.

Uninformed bidders and the gains from information acquisition.

To assess uninformed bidders' incentives to acquire information, we introduce the following notation.

First, for any price $p \in [0, \bar{\theta}]$, we define:

$$h(p) = E_{\theta_i} \max(\theta_i - p, 0). \tag{3}$$

$h(p)$ corresponds to the expected payoff bidder i would obtain if he were to learn (for free) his valuation θ_i and were offered to buy at price p.

We next define functions that will allow us to derive a lower bound on the gains made by an uninformed bidder should he acquire information when there is only one other active bidder and the current price is p. We let

$$H_\Delta(p) = E[h(\theta + \Delta) \mid \theta > p + \Delta],$$

which will be relevant for the case in which the other active bidder is informed. And

$$\begin{aligned} G(p) &= \sum_{\theta^k < p} \frac{\theta^k - p}{2} f(\theta^k) + \sum_{\theta^k \geq p} (\theta^k - p) f(\theta^k) \\ &= \frac{v - p}{2} + \frac{h(p)}{2} \end{aligned}$$

which will be relevant for the case in which the other bidder wishes to drop out.[20]

[20] If bidder i stays in whenever $\theta_i \geq p$, he wins the object at price p with probability one if $\theta_i \geq p$ (because the other drops out), and with probability $\frac{1}{2}$ otherwise (because there is a $\frac{1}{2}$ chance that the other bidder is given the option to drop out first.

Observe that the functions H_Δ and G are decreasing in their arguments.[21]

Finally, we let

$$\widetilde{H}_\Delta(p) \equiv \min(H_\Delta(p) - \Delta, G(p))$$

and define p^* to be the largest price p, multiple of Δ,[22] for which $\widetilde{H}_\Delta(p) > c+\Delta$. Throughout the paper, we shall assume that:[23]

Assumption 1 *$c + \Delta < \widetilde{H}_\Delta(v + \Delta)$.*

This assumption guarantees that there are some gains to information acquisition when there are two bidders in competition. Specifically,

> **Claim A:** Under Assumption 1, in equilibrium, in events where the current price is p^- with $p \in (v - \Delta, p^*)$ and $m = 2$, any still uninformed bidder acquires information and obtains a payoff at least equal to $H_\Delta(p) - c > 0$.

To prove this Claim, we first show that in equilibrium, once there are only two bidders active in the auction, and the current price lies above v, then any still uninformed bidder either drops out or acquires information immediately.

Lemma 1 *Assume that $c < G(v)$.*[24] *Consider any event where the current price is $p^- > v$ and $m = 2$. Then in equilibrium, any still uninformed bidder either drops out at p without acquiring information, or he acquires information immediately.*

The intuition for the result is as follows. If bidder i is supposed to drop out without acquiring information, then the earlier the better because, since $p > v$, this would allow

[21] Indeed, $h'(p) = -\Pr\{\theta_i > p\} \in (-1, 0)$. So G is decreasing. Now choose $p' > p$ and let $Q = \Pr\{\theta \in (p + \Delta, p' + \Delta) \mid \theta > p + \Delta\}$. We have

$$H_\Delta(p) > Qh(p' + \Delta) + (1 - Q)H_\Delta(p') > H_\Delta(p')$$

[22] Only prices that are multiples of Δ may be reached.

[23] This assumption ensures that $p^* > v$. To hold for some c and Δ sufficiently small, it is sufficient that there exist at least two types θ^k, θ^{k+1} lying above v. Indeed, for Δ small enough, $\theta^k > v + 2\Delta$ and $\theta^{k+1} > \theta^k + \Delta$. This implies that $h(\theta^k + \Delta) \geq f(\theta^{k+1})(\theta^{k+1} - \theta^k - \Delta)$ and $H_\Delta(v + \Delta) \geq f(\theta^k)h(\theta^k + \Delta)$, hence $H_\Delta(v + \Delta)$ lies above $c + \Delta$ for c and Δ small enough.

[24] This is implied by Assumption 1.

him to avoid acquiring the object at a loss. Delaying information acquisition is not a good strategy either. When bidder i postpones information acquisition till the price reaches $p' > p$, he avoids the information acquisition cost in the event the other bidder drops out between p and p'. However, by acquiring information right away, and in case $\theta_i < v$, he may drop out (if given the option to), and save at least $\frac{1}{2}E[v - \theta_i \mid \theta_i < v] = G(v)$. We provide a formal proof in the Appendix.

The second step consists in deriving a lower bound on the equilibrium payoff obtained from acquiring information when there is one other active bidder. Define the continuation strategy σ_p^*, defined from the current price p^-, as follows: acquire information immediately, and for any $l \geq 0$, drop out at $p_l = p + l\Delta$ where $p_l - \Delta < \theta \leq p_l$ (if given the option to).[25] We have:

Lemma 2 *In the ascending price auction, consider the event where $m = 2$, the current price is p^-, $p > v - \Delta$, and bidder i is still active and uninformed. Then bidder i can secure a payoff at least equal to $\widetilde{H}_\Delta(p) - c$ by following σ_p^*.*

Roughly, $G(p) - c$ corresponds to a lower bound from using σ_p^* when facing a bidder who wishes to drop out, while $H(p) - c$ corresponds to a lower bound from using σ_p^* when facing a bidder willing to remain active (thus informed by Lemma 1). So whether bidder i is facing an uninformed bidder (who by Lemma 1 either drops out or acquires information), or an informed bidder, bidder i is able to secure $\widetilde{H}_\Delta(p) - c$ by following σ_p^*. The details of the proof are in the Appendix.

We can now turn to the proof of Claim A:

Proof of Claim A: This Claim is a simple consequence of Lemma 1 and 2: under Assumption 1, bidder i can secure a payoff larger than Δ by following σ_p^*. It follows that dropping out is dominated (since it yields at most Δ). Hence in equilibrium, by Lemma 1, bidder i must acquire information, and thus he obtains at least what he can get by following σ_p^*, that is, $\widetilde{H}_\Delta(p) - c$. **Q.E.D.**

[25] Typically, σ_p^* will not be an equilibrium strategy. However, it permits us to derive a lower bound on the payoff a bidder can obtain when he acquires information.

3 Wait-and-see and information acquisition.

This Section illustrates that the incentives for information acquisition are very different in the sealed-bid and the ascending bid auction formats. Uninformed bidders will typically acquire information more often in the ascending price auction than in the sealed-bid auction, at least when the number of bidders is not too small. We will also illustrate that uninformed bidders may stay active in the ascending price auction much above their expected valuation. These two features will be exploited in the next section to establish that the ascending price auction generates more revenues than the sealed-bid auction when there are sufficiently many bidders.

Our first result concerns the sealed-bid second price auction.

Proposition 2 *Fix $c > 0$ and consider the sealed-bid second price auction in the symmetric case. Then when n is large enough, uninformed bidders do not acquire information, and accordingly they bid their expected valuation v.*

Intuitively, when bidder i acquires information at cost c, he obtains $h(p)$ in the event the price turns out to be p. When there are many bidders the distribution over price puts most weight on high values of p. And for high values of p, $h(p)$ is close to 0. As a consequence, this gain does not compensate the cost c of information acquisition, and there is no information acquisition in equilibrium.[26]

We now turn to the ascending price auction.

Proposition 3 *Assume that $c + \Delta < \widetilde{H}_\Delta(v + \Delta)$. Then, in equilibrium, information acquisition occurs with positive probability in the ascending auction.*

So, unlike in the second price auction for which information acquisition cannot occur when there are many bidders, in the ascending price auction, information acquisition always occurs with positive probability, even as the number of bidders gets very large.

The intuition for this difference is that, in the sealed bid auction, acquiring information costs c whether competition turns out to be fierce or not, i.e. whether the selling price

[26] Formally, an upperbound on the gain from information acquisition is

$$E[h(\theta^{(1)})] - c,$$

which tends to $-c$ when n gets large.

p turns out to be large or not. In contrast, in the ascending price auction, bidders see the number of competitors who are still active. Hence they can condition their decision to acquire information on the strength of competition. In particular, there is always a chance that competition turns out to be weak.[27] So information acquisition occurs with positive probability, even as the number of bidders gets arbitrarily large.

Proof of Proposition 3: Assume by contradiction that information acquisition occurs with probability 0. This implies that any uninformed bidder drops out as soon as $p > v$. Consider the event where $n_1 = n - 1$, and $\theta^{(2)} < v - \Delta$ and $v + \Delta < \theta^{(1)}$. Under that event, the auction continues until some price $p \in (v - \Delta, v]$ at which $m = 2$. By Claim A, the uninformed bidder should then acquire information, yielding a contradiction. **Q.E.D.**

Another implication of Claim A is that, in the ascending price auction, uninformed bidders may find it profitable to remain active above v, whether or not they eventually acquire information and irrespective of the number of bidders. This is because an uninformed bidder has the option to *wait and see whether or not competition turns out to be weak*. And he can do so at absolutely no (expected) cost until the price reaches p^* where $p^* > v$ (see above). Indeed, he can always decide to acquire information if at some price $p < p^*$ he faces a single other bidder, and to drop out at p^* otherwise. Such a strategy guarantees a positive (expected) payoff by Claim A.

Proposition 4 *Assume $c + \Delta < \widetilde{H}_\Delta(v + \Delta)$. Then, in equilibrium, as long as there are at least two active bidders ($m \geq 2$) and the next current price is no larger than p^* ($p \leq p^*$), any still uninformed bidder either acquires information, or he remains active in the auction.*

Proof of Proposition 4: First, we show that in any event where the next current price is $p \leq p^*$ and $m \geq 2$, a still uninformed bidder has a strategy that yields a strictly positive payoff. This will imply that dropping out is a dominated strategy, showing that he either acquires information or remains active without acquiring information.

Define the following wait-and-see strategy: wait until either $m = 2$ or $p = p^*$. Then, in case $m = 2$ arises before $p = p^*$, follow the information acquisition strategy σ_p^*. Otherwise, choose to drop out. In case one other bidder is also willing to drop out and given the option

[27] This chance remains positive no matter how many bidders are ex ante around (yet it typically decreases with this number).

to drop out first, continue to try dropping out until either given the option to, or $m = 2$. In the latter case, follow σ_p^*. Because (in the game with trembles) there is always a positive chance that the number of active bidders drops down to 2 without any price increase, this strategy yields a strictly positive payoff by Claim A.

Dropping out, which yields 0, is thus dominated. **Q.E.D.**

Comment: To conclude this Section, let us emphasize that Proposition 4 does not provide a full characterization of equilibrium behavior. It only provides an important feature of equilibrium behavior that will be sufficient to compare revenues across formats. In an earlier version of this paper, we were able to provide a full characterization of equilibrium behavior, in the "special case" where Δ is arbitrarily small and the number of initially uninformed bidders is known to be equal to 1. In that case, we can prove that the uninformed bidder does not acquire information unless $m = 2$. In the general case, the information acquisition strategy can be quite complex. For the sake of illustration, consider an uninformed bidder, say bidder i: if other uninformed bidders acquire information early on, then bidder i will typically be tempted to wait until $m = 2$ to acquire information (as if there were only informed bidders). However, if many bidders wait for $m = 2$ or some price $\widetilde{p}$ to acquire information, then there is a gain to acquiring information beforehand, because it reduces the risk of being stuck with the object (in the event the remaining bidders are all uninformed bidders and willing to drop out at the same price $\widetilde{p}$). So typically, in equilibrium, the information acquisition strategy will involve mixed strategies.

4 Revenues.

The following result establishes a revenue ranking between our two auction formats when there are sufficiently many bidders.

Proposition 5 *Consider the symmetric case and any n large enough. Then for Δ small enough, revenues in the ascending auction are larger than in the second price auction.*

Revenues increase through two channels: (i) There is more information acquisition in the ascending price auction (as Propositions 2 and 3 illustrate); (ii) Uninformed bidders who do not acquire information remain active in the ascending price auction beyond their expected valuation for the object, thereby making the winner pay a higher price (see Proposition 4).

Concerning the first channel, note that *a priori* the effect of information acquisition on revenues is *ambiguous*. For example, under the event where there is a single uninformed bidder and $\theta^{(1)} < v$. Then information acquisition by the uninformed bidder *decreases* revenues: without information acquisition, revenue is $\theta^{(1)}$; with information acquisition, the revenue cannot be larger, but it can be smaller (if the uninformed bidder learns that his valuation is actually below $\theta^{(1)}$).

For large enough number of bidders however, the effect of information acquisition is positive because then, on average, such events where the revenues decrease due to information acquisition by uninformed bidders have a small probability (compared to the events where the effect on revenues is positive).

We provide a full proof in the Appendix. Here we suggest some intuition for the result by considering the case where the distribution over (n_1, n_2) is concentrated (i.e. puts almost all weight) on some fixed (n_1^0, n_2^0). We will distinguish two cases: the case where n_2^0 is large (which will allow us to quantify the wait and see effect), and the case where n_2^0 is small (which will allow us to quantify the information acquisition effect).[28]

Consider first the case of a large n_2^0: Let us focus on events where competition is not too fierce, that is, events where $\theta^{(2)} < p^*$.[29] Under these events, revenue in the static auction cannot be larger than $\max(v, \theta^{(2)})$. Now observe that in the ascending auction, either at least $n_2^0 - 1$ uninformed bidders acquire information, in which case the price must rise to at least $\theta_u^{(3)}$; or at least two uninformed bidders do not acquire information, in which case, by proposition 4, price must raise to p^* at least. When there are many initially uninformed bidder (i.e. n_2^0 large), the event where $\theta_u^{(3)}$ is below p^* has a vanishing probability. Therefore, in expectation, as n_2^0 gets large, the ascending auction generates a gain in revenue at least equal to

$$\Pr(\theta^{(2)} < p^*) E[p^* - \max(v, \theta^{(2)}) \mid \theta^{(2)} < p^*]. \tag{4}$$

[28]In the rest of this Section, to simplify presentation of the intuition, we consider the case in which Δ is arbitrarily small.

[29]In events where $\theta^{(2)}$ is larger than p^*, revenue is equal to $\theta^{(2)}$ in the static auction, and it cannot be smaller in the dynamic auction, whether uninformed bidders acquire information or not.

This expression captures (and provides a lower bound on) the wait and see effect.[30]

Consider next the case of a small n_2^0. The argument above cannot be applied, because then there may be a substantial probability that all bidders acquire information and all learn that their valuation is below v. Depending on the value of $\theta^{(2)}$, the effect on revenue may be negative or positive. If $\theta^{(2)}$ is below v, information acquisition may then decrease revenues. If however $\theta^{(2)}$ is above v, then information acquisition cannot hurt, because revenue will still be at least equal to $\theta^{(2)}$ and possibly more when some bidders learn that their valuation is above $\theta^{(2)}$. In expectation, when n_1^0 gets large, the event $\theta^{(2)} < v$ is much less likely than the event $\theta^{(2)} \in (v, p^*)$, and the effect on revenues is thus positive.

5 Discussion

5.1 Multi-stage mechanisms.

Although our analysis has assumed that information acquisition may occur instantaneously all along the ascending price auction, our insight that dynamic auctions dominate static auctions would carry over to a wide range of mechanisms and information acquisition technologies. The key features of the dynamic format and the information acquisition technology that we have been exploiting is that contestants get feedback about the state of competition in the course of the procedure, and that they have enough time to use that feedback to adjust their information acquisition strategy.

Our insight may thus account for the prevalence of multi-stage mechanisms in which the number of contestants is reduced from one stage to another and enough time is given between the various stages so that contestants can effectively refine the estimate of their valuations.

As an illustration, let us briefly examine a selling procedure that consists of the following two stages: the first stage is a sealed-bid auction. The two bidders with highest bid are selected to participate in the second stage. Before the start of the second stage, acquisition of information may occur. The second stage is a second price auction where bidders submit a bid p_i' required to be at least as large as the third largest bid of the first stage. We have:

[30] Note that we omit here the additional effect from information acquisition, as eventually, the number of active bidders will drop down, and eventually uninformed bidders will acquire information, generating further gains in revenues.

Claim B: In equilibrium, uninformed bidders submit a bid at least equal to p^* in the first stage. If the third bid in the first stage belongs to $[v, p^*]$ and an uninformed bidder is selected for the second stage, he acquires information.

Proof of Claim B: Denote by $p^{(3)}$ the third largest bid in stage 1. Denote by η^* the strategy of an uninformed bidder in stage 2 which consists in acquiring information if $p^{(3)} \geq v$ (and then bidding $\max(p^{(3)}, \theta)$ in stage 2), and bidding v without acquiring information if $p^{(3)} < v$. For any $p^{(3)} \leq p^*$, this strategy generates a strictly positive payoff (this follows from Claim A).

Assume now that in equilibrium, an uninformed bidder, say bidder i, bids $p < p^*$ in stage 1; let $\sigma(p^{(3)})$ denote his continuation strategy, as a function of $p^{(3)}$, and denote by $\widehat{v}(p^{(3)})$ the corresponding (continuation) equilibrium payoff. Since $p^{(3)} \leq p < p^*$ when i is selected, we know that $\widehat{v}(p^{(3)})$ must be strictly positive (otherwise η^* would be a profitable deviation). We also know that if $p^{(3)} \geq v$, $\sigma(p^{(3)})$ must involve acquiring information (because not acquiring information yields a non-positive payoff).

Now consider the deviation that consists in bidding p^*, and then following $\sigma(p^{(3)})$ in events where $p^{(3)} \leq p$, and following η^* in events where $p^{(3)} > p$.

This deviation generates the same expected payoff in events where $p^{(3)} < p$ or where $p^{(3)} = p$ and bidder i was selected to participate in the second stage. In the event $p^{(3)} > p$, the gain is positive by the above argument. Finally, in the event $p^{(3)} = p$ and bidder i was not selected to participate, the gain is also positive because either $p < v$ and both strategies (acquisition or no acquisition) yield a positive payoff, or $p \geq v$, and it must be that $\sigma(p^{(3)})$ prescribes to acquire information, and thus bidder i gets a positive payoff (by claim A). So the deviation is profitable, yielding a contradiction. The second part of the claim follows from Claim A. **Q.E.D.**

5.2 Revenues: A numerical example

Proposition 5 is silent about the number of bidders sufficient to get an increase in revenues. It is also silent about the magnitude of the effects. We attempt to remedy this by looking at a numerical case. We assume that each θ_i is drawn from the uniform distribution on

$[0,1]$.[31] We choose a cost of information acquisition equal to 5% of the expected value, that is, $c = 0.025$. We assume that the distribution over (n_1, n_2) puts a weight close to 1 on some fixed (n_1^0, n_2^0).

With $n_1^0 = 5$, i.e. 5 initially informed bidders, uninformed bidders have no incentives to acquire information. Besides, under the assumptions above we have:[32]

$$p^* = 0.55$$

that is, uninformed bidders are ready to remain active at least 10% above their expected valuation without acquiring information. With a large number of uninformed bidders, these wait and see strategies *alone* generate an increase in revenue at least equal to 2.8%, a figure obtained by evaluating expression (4).

5.3 The role of information transmission

An important feature of the ascending price auction is that in the course of the auction, bidders learn about the number of bidders who are still interested in the object. (This is also true, though to a lesser degree, in the two-stage variant mentioned above.) To illustrate why this feature of the ascending format is important, we now briefly examine the *ascending price auction with secret drop out.* The auction is identical to the ascending auction, except that bidders do not observe whether and when other bidders drop out until the auction gets to a complete end (i.e. until there is only one bidder left).

Consider the case where n is large enough so that,

$$E[h(\theta^{(1)})] - c < 0. \tag{5}$$

which implies that in the static auction, uninformed bidders do not acquire information in equilibrium (see Section 3).

> **Claim C:** When condition (5) holds, then, in the ascending price auction with secret drop out, it is an equilibrium for uninformed bidders not to acquire information.

[31] More precisely, we consider the uniform distribution over $\{0, 1/K, ..., k/K, ..., 1\}$, and we consider the case where K is very large.

[32] With a uniform distribution, $h(p) = (1-p)^2/2$, and for $c = 0.025$, p^* is determined by the equality $(1/2 - p) + (1-p)^2/2 = 2c$.

Intuitively, learning that the price continues to raise may only be bad news. When there are no other uninformed bidders ($n_2 = 1$), or when $p > v$, observing the current price p signals that $\theta^{(1)} > p$, and it is readily verified that

$$E[h(\theta^{(1)}) \mid \theta^{(1)} > p] < E[h(\theta^{(1)})].$$

And when $n_2 \geq 2$ and $p \leq v$, observing the current price p conveys no information. It follows that when condition (5) is met acquiring information yields a negative expected payoff whatever the current price. Hence, there is no information acquisition in equilibrium.

Rezende (2005) in an independent work studies the ascending price auction with secret drop-out in a model where information acquisition costs may be arbitrarily small and heterogeneous among bidders. He shows that when the number of bidders is sufficiently large, the revenues generated by this auction is greater than the revenues generated by the sealed-bid second price auction. At first glance, Rezende's finding seems to contradict the message of claim C. Note however that in Rezende's setting there is always some amount of information acquisition, even in the sealed-bid auction (because information acquisition costs may be arbitrarily small). If these costs could not be arbitrarily small, the same insight as in claim C would arise even with heterogeneous costs.

Another important observation is that Rezende's insight entirely hinges on the feature that information acquisition costs need not be paid in case a bidder wins the auction. It is the prospect of buying the object without having to pay for the information acquisition cost that leads bidders to postpone the point at which they acquire information in Rezende's auction format.[33] In a number of applications though, it seems natural to assume that these costs have to be paid anyway by the winning bidder (see the Remark after the description of the model). In such applications, the ascending price auction with secret drop out would lead bidders willing to acquire information to do so at the start of the auction, and thus this format would be equivalent to the sealed-bid second price auction. By contrast, our main insight does not hinge on this feature: In our setup, it is the prospect of a higher chance of winning (as revealed by the number of active bidders) that leads bidders to acquire information more often in the ascending price auction.

[33] It also leads some bidders who would not acquire information in the sealed-bid format to acquire information in Rezende's ascending format.

Finally, note that other variants of the ascending price auction include the possibility (see Harstad and Rothkopf 2000 and Izmalkov 2003) that bidders might re-enter after dropping out.[34] In such formats, the observation that there are few bidders around may not be as reliable as in the ascending format analyzed in Sections 2 to 4, and it seems likely that such formats will generate less information acquisition than the one studied here.

5.4 Multi-object auctions.

While our formal analysis has been based on a single object problem, we show in Compte-Jehiel 2002a (in a simple model) that our basic insight carries over to multi-object auctions. In these auctions, not only do bidders have to decide whether or not to acquire extra information; but when they do so, they have to decide *on which object* to acquire information. This suggests that ascending formats that generate information on which object a bidder has better chances of winning are likely to perform better than formats such as static ones that do not generate such information: when a bidder is not guided as to which object(s) to focus on, he takes poor information acquisition decisions, which in turn is likely to discourage him from acquiring any information. In contrast, when a bidder is guided as to which objects he should focus on, he makes good information acquisition decisions, which in turn leads him to acquire extra information more often. Such a principle may guide the practitioner in his choice of activity rule (see Ausubel and Milgrom 2001 for an account of why this may be of practical importance in package auctions.)

5.5 The continuum type case

Our assumptions that price increases by increments and that valuations can only take a finite number of values allowed us to show the existence of equilibrium using standard existence results. This subsection discusses the existence issue when there is a continuum of valuations θ_i.[35] Note that the main difficulty is about determining the equilibrium strategy of bidders who know their valuations θ_i.[36] With a continuum of types, existence results can be found in the literature for static games of incomplete information (see, in particular, Athey 2001).

[34] Alternatively, bidders may sometimes have the possibility to hide that they are still around.

[35] We do not discuss the case in which the price increases continuously.

[36] There is only type of uninformed bidders, and since the number of (relevant) histories is finite there is no problem dealing with them.

Similar ideas can be transposed to our multi-stage framework. The key idea would be to show that informed bidders' optimal response (to any strategy profile of other bidders) is a threshold strategy. That is, if it is optimal for bidder i with valuation θ_i to remain active at some stage, then *a fortiori*, it should be optimal for bidder i with valuations $\theta_i' > \theta_i$ to remain active as well. Such a monotonicity property is referred to as single-crossing in the literature, and whenever it holds, every informed bidder i's strategy can be summarized by a scalar. This in turn ensures that standard fixed point theorems can be used to establish the existence of an equilibrium (see Athey 2001).[37]

As a matter of fact, in the context of the ascending price auction as described in Section 2, there is no problem applying that technique. Bidder i with valuation θ_i has to compare (after some history) the value derived from remaining active to the value derived from dropping out (which is 0). It can easily be proved that the value derived from remaining active is an increasing function of the valuation θ_i,[38] so the difference in the value derived from these two options (remaining active or dropping out) is also increasing in θ_i. So the optimal response necessarily takes the form of a threshold strategy.

Finally, let us emphasize that our result does not require pinning down exactly the behavior of informed bidders. The reason is that as Δ tends to 0, the behavior of informed bidders is almost entirely determined by dominance arguments (see Proposition 1). We thus conjecture that our result would continue to hold for weaker solution concepts, such as ones based on a few rounds of elimination of dominated strategies.

[37] When this single crossing property holds, a bidder's optimal threshold varies continuously with the thresholds used by others. Existence then follows because there is a finite number of possible histories (price increments are discrete, and after a while, it is a dominant strategy to drop out), hence a finite number of thresholds.

[38] The reason is as follows. Consider an history of the game for player i, and any continuation strategy σ involving remaining active at the current date. If bidder i with valuation θ_i obtains the object with probability p and makes an expected payment t by following σ, thereby obtaining an expected payoff equal to $\theta_i p - t$, then bidder i with valuation θ_i', by following σ can secure an expected payoff equal to $\theta_i' p - t$. Because we consider trembling hand equilibria, $p > 0$, and thus the value is strictly increasing in θ_i.

5.6 Dynamic screening.

We conclude by taking a broader perspective, and we consider the more general issue of how one should organize screening among agents who may acquire further information about their types.

From a general Principal-Agent perspective, we have dealt with a setting in which a principal (the seller) attempts to screen among agents (potential buyers) who may acquire information about their types (valuations), or invest so as to affect their types (as in the alternative formulation – see expression (2) in Section 2), *prior to signing a contract (the sale).* As in our sale example, the principal may find it desirable to provide agents with incentives to acquire information or invest prior to signing a contract, because this improves the chance of selecting a more able agent; that is, an agent with a better type (See Compte-Jehiel 2002b). In this context, our analysis suggests that a dynamic screening procedure, that leaves the competing agents some time to acquire information about their types, would outperform static screening procedures.[39]

Another example along these lines is the case of a sponsor who wishes to induce potential contestants (and possibly the ablest one) to exert high research effort. When research outcomes are not measurable or contractible, one option for the sponsor is to organize a tournament in which the winner gets a fixed prize. As mentioned in the introduction, inducing high research effort is sometimes more economically achieved by reducing the number of contestants (rather than increasing the prize). In such settings, it is important for the sponsor to screen among potential contestants so as to induce the participation of the ablest ones only.[40]

How should one organize screening? This issue is of primary importance, as failing to

[39] Cremer and Khalil (1992) consider a principal-agent setup in which the agent can learn his type before signing the contract. Most of their analysis bears on the one-agent case in which no screening is needed (their main insight is that the principal should not induce the agent to learn his type before the contract is signed). In the multi-agent section of their paper, Cremer and Khalil restrict attention to contracts in which the agents do not have incentives to learn their type. However, (unlike in the one-agent case) such contracts need not be optimal.

[40] When contestants have identical abilities (as in Che and Gale), this can easily be achieved by a random selection fo a subset of contestants. However, when they are not ex ante identical, a finer screening device is required.

screen good contestants may jeopardize the success of the tournament, and Fullerton and McAfee have identified why some screening procedures (that would auction rights to participate in the tournament) could fail to screen properly.[41]

Though screening is often thought of as a pure adverse selection problem, it seems plausible that some contestants may only have a rough idea of how successful their research effort will be, and that by investing a bit *prior to the tournament*, they could have a much better idea of how able they are. Besides, the sponsor could clearly benefit from such investments, since this should help him select the *truly* ablest contestants. Here again, a dynamic screening procedure would outperform static screening procedures.[42]

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[41] They suggest to consider all-pay auctions.

[42] To illustrate, and in order to abstract from the issues raised in Fullerton McAfee, assume that the outcome of research is measurable, and that it can be contracted on. Formally, the outcome of research for contestant i is a random variable x_i, and we assume that the sponsor auctions (through an ascending auction) a contract that pays $P(x)$ for the outcome x. The cost of research is assumed to be identical across contestants, and equal to γ. The bidders differ in how likely they are to produce good research, that is, contestant i's distribution over research outcome is $f(. \mid \theta_i)$ where θ_i is interpreted as contestant i's ability. When all bidders are aware of their ability, the auction selects the bidder for which $E[P \mid \theta_i]$ is highest. What if some bidders do not know θ_i precisely, but only have a rough idea of it, and if at some cost c, they could get to know θ_i? This is precisely the setting we have analyzed, with the conclusion that it is generally worthwhile to design a procedure that allows participants to take some time to acquire information during the process.

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Appendix

Proof of Proposition 1: Consider $p < \theta - \Delta$. By remaining active and waiting $p + \Delta$ to drop out, a bidder with valuation θ gets a non-negative payoff in the event the price rises to $p + \Delta$, and he gets a positive payoff in the event the price does not rise. Since the latter event has positive probability (in the game with trembles), dropping out at $p < \theta - \Delta$ is a dominated strategy.

Consider $p > \theta$ and assume bidder i is given the option to drop out. Bidder i obtains 0 by dropping out. If he remains active, there is a positive chance (in the game with trembles) that bidder i will eventually get the object (thus at a loss). So remaining active at $p > \theta$ is a dominated strategy. So it cannot be that in a trembling hand perfect equilibrium, bidder i with valuation θ drops out at some price $p > \theta + \Delta$ (he drops out earlier).

Proof of Lemma 1: Consider the event where at price p^-, there are two bidders left ($m = 2$), and one bidder, say bidder i is still uninformed. Conditional on this event, we let ϕ_l , $l \geq 0$, denote the (equilibrium) probability that his opponent drops out at $p_l = p + l\Delta$ (if given the option to drop out)[43]. Assume by contradiction that in equilibrium, bidder i has incentives to acquire information at p_{l+1}^-, and let $\sigma(\theta_i)$ denote the strategy followed by bidder i once he learns his valuation θ_i. Assume that instead, bidder i acquires information at p_l^-, and then depending on the realization of θ_i: either $\theta_i \leq v$, and he drops out at p_l if given that option, or he remains active one more period (at least) and follows $\sigma(\theta_i)$ at p_{l+1} otherwise. In expectation, this alternative strategy generates an extra acquisition cost, equal to $\phi_l c$ (because by waiting p_{l+1}^- to acquire information, with probability ϕ_l, bidder i's opponent would have dropped out and bidder i would have avoided acquiring information). However, under that event where bidder i's opponent drops out at p_l if given that option, and under the event where $\theta_i < v$, bidder i saves $v - \theta_i$ with probability 1/2, so in expectation, bidder i gains at least:[44]

$$\phi_l \sum_{k, \theta^k \leq v} \frac{v - \theta^k}{2} f(\theta^k) = \phi_l G(v).$$

which is greater than $\phi_l c$ by assumption. So it is a profitable deviation, hence it cannot be

[43] If the opponent is not given the option to drop out, it must be because bidder i was given that option before and decided to drop out.

[44] Bidder i also gains under the event where the opponent drops out p_{l+1}, because acquiring information at p_l^- allows him to drop out before his opponent.

that bidder i acquires information at p_{l+1}^- with positive probability. (Note that in the game with trembles ϕ_l is positive for all l).

Similarly, assume by contradiction that in equilibrium, bidder i has incentives to drop out at p_{l+1} (without having acquired information before). By dropping at p_l instead, bidder i would save $\frac{p_l - v}{2}$ in the event the opponent was willing to drop out at p_l, and $\frac{p_{l+1} - v}{2}$ in the event the opponent was willing to drop out at p_{l+1}. So again, this is a profitable deviation. It cannot be that bidder i drops out at p_{l+1} with positive probability.

Proof of Lemma 2: Under the conditions of Lemma 1, assume that bidder i follows σ_p^*. Since $p + \Delta > v$, we know from Lemma 1 that the other bidder, say bidder j, must either have dropped out by $p + \Delta$ (that is, at p or $p + \Delta$) or he must have acquired information by $p + \Delta$. We derive bounds in each case.

In case bidder j drops out at p, bidder i gets at least $G(p)$. Indeed, in the event $\theta_i < p$, both bidders wish to drop out (if given the option), hence bidder i has a chance $1/2$ of getting the object, so he gets $(\theta_i - p)/2$ in expectation; and in the event $\theta_i > p$, bidder i remains active and obtains the object at price p.

Similarly, in case bidder j drops out at price p_l, $l \geq 1$, bidder i (who drops out at $p_{l'}$ if $\theta_i \in (p_{l'-1}, p_{l'}]$) obtains:

$$\sum_{k, \theta^k \in (p_{l'-1}, p_{l'}]} \frac{\theta^k - p_l}{2} f(\theta^k) + \sum_{k, \theta^k > p_l} (\theta^k - p_l) f(\theta^k) \geq h(p_l) - \Delta/2$$

We use this lower bound to deal with the case where bidder j is informed at p or gets informed at $p + \Delta$.

(a) If bidder j has a valuation $\theta < p + \Delta$, he either drops out at p, in which case bidder i gets $G(p)$, or he drops out at $p + \Delta$, in which case bidder i gets at least $h(p + \Delta) - \Delta/2$, which is larger than $H_\Delta(p) - \Delta/2$ (because $h(.)$ is decreasing).

(b) If bidder j has a valuation $\theta > p + \Delta$, bidder j drops out at some price p_l with $l \geq 1$, with $p_l \leq \theta + \Delta$. Thus, since $h(.)$ is decreasing, bidder i obtains a payoff at least equal $h(\theta + \Delta) - \Delta/2$, hence at least equal to $H_\Delta(p) - \Delta/2$ in expectation.**Q.E.D.**

Proof of Proposition 5: In what follows, we denote by n_1 (respectively n_2) the number of initially informed (respectively initially uninformed) bidders, and, in case $n_2 \geq 1$, and remember that $\theta_u^{(1)}$ (respectively $\theta_u^{(2)}$) denotes the largest (respectively second largest) valuation among the initially uninformed bidders (in case $n_2 \leq 1$, we set $\theta_u^{(2)} = 0$). We denote by R^s (respectively R^d) the realized revenue in the static auction (respectively the dynamic auction). We also let $\widehat{p} = (v + p^*)/2$.

We are going to show that for any n large enough, we can choose Δ small enough so that the expected difference $R^d - R^s$ is strictly positive (and bounded away from 0 as Δ tends to 0).

Step 1: We first identify events in which revenue decreases by no more than Δ. That is, consider the events $B_0 = \{n_2 = 0\}$ and $B_1 = \{n_2 \geq 1, \theta^{(2)} \geq v \text{ or } \theta_u^{(2)} \geq v\}$. We show that in any event in $B_0 \cup B_1$ the loss is at most Δ, i.e.

$$R^d - R^s \geq -\Delta$$

Under B_0, all bidders are informed. Thus $R^s = \theta^{(2)}$. Since, in the ascending auction, any informed bidder i remains active until $\theta_i - \Delta$ at least, we have $R^d \geq \theta^{(2)} - \Delta$, hence $R^d - R^s \geq -\Delta$.

Under B_1, uninformed bidders bid v in the static auction, so we have $R^s \leq \max(v, \theta^{(2)})$. In the ascending price auction, Proposition 4 shows that any uninformed bidder i either acquires information (and does not drop out before $\theta_i - \Delta$) or he remains active until at least p^*. So, either all uninformed bidders eventually acquire information, in which case revenue is at least equal to $\max(\theta^{(2)}, \theta_u^{(2)}) - \Delta$, or at least one uninformed bidder remains active until p^*, in which case revenues is at least equal to $\max(\theta^{(2)}, p^*)$. In either case, $R^d - R^s \geq -\Delta$.

Step 2: We now identify events under which the gain is substantial. That is, consider the event $C = \{\theta^{(2)} \in [v, \widehat{p}], n_2 \geq 1\}$ and the event $D = \{n_2 \geq 1, \theta^{(2)} < v, \theta_u^{(2)} > p^*\}$. Also let $r_C = [1 - F(p^*)]^2(p^* - \widehat{p})$ and $r_D = [1 - F(p^*)](p^* - v)$. We show that for $X = C$ or D:

$$E[R^d - R^s \mid X] \geq r_X - \Delta$$

Under C, $R^s = \theta^{(2)} \leq \widehat{p}$. Now conditional on C, the event where $\theta^{(1)} > p^*$ and one uninformed bidder has a valuation at least equal to p^*, has strictly positive probability, at least equal to $[1 - F(p^*)]^2$. Under the latter event, either all uninformed acquire information, and price rises until p^* at least, or one uninformed bidder does not acquire information, but by Proposition 4, and since $\theta^{(1)} > p^*$, he must be remaining active until p^*. In either case, $R^d \geq p^* - \Delta$. Since $C \subset B_1$, it follows that

$$E[R^d - R^s \mid C] \geq [1 - F(p^*)]^2(p^* - \widehat{p}) + (1 - [1 - F(p^*)]^2)(-\Delta) \geq r_C - \Delta.$$

Under D, $R^s \leq v$. Now conditional on D, consider the event where $\theta^{(1)} > p^*$. (This event has probability at least equal to $1 - F(p^*)$). Under that event, price rises up to p^* at least, hence revenue increases by at least $p^* - \widehat{p}$, which concludes step 2 since $D \subset B_1$.

Final step: There are events where revenues may decrease (by more than Δ). By step 1, this may only happen under event $A = \{\theta^{(2)} < v, \theta_u^{(2)} < v\}$, that is, when competition is weak and uninformed bidders learn that their valuation is low. The loss however cannot be larger than v (since $R^s \leq v$ under A).

Defining $Q = \Pr(A \cup C \cup D)$ and $\pi = \Pr(A \mid A \cup C \cup D)$, we can thus bound the expected difference in revenues as follows.

$$\begin{aligned} E[R^d - R^s] &\geq Q(\pi(-v) + (1-\pi)(r_0 - \Delta) + (1-Q)(-\Delta) \\ &\geq Qr_0 - \pi Q(v + r_0) - \Delta \end{aligned}$$

where $r_0 = \min(r_C, r_D)$. To conclude on whether revenues increase in expectation, we just need to evaluate the probability π. Our result follows from the observation that π vanishes when n gets large. To see this, observe that since either $n_1 \geq n/2$ or $n_2 \geq n/2$, we have

$$\pi \leq \max(\Pr[A \mid A \cup C \cup D, n_2 \geq n/2], \Pr[A \mid A \cup C \cup D, n_1 \geq n/2])$$

π vanishes when n gets large because

$$\Pr\{A \mid A \cup C \cup D, n_2\} \leq \frac{\Pr\{\theta_u^{(2)} < v \mid n_2\}}{\Pr\{\theta_u^{(2)} \geq p^* \mid n_2\}},$$

which vanishes when n_2 gets large, and because

$$\Pr\{A \mid A \cup C \cup D, n_1\} \leq \frac{\Pr\{\theta^{(2)} < v \mid n_1\}}{\Pr\{\theta^{(2)} < \widehat{p} \mid n_1\}}$$

which vanishes when n_1 gets large. **Q.E.D.**