

# Bargaining and Majority Rules: A Collective Search Perspective\*

Olivier Compte<sup>†</sup> and Philippe Jehiel<sup>‡</sup>

revised, February 2008

## Abstract

We study a collective search process in which tentative proposals arrive sequentially and members of a committee decide whether to accept the current proposal or continue searching. The acceptance decision is made according to a (qualified) majority rule. The effects of patience, of the distribution over tentative proposals and of the majority requirement are examined. In contrast to standard models of bargaining, we find a tradeoff between majority and unanimity rules: As the majority requirement becomes more stringent, some inefficient outcomes are avoided, but it takes more time to reach a decision. We also study which members have more impact on the decision, and we relate the equilibrium outcomes obtained to some classic results in bargaining and voting including the Nash bargaining solution and the median voter outcome.

## 1. Introduction

We study a model of collective decision making. A committee is entitled to adopt and implement a proposal. Proposals arrive sequentially, exogenously, and are drawn from

---

\*This paper has circulated under the title “Bargaining over randomly generated offers”. The authors are grateful to seminar audiences at University of Essex, Barcelona (JOCS), Paris (Roy), UCL (London), Stockholm, Yale, Princeton, University of Pennsylvania, NYU, Northwestern University, University of Chicago, Columbia.

<sup>†</sup>PSE, 48 Boulevard Jourdan, 75014 Paris. e-mail:compte@enpc.fr

<sup>‡</sup>PSE and UCL; e-mail: jehiel@enpc.fr

identical and independent distributions. The members of the committee have different preferences over the possible proposals. Upon arrival of a proposal, each member has to decide whether he votes for it or whether he prefers to wait. The examined proposal is implemented whenever it receives the required majority of support. Our analysis focuses on the role of the majority requirement in determining the speed and (ex ante) efficiency of the decision process, as well as on the role of the distribution from which proposals are drawn and the role of impatience.

Many recruitment decisions take this form. A candidate is examined by a committee composed of members with diverse preferences over potential candidates, and the committee has to decide whether to recruit now or pass this opportunity and wait for another candidate. A housing decision, in which both members of the household have a say in the decision, and in which potential houses arrive sequentially, is another example. These situations can be viewed as collective search problems. They differ from standard search models (see Diamond (1982)) in that the decision to stop searching has to be approved by sufficiently many people (and not just one agent).<sup>1</sup>

Our model applies to collective search problems, but it can also be viewed as a *stylized model of bargaining*. In real negotiations involving complex issues or many individuals, agents have some influence over the proposals put to a vote. Yet the elaboration of a given proposal is rarely under a single agent's control. As for the generation of new ideas, the process generating proposals surely contains stochastic elements in it. While standard models of bargaining portray each party as having *full* control over the proposals *he* makes, our search model can be interpreted as a bargaining model in which the process generating proposals is not under parties' control.<sup>2</sup>

We report three kinds of result.

---

<sup>1</sup>In a recent paper, Strulovici (2007) considers the issue of experimentation by voting. He extends the literature on experimentation in the same way as we extend the search literature by considering a multi-agent setup with voting procedures rather than a single agent setup.

<sup>2</sup>By focusing on the case in which parties have no influence over proposals, we emphasize the stochastic nature of the proposal process. But our model also paves the way to studying intermediate situations in which parties devote resources to influence the proposal process, or to put it differently, to studying the effect of lobbying efforts in negotiations. See Section 7 for further discussion.

- We identify a trade-off between unanimity and majority rules and show that majority rules may be ex ante more efficient than the unanimity rule. We also show that in general, patience makes the unanimity rule more attractive.
- We show that if agents are patient, the support of possible proposals is sufficiently rich and the unanimity rule applies, our search problem has the same prediction as a standard bargaining model: only proposals close to the Nash bargaining outcome are accepted.
- Under single-peakedness (with preferences and proposals defined over a single dimension) and with patient agents, we find that as the majority requirement rises, the set of accepted proposals is determined by members with more extreme taste (as compared to the median member).

#### **Detailed overview and intuitions.**

*Majority versus unanimity.* (Section 3) Compared with the unanimity rule, majority rules ensure a faster decision process, but they may lead to less efficient decisions. In essence, the decision process is comparatively fast under majority rules because finding a proposal that pleases fewer members is easier. However the adopted proposal may harshly hurt the minority, hence the decision may be very inefficient. By contrast, under the unanimity rule no minority can be hurt as every member has the option to reject the proposal.<sup>3</sup> But this very option results in a slower decision process because it is harder to please everyone. Which of the unanimity or the majority rule dominates will thus depend on the magnitude of the waiting costs and on the distribution over offers. If the support of possible proposals contains mostly (welfare) efficient proposals, then majority rules dominate unanimity rules, as the main issue is the speed of the decision process. If proposals may be inefficient, then the unanimity rule may be preferable, at least when members are patient enough.

The cost associated with the unanimous rule is alluded to in a number of classic writings. For example, Black (1958, page 99) writes:<sup>4</sup>

---

<sup>3</sup>Absent waiting costs, this observation ensures a form of Pareto efficiency of the unanimity rule that agrees with the early observation made by Wicksell (1896) (in favor of unanimity rules).

<sup>4</sup>The smaller the likelihood of reaching a decision translates in a slower decision process in our

"The larger the size of majority needed to arrive at a new decision on a topic, the smaller will be the likelihood of the committee reaching a decision that alters the existing state of affairs."

In a similar vein, Buchanan and Tullock (1962) argue against the unanimity rule emphasizing the "decision-making costs" associated with it. While Buchanan and Tullock are not explicit about the nature of these "decision-making costs", our setting offers a formalization of these in terms of waiting costs.

Our result contrasts with that obtained in classical bargaining models in which utilities are transferable, proposals are freely chosen by parties and the pie over which parties are bargaining has a fixed size. Such models would instead conclude that majority and unanimity rules are welfare-equivalent: an agreement would be reached immediately on an efficient outcome whatever the decision rule, thereby ensuring that every decision rule is equally efficient (see Baron and Ferejohn (1989)).<sup>5</sup>

Our result also differs from that obtained when the size of the pie may vary stochastically and yet offers are perfectly controlled by the party selected to make an offer. In such models, Merlo and Wilson (1998) have shown that the unanimity rule results in an ex ante welfare efficient outcome while Eraslan and Merlo (2002) have pointed out that majority rules may lead to inefficiencies.<sup>6</sup> Thus, such models conclude that the unanimity rule dominates majority rules whatever the (im-)patience of parties.<sup>7</sup>

Our insight that majority rules may be preferable to the unanimity rule when parties are not arbitrarily patient is -we believe- of practical importance, as it helps understand, for example, why the European Union has been so eager to alleviate the majority requirement imposed on many decisions after several new countries have joined the E.U. 

---

<sup>5</sup>The reason why agreement is immediate is that, whatever the decision rule, the maximum surplus the proposer can extract is always by having a proposal approved (a disagreement is necessarily suboptimal).

<sup>6</sup>Majority rules may lead to too early agreements in response to the fear that a bigger pie to be shared next may benefit too much a left aside party whose consent is not required to get an agreement now.

<sup>7</sup>The conclusion that the unanimity rule dominates majority rules also applies to the class of models considered in Gomes and Jehiel (2005) in which parties repeatedly bargain/vote over the transition to a new state in exchange for side-payments.

(see the treaty of Lisbon signed on Dec. 13th 2007).<sup>8</sup>

*Search versus Bargaining under unanimity.* (Section 4) If all members have different tastes and the set of possible proposals is sufficiently rich (in the utility space, the set of proposals has as many dimensions as there are members), we show that under the unanimity rule, sufficiently patient members will agree only on proposals close to the Nash bargaining solution (i.e., a proposal that maximizes the product of payoffs of all members), and we show that this is true for *any* given (non-degenerate) distribution over proposals.

Such an insight has precedents in the literature. In a two-person bargaining game à la Rubinstein, Binmore et al (1986) show how the unique subgame perfect Nash equilibrium outcome approaches the Nash bargaining solution as parties get patient. Compared to Binmore et al., our analysis reveals that what is key for this insight is parties' ability to veto proposals rather than their ability to control the proposal process. Even more closely related to this insight, Wilson (2001) considers the two-person version of our model under the unanimity rule and he also obtains the link to the Nash bargaining solution as parties get very patient. In the discussion section of his paper, Wilson also suggests how his insight could be extended to the more than two player case under additional assumptions on the distribution of offers. The insight we obtain in the unanimity rule case (for the case of rich proposal space) generalizes Wilson's insight for the case of more than two players, in particular revealing that there is no need for further assumption on the distribution of offers.

*Varying the majority requirement under single-peakedness.* (Section 5) If proposals vary only according to a single dimension (as in most static voting models), we show under the simple majority rule that sufficiently patient members will agree on decisions close to that preferred by the median member. Thus, our approach agrees with the classic median voter theory (Black 1958), when it applies.<sup>9</sup> We further characterize who

---

<sup>8</sup>As we will illustrate, our framework explains why as more members join the collective search problem, a less stringent majority requirement may be preferable (in short, this is because the waiting costs attached to the unanimity rule then become prohibitive).

<sup>9</sup>Note that we do not need that proposals be distributed along a single dimension to study the simple majority rule (the same comment applies also to the bargaining approach followed by Baron

drives the decision in such a one-dimensional setting when preferences are concave, and one moves from the simple majority rule to more stringent majority rules. Specifically, we find that members with more extreme tastes (as compared with the median voter) become pivotal as more stringent majority rules are required. At the extreme, in the unanimity case, the decision is solely driven by the most extreme types.

*Applications.* (Section 6) We address three questions. Is it necessarily good that other members share one's own taste? No. Can one benefit from participating and letting other members decide? Yes. Can members benefit from ignoring a characteristic of the proposals? Yes.

*Extensions.* We discuss the case in which parties have some control or influence over the proposals put on the table (Section 7), and the case of asymmetric discount factors (Section 8). We also compare the insights obtained in our model with those obtained in the well-studied random proposer model (Section 9).

## 2. The Model

We consider a committee consisting of  $n$  members, labeled  $i = 1, \dots, n$ .

*Timing.* At any date  $t = 1, \dots$ , if a decision has not been made yet, a new proposal is drawn and examined. The set of proposals varies in a space of dimension  $m \geq 1$ . We denote by  $x$  a proposal, by  $X$  the set of proposals, and we assume that  $X$  is isomorphic to  $[0, 1]^m$ . We also assume that proposals at the various dates  $t = 1, \dots$  are drawn *independently* from the same distribution with continuous density  $f(\cdot) \in \Delta(X)$ .<sup>10</sup>

Upon arrival of a new proposal  $x$ , each member decides whether to accept that proposal. We consider various majority rules. Under the  $k$ -majority rule, the game stops whenever at least  $k$  out of the  $n$  members vote in favor of the proposal.

*Preferences.* We let  $u_i(x)$  denote the utility that member  $i$  derives from decision  $x$  at the time it is implemented. We assume that  $u_i(\cdot)$  is continuous, and we normalize to

---

and Ferejohn 1989). See also Banks and Duggan (2000) for a relation of the median voter approach to Baron-Ferejohn's model.

<sup>10</sup>So members have no influence over the proposals that are examined. In Section 7, we will discuss an extension of the model where members have partial control over the distribution of proposals.

0 the payoff that parties obtain under perpetual disagreement. We let  $X_0$  denote the set of proposals that are individually rational for all players:

$$X_0 = \{x \in X, u_i(x) > 0 \text{ for all } i \in \{1, \dots, n\}\}$$

Throughout the paper, we will assume that  $X_0$  is non-empty.

We assume that payoffs are discounted with a common discount factor  $\delta$ , so that viewed from date 1, a decision  $x$  agreed upon at date  $t$  yields party  $i$  a discounted payoff equal to  $\delta^{t-1}u_i(x)$ .<sup>11</sup>

*Strategies and equilibrium.* In principle, a strategy specifies an acceptance rule that may at each date be any function of the history of the game. We will however restrict our attention to *stationary* equilibria of this game, where each member adopts the same acceptance rule at all dates.<sup>12</sup>

Given any stationary acceptance rule  $\sigma_{-i}$  followed by members  $j, j \neq i$ , we may define the expected payoff  $\bar{v}_i(\sigma_{-i})$  that member  $i$  may derive given  $\sigma_{-i}$  from following his (best) strategy. An optimal acceptance rule for member  $i$  is thus to accept the proposal  $x$  if and only if

$$u_i(x) \geq \delta \bar{v}_i(\sigma_{-i}),$$

which is stationary as well (this defines the best-response of member  $i$  to  $\sigma_{-i}$ ).

Stationary equilibrium acceptance rules are thus characterized by a vector  $v = (v_1, \dots, v_n)$  such that member  $i$  votes in favor of  $x$  if  $u_i(x) \geq \delta v_i$  and votes against it otherwise.<sup>13</sup> For any  $k$ -majority rule and value vector  $v$ , it will be convenient to refer to  $A_{v,k}$  as the corresponding acceptance set, that is, the set of proposals that get support from at least  $k$  members when failing to agree today yields member  $i$  a continuation

---

<sup>11</sup>In Section 8, we discuss the role of asymmetric discount factors.

<sup>12</sup>To avoid coordination problems that are common in voting (for example, all players always voting "no"), we will also restrict attention to equilibria that employ no weakly dominated strategies (in the stage game). These coordination problems could alternatively be avoided by assuming that votes are sequential.

<sup>13</sup>Strictly speaking, member  $i$ 's behavior is indeterminate when  $u_i(x) = \delta v_i$ . Yet, cases of indifferences, i.e. such that  $u_i(x) = \delta v_i$ , are irrelevant, as they have measure 0 (according to  $f(\cdot)$ ).

payoff of  $v_i$  (from the viewpoint of next period):<sup>14</sup>

$$A_{v,k} = \{x \in X, \exists K \subset \{1, \dots, n\}, |K| = k, u_i(x) \geq \delta v_i \text{ for all } i \in K\}. \quad (2.1)$$

Equilibrium consistency then requires that

$$v_i = \Pr(x \in A_{v,k})E[u_i(x) | x \in A_{v,k}] + [1 - \Pr(x \in A_{v,k})] \delta v_i \quad (2.2)$$

or equivalently

$$v_i = \frac{\Pr(x \in A_{v,k})}{1 - \delta + \delta \Pr(x \in A_{v,k})} E[u_i(x) | x \in A_{v,k}]. \quad (2.3)$$

A stationary equilibrium is characterized by a vector  $v$  that satisfies (2.1)-(2.2). We have:

**Proposition 1 (*Existence*)** *Whatever the majority requirement, a stationary equilibrium exists.*

**Proof:** Define the function  $v \rightarrow \phi(v)$ , where  $\phi_i(v)$  coincides with the RHS of Equation (2.2), and let  $\bar{u} = \max_{i,x} u_i(x)$ . The function  $\phi$  is continuous from  $[0, \bar{u}]^n$  to itself, hence it has a fixed point. **Q. E. D.**

To conclude this Section, we provide a simple two-player example that illustrates how one computes an equilibrium.

*A two-player example.* We consider the following example where (i) decisions require unanimity; (ii) proposals are drawn uniformly on the simplex  $X = \{(x_1, x_2), 0 \leq x_i, x_1 + x_2 \leq 1\}$ ; (iii) preferences are linear, i.e.  $u_i(x) = x_i$ . As shown in Figure 2.1, the acceptance set takes the form of a small simplex  $A = \{x = (x_1, x_2) | x_i \geq \delta v_i \text{ for } i = 1, 2\}$  where each  $v_i$  is a weighted average between  $E[x_i | x \in A]$  and  $\delta v_i$ , with a weight on the former equal to  $\Pr(A)$ . Since  $E[x_i - \delta v_i | x \in A] = \frac{1}{3}[1 - \delta v_1 - \delta v_2]$  and  $\Pr A = (1 - \delta v_1 - \delta v_2)^2$ , the equilibrium requirement (2.2) boils down to finding  $v$  and  $u = \delta v$  such that  $\frac{1-\delta}{\delta}u = \frac{1}{3}(1 - 2u)^3$ .

---

<sup>14</sup>For any finite set  $B$ ,  $|B|$  denotes the cardinality of  $B$ .

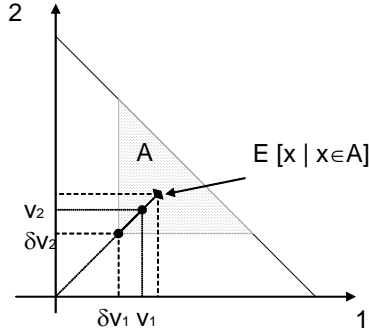


Figure 2.1:

### 3. Unanimity versus Majority

We wish to understand how decision rules compare in term of ex ante welfare as a function of the discount factor  $\delta$  and the number  $n$  of members. We first analyze a simple symmetric model for which the solution can be characterized simply. In that model, we show for any size  $n$  of the committee that, as  $\delta$  gets smaller, the optimal majority requirement  $k$  gets smaller, and we provide numerical simulations showing the loss associated with suboptimal choices of the majority requirement. Next, we go back to the general model and we explain what drives the comparison between majority and unanimity rules.

#### 3.1. A simple symmetric model.

We assume that proposals are drawn uniformly from the product set  $X = \times_i X_i$  with  $X_i = [-1, 1]$ , and that  $u_i(x) = x_i$  where  $x = (x_i)_{i=1}^{i=n}$ . This represents situations in which news proposals have expected value that coincides with the status quo, and where the effect of new proposals is independently distributed across members.

For any discount factor  $\delta$  and  $k$ -majority rule, we look for a symmetric equilibrium (i.e., the acceptance threshold of every member  $i$  is the same). We will show that the symmetric equilibrium is uniquely defined, and we denote by  $v(\delta, k)$  the corresponding equilibrium expected payoff obtained by each member. We also let  $\underline{v}(\delta, k) = \delta v(\delta, k)$

denote the corresponding threshold such that member  $i$  votes “yes” to proposal  $x = (x_i)_{i=1}^{i=n}$  if and only if  $x_i > \underline{u}(\delta, k)$ . A majority requirement  $k^*$  is optimal (given  $n$  and  $\delta$ ) if it maximizes  $v(\delta, k)$  over all  $k = 1, \dots, n$ .

To characterize the equilibrium, it is convenient to let  $P^m(u)$  be the probability that exactly  $m$  members out of  $n$  get a draw above  $u$  for arbitrary  $u \in (-1, 1)$ .<sup>15</sup> Let  $v$  be a tentative (symmetric) equilibrium value and  $u = \delta v$  the corresponding threshold. The consistency condition (2.2) can be rewritten as

$$\frac{(1 - \delta)}{\delta} u = \sum_{m \geq k} P^m(u) E[u_i(x) - u \mid m \text{ members vote yes}]$$

Define  $\phi(k, u)$  as the right hand side of the above expression. It is easy to check (see the Appendix) that

$$\phi(k, u) = \frac{1}{2} \sum_{m=k}^n P^m(u) \left[ \frac{2m}{n} - 1 - u \right]$$

**Proposition 2:** let  $u_k = \frac{2k}{n} - 1$  and  $\delta_k = \frac{u_{k-1}}{u_{k-1} + \phi(k, u_{k-1})}$ . For any  $\delta, k$ , the symmetric equilibrium is uniquely defined and the threshold  $\underline{u}(\delta, k)$  is characterized by the solution to

$$u = \frac{\delta}{1 - \delta} \phi(k, u). \tag{3.1}$$

$\underline{u}(\delta, k)$  is increasing in  $\delta$ . Besides, for any  $\delta \in (\delta_k, \delta_{k+1})$ , the optimal majority rule is the  $k$ -majority rule.<sup>16</sup>

Proposition 2 makes it explicit that for every number  $n$  of players as  $\delta$  gets smaller the best majority requirement  $k$  gets smaller.

Figure 3.1 and 3.2 provide numerical simulations for the equilibrium value  $v(\delta, k)$  as a function of  $k$  when  $\delta = 0.9999$ . We consider two cases:  $n = 15$  and  $n = 27$ .<sup>17</sup>

<sup>15</sup>That is, letting  $B^+(u, x) \equiv \{i \mid x_i \geq u\}$ , we define  $P^m(u) \equiv \Pr(|B^+(u, x)| = m)$ .

<sup>16</sup> For that rule, the equilibrium value belongs to the interval  $(u_{k-1}, u_k)$ .

<sup>17</sup>Note that in a short period of time, the number of countries forming the EU has increased from 15 to 27, which has led to the proposal of a new european constitution (now being considered as the Lisbon treaty).  $\delta = 0.9999$  corresponds roughly to the case in which new proposals (on a given prototypical issue) would arrive daily and the yearly interest rate is 3 per cents.

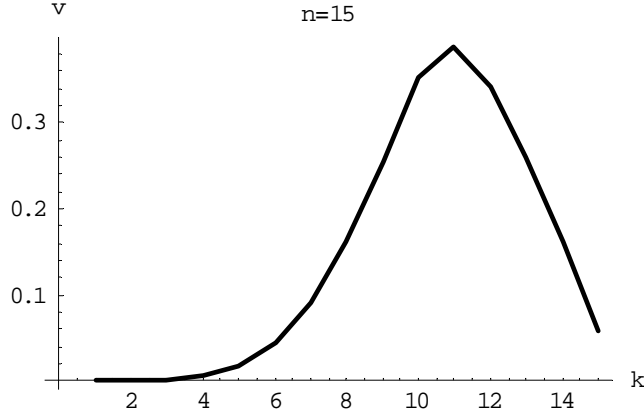


Figure 3.1:

Note that the optimal value of  $k$  corresponds to a 73% majority rule in the first case ( $k = 11$ ), and to a 66% majority rule in the second case ( $k = 18$ ). Also note that the cost associated with keeping the unanimity rule is significant.

### 3.2. Basic trade-off in the general model

We go back to the general case, and explain what drives the trade-off between majority and unanimity rules. To that end, we decompose the various effects of the majority requirement on the ex ante welfare (as measured by the expected sum of utilities obtained by all committee members in equilibrium.)

Specifically, define  $W(x) \equiv \sum_i u_i(x)$  as the welfare associated with proposal  $x$ , by  $v^k$  the equilibrium value profile associated with the  $k$ -majority rule, and by  $W_k \equiv \sum_i v_i^k$  the associated ex ante welfare. Summing expression (2.3) over all members  $i$  yields:

$$W_k \equiv \frac{\Pr(x \in A_{v^k,k})}{1 - \delta + \delta \Pr(x \in A_{v^k,k})} E[W(x) \mid x \in A_{v^k,k}] \quad (3.2)$$

Expression (3.2) shows that there may be two factors reducing welfare as compared with the maximum possible welfare level  $\bar{W} = \max_{x \in X} W(x)$ :

- There may be delays, because it may take some time before a proposal gets accepted: the smaller the term  $\frac{\Pr(A_{v^k,k})}{1 - \delta + \delta \Pr(A_{v^k,k})}$ , the more severe the decrease in welfare due

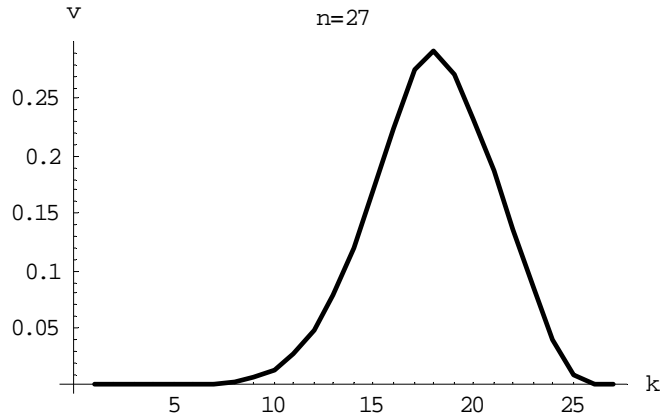


Figure 3.2:

to delays.

- The acceptance set may contain inefficient proposals in the sense that some proposals in  $A_{v^k, k}$  may not belong to  $\arg \max_{x \in X} W(x)$ . The reduction in welfare is all the more severe that  $A_{v^k, k}$  is far away from  $\arg \max_{x \in X} W(x)$ .

*Why majority rules may dominate?*

Intuitively, for a given profile of acceptance thresholds, as one reduces the majority requirement  $k$ , the acceptance set increases, hence delays are reduced. However, the acceptance set may now allow for proposals that are further away from the Pareto frontier, thereby inducing a welfare loss.

In case proposals are mostly efficient, the term  $E[W(x) \mid x \in A_{v^k, k}]$  in Expression (3.2) remains close to  $\bar{W}$  for all  $k$ , hence the only source of decrease in welfare is delay. Majority rules then dominate the unanimity rule because they reduce delay. Of course the comparison may be reversed when proposals may be significantly inefficient and members are patient enough (so that delay costs become negligible).

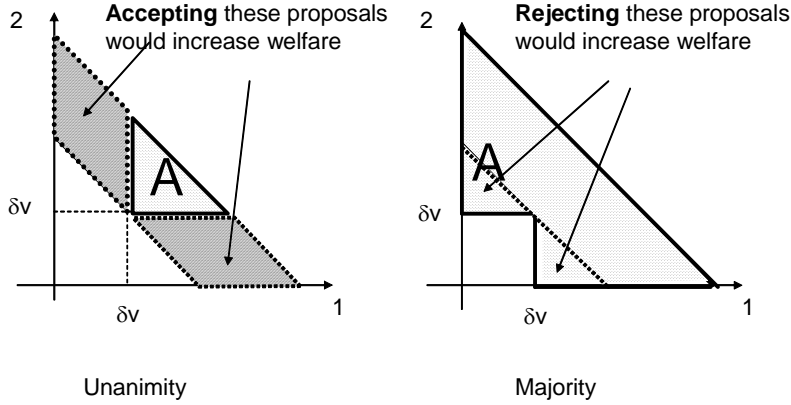


Figure 3.3:

*Comparison with first-best.*

Given the search friction (i.e. the randomness of proposals), waiting may be socially desirable,<sup>18</sup> and the maximum welfare that members can *jointly* obtain may be strictly smaller than  $\bar{W}$ . Let  $\bar{w}$  be that ex ante welfare value.<sup>19</sup> The socially efficient acceptance set  $\bar{A}$  would thus consist of the proposals  $x$  for which  $W(x) \geq \delta\bar{w}$ :

$$\bar{A} = \{x \mid W(x) \geq \delta\bar{w}\}.$$

All  $k$ -majority rules whatever  $k$  are socially inefficient because they induce acceptance sets that cannot take the form  $\bar{A}$ . For any tentative  $v$ ,  $k$ -majority rules either exclude proposals that are welfare superior to  $\delta W(v)$  (this is for example the case under the unanimity rule), or they include proposals that are welfare inferior to  $\delta W(v)$ . When the majority requirement is increased, inefficiencies of the second type are reduced, but inefficiencies of the first type are generated.

Figure 3.3 provides an illustration in a simple three player case in which player 3 would always accept (because, say, all proposals are equivalent for him). The figure

<sup>18</sup>Because the current proposal may be very inefficient.

<sup>19</sup>It is the value obtained in a standard one-agent search model in which this agent's utility is  $W(x)$ ,  $x$  arrives according to  $f(\cdot)$  and the discount factor is  $\delta$ .

draws the acceptance set in the space of player 1 and 2's preferences under the unanimity rule (Left) and under the majority rule (Right).

*The effect of patience. The limit case.*

Majority rules may be more efficient than the unanimity rule, but in general, this conclusion will be reversed when members are patient enough.

First consider the unanimity rule. We observe that the size of the acceptance set must tend to 0 as players get very patient (see below). It follows that the equilibrium value profile must get close to the Pareto frontier, since otherwise the size of the acceptance set would not be small.

To understand why the agreement set is small, assume by contradiction it is not. Then any member  $i$  could increase his demand significantly beyond his supposed continuation value  $\delta v_i$  while keeping the agreement set not too small. A favorable vote would still be obtained quickly (because the acceptance set would still be of significant size) and member  $i$  would be strictly better off. Such a deviation cannot be profitable in equilibrium. So the acceptance set must be small.

Now consider any  $k$ -majority rule with  $k < n$ , and assume that  $u(X)$  is  $n$ -dimensional, smooth and convex and that  $f(\cdot)$  has full support on  $X$ . Consider any candidate equilibrium value profile  $v$  and define

$$X(v) = \{x \in X, u(x_i) > v_i \text{ for at least } k \text{ members}\}.$$

Observe that no matter what the discount factor  $\delta$  is, the acceptance set includes  $X(v)$  (see the expression (2.1) of  $A_{v,k}$ ). Because  $u(X)$  is  $n$ -dimensional, smooth and convex, the set  $X(v)$  is also  $n$ -dimensional and has a size bounded away from 0 for any  $v$  and any  $k < n$ . Since  $f(\cdot)$  has full support, the expectation profile

$$E[u(x) \mid x \in X(v)]$$

is bounded away from the Pareto frontier. Hence, by (2.3), so must be the equilibrium value.

It follows from the above observations that for symmetric problems,<sup>20</sup> the unanimity rule dominates any  $k$ -majority rule (with  $k < n$ ) when members are patient enough.

#### 4. Unanimity with patient members.

Our model has two frictions: discounting and the noisiness of the proposal process. In this Section and the next, we will concentrate on the second friction, and provide predictions of our model assuming that the discount factor  $\delta$  is close to 1.

In this Section, we consider the *unanimity rule* and we assume that the *space of proposals is rich*. That is, we assume that local variations in the space of proposals generate all possible variations in the utility space. So in particular, the dimension of the space of proposals ( $m$ ) must be at least equal to the number of committee members ( $n$ ).

Formally, we make the following assumption, which not only ensures that the space of proposals is rich, but also that the Nash bargaining solution is uniquely defined, and that it is a non-degenerate point of the Pareto frontier:

**Assumption 1:** Assume that (i)  $u(X) = (u_i(X))_{i=1}^{i=n}$  is a smooth  $n$ -dimensional convex set, and (ii) the generalized  $n$ -person Nash bargaining solution,  $v^* = u(x^*)$  where  $x^* = \arg \max_{x \in X} \prod_i u_i(x)$ , is not a boundary point of the Pareto-frontier of  $u(X)$ .

We show that when the discount factor is close to 1, equilibrium outcomes must get close to the generalized  $n$ -person Nash bargaining solution (hence to the Rubinstein (1982) solution as well in the two person case).<sup>21</sup>

**Proposition 3:** *Let Assumption 1 holds, and assume that the distribution*

---

<sup>20</sup>Assuming  $X = [0, 1]^n$ ,  $f(\cdot)$  is symmetric in  $x$  if  $u_i(x) = u_j(x')$  whenever  $x$  and  $x'$  are obtained by permuting the  $i^{th}$  and  $j^{th}$  component.

<sup>21</sup>This can be viewed as the analog of Binmore et al. (1986) in our random offer bargaining setup. Note that we allow for more than two players, but yet restrict attention to stationary equilibria. Wilson (2001) obtains a similar characterization for the case of two players and he makes conjectures for the more than two player case.

of proposals  $f(\cdot)$  is bounded away from 0 and smooth on  $X$ . When  $\delta$  tends to 1, equilibrium values tend to the generalized  $n$ -person Nash bargaining solution  $v^*$ .

Several points are worth emphasizing. First, there is convergence to the generalized Nash bargaining solution, independently of the distribution from which proposals are drawn. Second, as already noted, the size of the acceptance set gets small as members get very patient. (This is unlike what happens for  $k$ -majority rules when  $k < n$ , see Section 3). Finally, compared to standard bargaining models where typically players have two strategic decisions to think of, i.e. what proposals to make, and what proposals to accept, our model restricts attention to the second strategic decision, and it shows that the right to veto alone is sufficient to drive the solution to the generalized Nash bargaining outcome.

We now provide the basic steps yielding the result.

The argument as to why as  $\delta$  tends to 1, equilibrium values must tend to the Pareto frontier has already been explained in Section 3.<sup>22</sup>

Now observe that since  $f(\cdot)$  is bounded away from 0 and smooth on  $X$ , conditional on acceptance, the distribution over proposals gets close to being uniform on the acceptance set when  $\delta$  is close to one (because as  $\delta$  tends to 1, the acceptance set becomes a small set, on which variations of  $f(\cdot)$  become tiny). This is essentially why as  $\delta$  tends to one, the solution is independent of the distribution  $f(\cdot)$ .

Let us now characterize the limit equilibrium vector, and explain why it coincides with the generalized Nash bargaining solution. For any given  $\delta$ , consider an equilibrium value profile  $v^\delta$ , and define  $\underline{u}_i^\delta = \delta v_i^\delta$ . Equation (2.2) implies

$$\frac{v_i^\delta}{v_1^\delta} = \frac{E(u_i(x) - \underline{u}_i^\delta \mid u_j(x) \geq \underline{u}_j^\delta \text{ for all } j)}{E(u_1(x) - \underline{u}_1^\delta \mid u_j(x) \geq \underline{u}_j^\delta \text{ for all } j)}. \quad (4.1)$$

---

<sup>22</sup>Note that as  $\delta$  tends to 1, equilibrium values cannot tend to the Pareto frontier too fast. The intuition is simple. If  $v$  were an equilibrium value close to the frontier, (away from the frontier by  $\Delta \ll (1 - \delta)^{1/n}$ ), then the acceptance set  $A$  would be small, the probability of agreement would be small as well (i.e.  $P(v) \ll 1 - \delta$ ), implying that players' expected payoff cannot lie too close to the frontier. Thus, yielding a contradiction. It follows that equilibrium values lie away from the frontier by a distance at least of the order of  $(1 - \delta)^{1/n}$ .

Since  $u$  is smooth, the distribution over proposals induces a distribution over joint utilities  $(u_1, \dots, u_n)$  that is close to the uniform distribution on the set

$$D_\delta = \{u \in u(X), u_i \geq \underline{u}_i^\delta \text{ for all } i\}.$$

Let  $g(u) = 0$  be a parameterization of the frontier. The set  $D_\delta$  is not a simplex, but it is close<sup>23</sup> to the simplex

$$\bar{D}_\delta = \{u, u_i \geq \underline{u}_i^\delta, \sum_i a_i^\delta (u_i - \bar{v}_i^\delta) \leq 0\},$$

where  $\bar{v}^\delta$  is the point on the frontier closest to  $v^\delta$ , and where  $a_i^\delta = g'_i(\bar{v}^\delta)$ .

This implies that the right hand side of (4.1) is close to  $\frac{a_1^\delta}{a_i^\delta}$ , hence

$$\frac{v_i^\delta}{v_1^\delta} = \frac{a_1^\delta}{a_i^\delta}.$$

In the limit as  $\delta$  tends to 1,  $\bar{v}^\delta$  must get close to  $v^\delta$ . Therefore, the limit equilibrium value must be the vector  $v$  on the Pareto frontier that satisfies:

$$\frac{v_i}{v_1} = \frac{a_1}{a_i} \text{ for all } i,$$

where  $a_i = g'_i(v)$ . But, these are precisely the conditions characterizing the generalized Nash bargaining solution.<sup>24</sup>

*Comment.* For ease of presentation, we have restricted attention to distributions  $f(\cdot)$  that induce positive weight on the whole set  $u(X)$ . It should be clear from the above argument that Proposition 3 also holds when  $f(\cdot)$  induces a distribution in the utility space that has full support *only* on the Pareto frontier of  $u(X)$ .

## 5. Single-peaked preferences.

In the previous Section, we have assumed that the set of alternatives was rich in the sense that local changes in the alternative picked could generate all possible variations

---

<sup>23</sup>By close, we mean here by order of magnitude small compared to the size of  $D^\delta$ .

<sup>24</sup>The Nash solution solves  $\max_{g(u)=0} \prod_i u_i$ .

in the utility space. When utility is not transferable and bargaining takes place over physical alternatives (other than money), there are a number of applications in which local variations in the alternative picked need not generate all possible variations in the utility space. In this Section we restrict our attention to situations in which the set of feasible agreements is one-dimensional, and preferences over these agreements are single-peaked. More precisely, we assume:

**Assumption 2:** *Assume  $X = [0, 1]$ ,  $0 \leq \theta_1 < \dots < \theta_n \leq 1$ ,  $u_i(x) = v(x_i - \theta_i)$ , where  $v$  is smooth, single-peaked with a maximum at 0, concave and positive on  $[-1, 1]$ .*<sup>25</sup>

We will refer to the parameter  $\theta_i$  as member  $i$ 's bliss point.

As in the previous Section, we are interested in the limit case where members are very patient. We consider majority requirements  $k \geq n/2$ . We find that as  $\delta$  approaches 1: (i) The acceptance set gets small. (ii) The set of accepted proposals is determined by the preference of two agents at most. (iii) As the majority requirement rises, the preferences of these two agents are more extreme (compared to the median).

### 5.1. Simple majority rule.

We examine here the majority rule case with an odd number of members.<sup>26</sup> Under the assumption that the set of feasible alternative is one-dimensional, and that preferences over these alternatives are single-peaked, the standard voting model is quite predictive when there is an odd number of agents: a unique outcome turns out to be stable,<sup>27</sup> the one preferred by the median voter. What Proposition 4 shows is that the same prediction obtains in our collective search process.<sup>28</sup>

**Proposition 4:** *Let Assumption 2 holds.<sup>29</sup> Let  $m = \text{int}(n + 1)/2$  be the*

---

<sup>25</sup>The concavity of  $v(\cdot)$  can be interpreted as reflecting the risk aversion of members.

<sup>26</sup>In case  $n$  is even, Section 5.3 applies.

<sup>27</sup>An outcome is stable if there is no alternative outcome that a majority would prefer.

<sup>28</sup>This insight should be compared with that of Baron (1996), who also obtains that same prediction, using the bargaining model of Baron and Ferejohn (1989) (based on the random proposer model of Binmore (1987)). For further work on the relationship between Baron and Ferejohn's model and the core, see Banks and Duggan (2000).

<sup>29</sup>The assumption that  $v$  is concave is not necessary for that proposition. It greatly simplifies the proof however.

largest integer less than or equal to  $\frac{n+1}{2}$ . If  $n$  is odd, then  $m$  is the median voter and when  $\delta$  tends to 1, in equilibrium, only proposals close to  $\theta_m$  are accepted.

We sketch the proof here. Details are in the Appendix. Assume that the set of accepted outcomes consists of an interval  $A = [\underline{x}, \bar{x}]$ .<sup>30</sup>  $A$  must contain  $\theta_m$ . Indeed assume by contradiction that  $\bar{x} < \theta_m$ . Then any individual  $i \geq m$  would get utility at most  $u_i(\bar{x})$ , hence they would also accept any outcome in  $[\bar{x}, \theta_m]$ . Since these individuals form a majority, such offers are accepted, leading to a contradiction. A symmetric argument implies that  $\underline{x} \leq \theta_m$ , thereby showing that  $A$  must contain  $\theta_m$ . But, how large can  $A$  be? We will show that  $A$  must be small as  $\delta$  tends to 1. To see this, assume by contradiction that  $A$  is large. Then for very patient individuals, the expected value of voting against the proposed alternative is approximately:

$$E[u_i(x) \mid x \in A]$$

which, because preferences are single-peaked, is strictly larger than  $\min(u_i(\underline{x}), u_i(\bar{x}))$ .<sup>31</sup> So no individual votes for both  $\underline{x}$  and  $\bar{x}$ , which in turn implies that there cannot be a (strict) majority for both  $\underline{x}$  and  $\bar{x}$ . This violates the assumption that both  $\underline{x}$  and  $\bar{x}$  belong to  $A$ , thereby showing that  $A$  must be small as  $\delta$  goes to 1.

*Connection with standard voting model.* Our model is different: it is dynamic, and agents have the choice between voting for one particular alternative (randomly selected), or remaining at the status quo. Of course, voting against the proposed alternative does not imply that agents will remain at the status quo for ever. Other proposals will later be put to a vote. This is why there is a connection between the two models: when an outcome far from the median voter's bliss point is proposed, a majority of players will prefer to reject the current proposal and wait for the arrival of a proposal closer to the median voter's bliss point.

---

<sup>30</sup>This will easily follow from the concavity of  $v(\cdot)$ .

<sup>31</sup>The assumption that  $\underline{x}$  and  $\bar{x}$  are far apart from each other as  $\delta$  goes to 1 guarantees that conclusion.

## 5.2. Unanimity rule.

We show that the solution is determined by the Nash bargaining solution among the two members with most extreme preferences.

**Proposition 5:** *Let Assumption 2 holds. Then when  $\delta$  tends to 1, only proposals close to some  $\theta^*$  are accepted. The position  $\theta^*$  is determined only by the bliss points  $\theta_1$  and  $\theta_n$  of members 1 and  $n$ , and it corresponds to the Nash bargaining solution obtained in case only members 1 and  $n$  are present. If  $v$  is symmetric around 0, then  $\theta^* = \frac{\theta_1 + \theta_n}{2}$ .*

As in the rich proposal space case (see Section 4), the set  $A$  of accepted proposals gets arbitrarily small as players get very patient. The reason is the same: the set  $A$  of accepted proposals must be small because otherwise any member would prefer to veto those proposals in  $A$  that lie furthest away from his bliss point. A notable difference with the rich proposal space case is that the locus of the accepted proposals no longer coincides with the generalized  $n$ -person Nash solution, in which all parties' preferences would matter. In the single-peaked case, only the extremists's preferences matter.

Intuitively, equilibrium conditions impose constraints on the set of proposals that may be accepted. When the set of possible proposals is one dimensional and  $v(\cdot)$  is concave, the set  $A$  of accepted proposals is an interval. This set is thus determined by two conditions (i.e. the limit points of the interval), thereby suggesting why the preferences of only two members matter.

The reason why only the extremists matter is as follows. Let  $A = [\underline{x}, \bar{x}]$  denote the set of outcomes accepted when only members 1 and  $n$  are present:  $\underline{x}$  and  $\bar{x}$  are determined so that (i) member 1 is just indifferent between accepting  $\bar{x}$  now and waiting for the arrival of another proposal in  $A$  - these proposals are better for him, but he has to wait -, and (ii) member  $n$  is just indifferent between accepting  $\underline{x}$  now and waiting for the arrival of another proposal in  $A$ . Now assume that there are other members around, with  $\theta_i \in (\theta_1, \theta_n)$  for each of them. Assume that  $\theta_i < \underline{x}$ . Because  $v(\cdot)$  is concave, and because  $\theta_i$  is closer to  $A$  than  $\theta_1$  is, member  $i$  cares less than member 1 about which alternative in  $A$  is picked. Given that only alternatives in  $A$  can be picked, he just does

not want to delay further the outcome, and  $i$  votes in favor of any proposal in  $A$ . Thus, the presence of  $i$  is irrelevant for the determination of the acceptance set, and only the extremists' preferences determine the outcome.<sup>32</sup>

### 5.3. Qualified majority rules.

We find that as one reduces the majority requirement, the decisive members become more and more moderate.

**Proposition 6:** *Let Assumption 2 holds. Consider the qualified majority rule  $k \geq \text{Int}(n/2)$ . When  $\delta$  tends to 1, the equilibrium outcome coincides with the Nash bargaining solution obtained when only members  $k$  and  $n-k+1$  are present.*

To get some intuition for the result, let  $A = [\underline{x}, \bar{x}]$  denote the acceptance set when only members  $k$  and  $i_0 = n - k + 1$  are present. When  $\delta$  tends to 1, this set tends to the Nash bargaining solution between these two players, and we have  $\theta_{i_0} < \underline{x} < \bar{x} < \theta_{n_0}$ . Let us check what happens when the other members are added. As for the unanimity case, members  $i \in \{n - k + 1, \dots, k\}$  accept all proposals in  $A$ . Now observe that since  $v(\cdot)$  is concave, rejecting a proposal cannot yield member  $i$  more than  $u_i(\tilde{x})$  where  $\tilde{x} = E[x \mid x \in A]$ . It follows that members with a bliss point lower than  $\theta_{i_0}$  do not accept all proposals in  $A$ , but accept all proposals in  $[\underline{x}, \tilde{x}]$ . And similarly, all members with a bliss point higher than  $\theta_{n_0}$  accept all proposals in  $[\tilde{x}, \bar{x}]$ . Hence for any  $x \in A$ , there is a qualified majority of at least  $k$  players that accepts  $x$ . The full proof is more complex, because we need to show that the result holds for any (stationary) equilibrium. The proof is relegated to the Appendix.

---

<sup>32</sup>The fact that the outcome is the Nash bargaining solution between 1 and  $n$  can then be viewed as a corollary of Proposition 3 in which attention is restricted to players 1 and  $n$ . Note that the comment at the end of Section 4 applies.

## 6. Applications

We address three questions. Is it necessarily good for a member that other members share his own taste? Can one benefit from not participating and letting other members decide? Can members benefit from ignoring a characteristic of the proposals?

These questions are explored by means of examples with 2-dimensional proposals. Proposals  $x = (x_a, x_b)$  are drawn from a smooth distribution  $f(\cdot)$  with support  $[-1, 1]^2$ , where  $f(\cdot)$  is assumed to be bounded away from 0 on its entire support. Member  $i$ 's preference is characterized by his bliss point  $\theta_i \in [-1, 1]^2$  and the further away proposal  $x$  is from  $\theta_i$  the lower is  $i$ 's utility  $u_i(x)$ .

### 6.1. The benefits of diversity

Under the unanimity rule, we observe that a committee member  $i$  may be strictly better off when member  $j$  has nearby rather than identical tastes.<sup>33</sup>

The reason is as follows. Consider the case of three members. In the unanimity case, two identical members vote identically, so when member 2 has exactly the same taste as member 1, whether or not member 2 is around is irrelevant for the vote outcome. By application of Proposition 3, it follows that as  $\delta$  gets close to 1 the solution gets close to the Nash bargaining solution between members 1 and 3. When member 2's taste is slightly different from member 1's, and when bliss points are not aligned, then the conditions of Proposition 3 apply, and the solution tends to the generalized Nash solution between all three members 1, 2 and 3, a solution which is more favorable to member 1.

### 6.2. A benefit to delegation

A member who *a priori* enjoys a right to vote can benefit from letting his fellow members decide on their own, thus suggesting a new channel through which one may benefit from delegation (or non-participation).

---

<sup>33</sup>This insight carries over to less stringent majority rules, when for example some members prefer the status quo to any possible proposal.

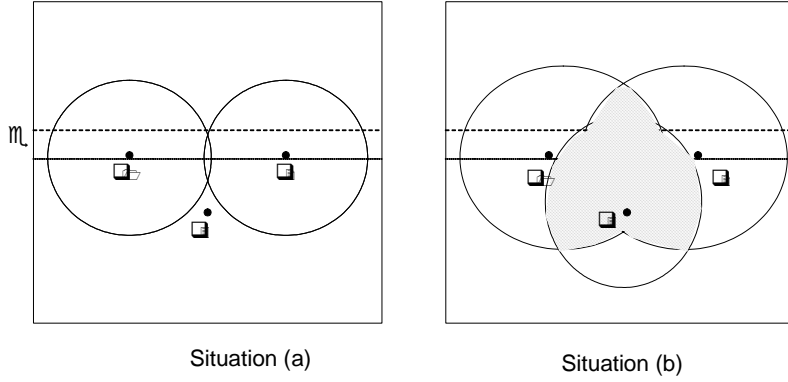


Figure 6.1:

We consider a three-agent ( $i = 1, 2, 3$ ) problem, and we compare two situations. In situation (a), agents 1 and 2 make the decision on their own (under unanimous consent between 1 and 2). In situation (b), all agents participate, and the decision is made under majority rule. We explain below that agent 3 may be better off in situation (a) than in situation (b).

Intuitively, the participation of agent 3 (situation (b)) generates a fear for agent 1 that agents 2 and 3 would agree on a proposal that 1 dislikes, and symmetrically a fear for agent 2 that agents 1 and 3 would agree on a proposal that 2 dislikes. As a result, compared to situation (a), agents 1 and 2 accept more proposals, including proposals that agent 3 may dislike very much.

Formally, let  $\theta_1 = (-\eta, 0)$ ,  $\theta_2 = (\eta, 0)$ , and  $\theta_3 = (0, -\mu)$  for some  $\eta, \mu$  in  $(0, 1]$ . Let the utility of members 1 and 2 take the quadratic form  $u_i(x) = 4 - (d(\theta_i, x))^2$  where  $d(\theta_i, x)$  denotes the Euclidian distance between  $\theta_i$  and  $x$  while member 3 is assumed to derive a very low utility from  $x = (x_a, x_b)$  as soon as  $x_b \geq \varepsilon > 0$ , where  $\varepsilon$  is small. In situation (a), the acceptance set is a small neighborhood of  $(0, 0)$  as  $\delta$  gets close to 1 (this is a corollary of Proposition 3), hence accepted proposals have  $x_b < \varepsilon$  when  $\delta$  is close enough to 1. In situation (b), the acceptance set may get large including agreements on  $(x_a, x_b)$  with  $x_b > \varepsilon$ . To the extent that member 3 dislikes proposals with  $x_b > \varepsilon$  very much she is better off delegating the decision to 1 and 2.

### 6.3. The benefits of ignorance

In recruitment decisions, can there be some benefits to not examining fully all characteristics of the candidates. In a research department, one generally looks at the research record of the candidate. But one may also consider the (in)ability of the researcher to generate a flow of students towards his research agenda, as well as his ambition (or lack of ambition) to eventually run the department. And members of the committee may have conflicting preferences about these characteristics.

With very patient members, information cannot hurt under the unanimity rule. As we have shown, the outcome is close to the Pareto frontier, and more information is good because it expands the Pareto frontier, hence helps achieve welfare-superior outcomes.

With less patient members however, too much information can hurt under the unanimity rule. The intuition is similar to that developed in Section 3 for the comparison of majority vs unanimity rules. Learning the realization in all dimensions will make it more difficult that all committee members are pleased with the current candidate. Not learning a characteristic on which there are conflicting preferences but little efficiency loss will be preferable insofar as it speeds up the decision process.

## 7. Controlling the proposal process.

We consider now situations in which committee members have some influence over the distribution of proposals. We see two motivations. (i) In many recruitment processes, candidates are not drawn from exogenous distributions: committee members may have influence over the pool from which candidates are drawn. (ii) It is a reasonable feature of any bargaining process that parties have at least partial control over the proposal process.<sup>34</sup>

We formalize influence by assuming that at any date each member  $i$  may take an action  $a_i$ , not observable to other members, that affects the distribution over the proposals made in the period, and we denote by  $f(\cdot | a)$  the distribution over proposals

---

<sup>34</sup>>From a more theoretical perspective, this Section also prepares us to further discuss the connection between search and bargaining models, and to compare our model with the random proposer model (Section 8).

induced by the action profile  $a = (a_1, \dots, a_n)$ .<sup>35</sup> The following Proposition shows that in the face of extremely patient players, the ability to influence offers has no value under the unanimity rule in the rich proposal space case (see Proposition 3).

**Proposition 7:** *Let Assumption 1 holds (see Section 4), and assume  $f(\cdot | a)$  is smooth and bounded away from 0 for all  $a$ . Under the unanimity rule, when  $\delta$  tends to 1, equilibrium values tend to the generalized Nash bargaining solution  $v^*$  (see Proposition 3 for the definition of  $v^*$ ).*

The main intuition behind this result is that if the set of accepted proposals is very small, then conditional on acceptance, the distribution over proposals looks like a uniform distribution, whatever actions players undertake to influence the distribution. So these actions have no effect on the outcome as players get very patient.<sup>36</sup> For the same reason, the results of Section 4 and 5 would be unaffected by the possibility of influence.

When players are not patient, or when the space of proposal is rich and the decision rule is different from the unanimity rule, the agreement set is not small, and influence activities may then have some value. So even when these are costly, we should expect players to undertake them in equilibrium.

In the presence of influence activities, the result of Section 3 thus has to be qualified. To the extent that these activities are costly, we should expect that more of these will be performed as the majority requirement is decreased (the agreement set is larger).<sup>37</sup> So, when members are not patient, the comparative advantage of majority rules may be reduced (costly influence activities will be undertaken whatever the decision rule, but even more so when the majority requirement is decreased). When members are very patient, the comparative advantage of the unanimity rule should be reinforced (since

---

<sup>35</sup> An alternative model of influence activity is developed in Yildirim (2007) who assumes in the context of the random proposer model that influence affects the probability of being the proposer.

<sup>36</sup> One interpretation is that patience and veto power from other parties limit the gains to influence.

<sup>37</sup> Yildirim (2007) makes a related observation in his model of influence activities. In the symmetric case, he shows that parties exert more effort to be the proposer as the majority requirement is less strong (see his Proposition 5).

even when members are very patient, the agreement set need not vanish under the majority rule).

## 8. Asymmetric waiting costs

In our main model, waiting costs are the same for all committee members. Our analysis can be extended to the case of asymmetric waiting costs. The following Proposition establishes the connection with the generalized Nash bargaining solution under the unanimity rule where  $\alpha_i$  is defined so that each member  $i$ 's discount factor  $\delta_i$  satisfies:  $(1 - \delta_i) = (1 - \delta)/\alpha_i$ .<sup>38</sup>

**Assumption 3:** Assume that (i)  $u(X) = (u_i(X))_{i=1}^{i=n}$  is a smooth  $n$ -dimensional convex set, and (ii)  $v^{**} = u(x^{**})$  where  $x^{**} = \arg \max_{u \in u(X)} \prod_i (u_i)^{\alpha_i}$  is not a boundary point of the Pareto-frontier of  $u(X)$ .

We have:

**Proposition 8:** *Let Assumption 3 holds, and assume that  $f(\cdot)$  is bounded away from 0 and smooth on  $X$ . When  $\delta$  tends to 1, equilibrium values tend to  $v^{**}$ .*

Intuitively, it is not surprising that more patient committee members get a higher expected payoff in equilibrium. For example assume that all members have a fixed discount factors  $\delta_i$ , and consider the case where member 1 would become arbitrarily patient. It is readily verified that as  $\delta_1$  approaches one, member 1 must get all the surplus. Indeed, as  $\delta_1$  approaches 1, it cannot be that in equilibrium the probability of agreement  $P = P(v)$  remains bounded away from 0, because otherwise, member 1 would prefer to demand (substantially) more, at the risk of decreasing the probability of agreement to, say  $P/2$ . This would increase for him the expected value of agreement, conditional on agreeing. This increase would be at the cost of more delays, but since his waiting costs are small, the deviation would be profitable. So  $P$  must tend to 0. As

---

<sup>38</sup>Wilson (2001) also considers the case of asymmetric waiting costs but his formal claims are restricted to the two-player case.

before,  $P$  close to 0 implies that the equilibrium value is close to the Pareto frontier (otherwise  $P$  would be large). But,  $P$  close to 0 (as  $\delta_1$  approaches 1) also implies that members  $i \neq 1$  get a payoff close to 0 in this limit (this is seen simply by writing the expected payoff obtained by player  $i$  and observing that for a fixed  $\delta_i$  this payoff tends to 0 as  $P$  tends to 0).

## 9. Comparison with the random proposer model.

In the random proposer model (Binmore 1987), each party is selected with probability  $1/n$  to make an offer. Starting with Baron and Ferejohn's seminal paper, this model has been used in numerous political science applications. We review how our model compares with the random proposer model.

As mentioned in the introduction, when players are patient, both models yield the same prediction (the Nash bargaining outcome) when the set of proposals is rich and the decision rule is the unanimity. They also yield the same prediction (median voter outcome) under single-peaked preferences and the simple majority rule (see Banks and Duggan (2000) for the random proposer model).

Otherwise, the models generate different predictions. The random proposer model predicts that all decision rule are equally efficient, while we find a trade-off. The predictions also differ under single-peaked preferences when the decision rule is not the majority rule. In the unanimity case, our search model predicts that only the preferences of the extremists matter. In the random proposer model, the solution would not coincide with that of our model, because the way bliss points are distributed over the segment  $[0, 1]$  would matter.<sup>39</sup> An equilibrium would consist of a pair  $\{\underline{x}, \bar{x}\}$  of proposals: members with bliss point below  $\underline{x}$  would offer  $\underline{x}$ , and members with bliss point above  $\bar{x}$  would offer  $\bar{x}$ . The relative frequency with which  $\underline{x}$  and  $\bar{x}$  are proposed would thus depend on the number of parties with bliss points below and above  $\underline{x}$  and  $\bar{x}$ , and so would the locus of  $\underline{x}$  and  $\bar{x}$ . As  $\delta$  tends to 1, the solution would tend to the weighted Nash bargaining solution among the two most extreme members ( $\theta_1$  and  $\theta_n$ ), in which weights are determined endogenously by the distribution of members along the segment

---

<sup>39</sup>Note that the equilibrium value vector would not coincide with the generalized Nash solution either.

$(\theta_1, \theta_n)$ .

What about intermediate models of search or bargaining, in which members do not perfectly control proposals. One implication of the analysis in Section 7 is that under the assumptions of Sections 4 and 5, when players are very patient, these models have the same prediction as the basic search model.

In particular, one feature of the random proposer model is that the relative frequency with which parties make proposals plays a key role. When one moves away from perfect control however, the relative frequency effect vanishes as players get patient, and the prediction of the search model applies.

Propositions 7 and 8 have interesting implications for the investigation of the robustness of two classic insights in bargaining theory. The traditional view is that bargaining power is driven by the relative impatience (Rubinstein (1982)) and by the relative frequency with which parties make offers. While the effect of the relative impatience is robust to the introduction of imperfect control of offers (Proposition 8), the frequency with which parties make offers plays little role when parties are sufficiently patient and only imperfectly control the offers being made (Proposition 7).<sup>40</sup>

## 10. Conclusion.

From a practical viewpoint, a first contribution of our paper is to show that when the members composing a committee do not control the process of offers, then majority rules may be preferable to the unanimity rule. A second contribution of our paper is to show that many insights obtained separately in models of voting and bargaining can be obtained simply, in a unified way, when considering that offers are generated according to an exogenous process as in search models.

We also contribute to the legislative bargaining literature, shedding some light on who drives the outcome as a function of the majority requirement (in the case of single-peaked preferences).

There are many avenues left for possible applications and extensions of our frame-

---

<sup>40</sup>Which prediction applies (the search model or standard bargaining) depends on the order of the limits, that is, how the noisiness of the proposal process compares to patience.

work. We have suggested some in the body of the paper including questions related to the analysis of delegation and/or the incentives to control the process of offers through lobbying and/or the incentives to gather information about the characteristics of the proposal. Further extensions include the analysis of collective search models in which the committee would evolve in a richer constitutional arrangement, for example considering situations in which the committee keeps searching (over a pre-specified amount of time) even after a proposal has been tentatively accepted in order to possibly improve upon it.

## References

- [1] Banks, J. and Duggan, J. (2000) "A Bargaining Model of Collective Choice", *American Political Science Review*, **94**, 73-88.
- [2] Baron, D. (1991) "A Spatial Bargaining Theory of Government Formation in Parliamentary Systems" *American Political Science Review*, **85** 137-164.
- [3] Baron, D. (1996) "A Dynamic Theory of Collective Good Programs", *American Political Science Review*, **83**, 1181-1206.
- [4] Baron, D. and Ferejohn (1989), "Bargaining in Legislatures", *American Political Science Review*, **90**, 316-330.
- [5] Binmore, K. (1987) "Perfect Equilibria in Bargaining Models", in *The Economics of Bargaining*, ed. K. Binmore and P. Dasgupta, Oxford: Basil Blackwell, 77-105.
- [6] Binmore, K., A. Rubinstein and A. Wolinsky (1986): "The Nash Bargaining Solution", *RAND Journal of Economics*, **17**, 176-188.
- [7] Diamond, P. (1982): "Wage Determination and Efficiency in Search Equilibrium," *Review of Economic Studies*, **49**, 217-227.
- [8] Eraslan, H. and A. Merlo (2002): "Majority Rule in a Stochastic Model of Bargaining," *Journal of Economic Theory*, **103**, 31-48.

- [9] Gomes, A and P. Jehiel (2005): "Dynamic Processes of Social and Economic Interactions: On the Persistence of Inefficiencies," *Journal of Political Economy* **113**, 626–667
- [10] Merlo, A. and C. Wilson (1998): "Efficient Delays in a Stochastic Model of Bargaining," *Economic Theory*, **11**, 39-55.
- [11] Nash, J. (1950), "The Bargaining Problem," *Econometrica* **18**, 155-162.
- [12] Rubinstein, A. (1982): "Perfect Equilibrium in a Bargaining Model", *Econometrica* **50**, 97-109.
- [13] Strulovici, B. (2007): "Voting and Experimentation," mimeo.
- [14] Wilson, C. (2001): "Mediation and the Nash bargaining solution," *Review of Economic Design*, **6**, 353-370.
- [15] Yildirim, H. (2007): "Proposal Power and Majority Rule in Multilateral Bargaining with Costly Recognition," *Journal of Economic Theory* **136**, 167-196.

## Appendix

**Proof of Proposition 2:** Let  $v$  be a tentative equilibrium value and  $u = \delta v$  the corresponding threshold. The consistency condition can be rewritten as

$$(1 - \delta)u = \delta \sum_{m \geq k} P^m(u) E[u_i(x) - u \mid m \text{ members vote yes}]$$

Conditional on acceptance by exactly  $m$  members, by symmetry, a member has a chance  $m/n$  of being among these members who got a realization  $x_i \geq u$ , and a chance  $(1 - m/n)$  of being in the minority with a realization  $x_i < u$ . Hence

$$\begin{aligned} E[u_i(x) - u \mid m \text{ members vote yes}] &= \frac{m}{n} \frac{1 + u}{2} + \left(1 - \frac{m}{n}\right) \frac{u - 1}{2} \\ &= \frac{1}{2} \left[ \frac{2m}{n} - 1 - u \right] \end{aligned}$$

and  $u$  must therefore solve equation (3.1).

For  $k \geq n/2$ , and  $u \leq 0$ ,  $\phi(k, u)$  is strictly positive and it can be checked that it is decreasing in  $u$ . Existence and unicity follows. It also follows that (i)  $\underline{u}(\delta, k)$  is increasing in  $\delta$ , and (ii) if  $\underline{u}(\delta, k) < u_{k-1}$ , then  $\underline{u}(\delta, k - 1) > \underline{u}(\delta, k)$ , and (iii) if  $\underline{u}(\delta, k) > u_{k-1}$ ,  $\underline{u}(\delta, k - 1) < \underline{u}(\delta, k)$ . By construction,  $\underline{u}(\delta_k, k) = u_{k-1}$ . Hence for any  $\delta \in (\delta_k, \delta_{k+1})$ ,  $\underline{u}(\delta, k) > u_{k-1}$ , and thus  $\underline{u}(\delta, k') < \underline{u}(\delta, k)$  for any  $k' < k$ . Also by construction  $\underline{u}(\delta_{k+1}, k + 1) = u_k$ , hence for any  $\delta < \delta_{k+1}$ ,  $\underline{u}(\delta, k + 1) < u_k$ , thus  $\underline{u}(\delta, k) > \underline{u}(\delta, k + 1)$ . **Q. E. D.**

**Proof of Proposition 5:** Let  $\underline{x}$  (respectively  $\bar{x}$ ) denote the lowest proposal accepted in equilibrium. Let  $d = \bar{x} - \underline{x}$ , and recall that  $A$  is the set of proposals accepted in equilibrium. We have shown in the main text that we must have  $\underline{x} \leq \theta_m \leq \bar{x}$ . We will show that when  $\delta$  tends to 1,  $d$  must shrink to 0.

We first show that  $A$  must be an interval. Let  $\tilde{x} = E[x \mid x \in A]$ . Since  $v$  is concave,  $\delta v_i \leq \lambda u_i(\tilde{x})$ , where  $\lambda = \frac{\delta \Pr A}{1 - \delta + \delta \Pr A} < 1$ . Hence all parties (unanimously) accept  $\tilde{x}$ . Since  $v$  is single-peaked, parties that accept  $\underline{x}$  and  $\tilde{x}$  also accept any proposal in  $[\underline{x}, \tilde{x}]$ .

So there must be a majority voting for these proposals. A similar argument applies to proposals in  $[\tilde{x}, \bar{x}]$ . So  $A = [\underline{x}, \bar{x}]$ .

We now show that  $\lambda \nearrow 1$  as  $\delta$  tends to 1. Assume  $\lambda$  does not tend to 1. Then there exists a sequence of discounts  $\delta_k \nearrow 1$  and equilibria such that  $\lambda_k < \lambda < 1$ . Since  $\delta v_i \leq \lambda u_i(\tilde{x})$ , hence all parties (unanimously) accept all proposals in a neighborhood of  $\tilde{x}$  (of size comparable to  $1 - \lambda$ ). It follows that  $\Pr A$  does not tend to 0, contradicting the premise that  $\lambda$  remains bounded away from 1.

Now assume that  $d$  does not tend to 0. Since  $v$  is concave, and since the distribution over proposals has full support, there must exist  $\mu > 1$  such that

$$E[u_i(x) \mid x \in A] > \mu \min(u_i(\bar{x}), u_i(\underline{x})). \quad (10.1)$$

Since, at least one party, say  $i_0$ , must accept simultaneously  $\underline{x}$  and  $\bar{x}$ , we must also have

$$\min(u_{i_0}(\bar{x}), u_{i_0}(\underline{x})) \geq \lambda E[u_{i_0}(x) \mid x \in A],$$

which, combined with inequality (10.1) implies that  $\lambda < 1/\mu$ , contradicting the fact that  $\lambda$  must tend to 1. So  $d$  must tend to 0.

**Proof of Proposition 6:** Because  $v(\cdot)$  is concave, the set of outcomes accepted by each individual is an interval. So the joint acceptance set is also an interval, say  $A = [\underline{x}, \bar{x}]$ .

Now let  $v_i$  denote player  $i$ 's equilibrium value. We have:

$$v_i = \Pr(A)E[u_i(x) \mid x \in A] + (1 - \Pr A)\delta v_i,$$

hence

$$\delta v_i = \lambda E[u_i(x) \mid x \in A]$$

where  $\lambda = \frac{\delta \Pr A}{1 - \delta + \delta \Pr A} < 1$ . Define  $g(\theta, x) = v(x - \theta) - \lambda E[v(x - \theta) \mid x \in A]$ . Equilibrium conditions therefore require that for all  $x \in A$ , and for all  $i$ ,  $g(\theta_i, x) \geq 0$ .

We wish to show that  $g(\theta_1, \bar{x}) = 0$ . Indeed, assume  $g(\theta_1, \bar{x}) > 0$ . Since  $v$  is concave, for any  $\theta < \bar{x}$ , we have

$$\begin{aligned} \frac{\partial g}{\partial \theta}(\theta, \bar{x}) &= -v'(\bar{x} - \theta) + \lambda E[v'(x - \theta) \mid x \in A] \\ &\geq -(1 - \lambda)v'(\bar{x} - \theta) > 0. \end{aligned}$$

Besides, for any  $\theta \geq \bar{x}$ ,  $g(\theta, \bar{x}) > 0$ . It would thus follow that  $g(\theta_i, \bar{x}) > 0$  for all  $i$ , hence proposals slightly larger than  $\bar{x}$  would be accepted unanimously as well. Similarly, we may show that  $g(\theta_n, \underline{x}) = 0$ . The interval  $A$  is thus solely determined by the preferences of members 1 and  $n$ . **Q. E. D.**

**Proof of Proposition 7:** Let  $v_i$  denote player  $i$ 's equilibrium value, and let  $A$  denote the equilibrium acceptance set. Also let  $\bar{x}$  (respectively  $\underline{x}$ ) denote the highest (respectively lowest) proposal accepted in equilibrium. Following the steps of Proposition 5, we have:

$$\delta v_i = \lambda E[u_i(x) \mid x \in A]$$

where  $\lambda = \frac{\delta \Pr A}{1 - \delta + \delta \Pr A} < 1$ .

Define  $g(\theta, x)$  as in Proposition 7 and let  $i_0 = n - n_0 + 1$ .

(i) We check that  $g(\theta_{i_0}, \bar{x}) = 0$ .

Indeed, if  $g(\theta_{i_0}, \bar{x}) > 0$ , then for any  $\theta \geq \theta_{i_0}$ ,  $g(\theta, \bar{x}) > 0$ .<sup>41</sup> Hence by continuity, there are proposals  $x > \bar{x}$  that are accepted by at least  $n_0$  individuals, contradicting the premise that  $\bar{x}$  is the largest proposal accepted in equilibrium. So  $g(\theta_{i_0}, \bar{x}) \leq 0$ . Now if  $g(\theta_{i_0}, \bar{x}) < 0$ , then  $g(\theta, \bar{x}) < 0$  for all  $\theta < \theta_{i_0}$ , hence  $\bar{x}$  would not be accepted by the qualified majority  $n_0$ .

(ii) Similarly, it is easy to check that  $g(\theta_{n_0}, \underline{x}) = 0$ .

**Proof of Proposition 8:** In a stationary equilibrium, the action chosen by players is the same in every period. For any fixed  $a$ , the argument of Proposition 3 applies, showing that equilibrium values must tend to the Nash solution. **Q. E. D.**

**Proof of Proposition 9:** The analysis is very similar to that of Proposition 3, so details are omitted. Just note that with asymmetric discount factors, the left hand side of Equations (4.1) becomes  $\frac{(1-\delta_i)v_i}{(1-\delta_1)v_1}$ . **Q. E. D.**

---

<sup>41</sup>This is because for any  $\theta < \bar{x}$ ,  $\frac{\partial g}{\partial \theta}(\theta, \bar{x}) > 0$ , and because for any  $\theta \geq \bar{x}$ ,  $g(\theta, \bar{x}) > 0$ .