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Constitutional Rules of Exclusion in Jurisdiction Formation

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The rules under which jurisdictions (nations, provinces) can deny immigration or expel residents are generally governed by a constitution, but there do not exist either positive or normative analyses to suggest what types of exclusion rules are best. We stylize this problem by suggesting four constitutional rules of admission: free mobility, admission by majority vote, admission by unanimous consent, admission by a demand threshold for public goods. In a simple model we characterize the equilibria that result from these rules, and provide a positive theory for which constitutional rules will be chosen.

1. INTRODUCTION

The constitution of each nation establishes who has a right to citizenship, and collectively, constitutions constrain migration among nations. In addition each nation's constitution typically defines the right of its citizens to migrate internally between provinces and cities. Westerners typically have the right to move freely within their countries, and the European Union has extended this right across national boundaries as well. However some countries have been reluctant to join the E.U. precisely because they fear immigration. Immigration policies are also controversial in other countries, such as Canada and the U.S.

Controversies over immigration lead us to ask what a good immigration policy would be, when one considers global efficiency, rather than national interests. Much of the current debate in the economics literature (see Borjas (1994, 1995), Raphael and Smolensky (1996)) takes the partial-equilibrium perspective of the host country, asking whether externalities on the labour-market and elsewhere are on net positive. In this paper we take a broader perspective that accounts for welfare effects in both the host country and countries of origin. From this broader perspective we have found few arguments, at least from economists, claiming that the rules we observe are good ones. This is a startling omission, give that economists implicitly study immigration rules whenever they study local public goods economies. Every equilibrium for jurisdiction formation, whether non-cooperative or cooperative, implicitly contains an immigration rule: Free mobility contains the rule that an individual can migrate to any jurisdiction, whether or not the members wish to accept him.[1] The core concept implicitly contains the rule that a group of individuals can

1. See Westhoff (1997), Epple, Filimon and Romer (1984, 1993), Guesnerie and Oddou (1981), Greenberg and Weber (1986), Konishi (1996), Jehiel and Scotchmer (1993, 1997).

be expelled from a jurisdiction if all the other members wish to exclude them (they can form a blocking coalition).[2]

The purpose of this paper is to introduce three new non-cooperative concepts of equilibrium that seem to reflect constitutional rules. These equilibrium concepts differ in the power of current residents to exclude immigrants, but all have the feature that any resident is free to leave. We compare the following, of which the first is already studied in the literature:

- No exclusion rights (free mobility);
- Admission by majority vote;
- Admission by unanimous consent;
- Admission below a threshold of demand for public services.

Throughout the paper we assume that citizens vote on public goods in each jurisdiction after it is formed, and share the costs equally. When an individual considers moving to another jurisdiction, or before residents admit an immigrant, they predict the effect on the public good. The internal rules of voting on public goods and sharing the costs are designed to reflect democratic ideals and redistributive concerns, but also create a disagglomerative force: Citizens with low demand for public goods will not want to share the high cost of public goods in a high-demand jurisdiction, and might therefore want to migrate to a low-demand jurisdiction or form a new jurisdiction.

The objective of the paper is to analyse how citizens are partitioned into jurisdictions under the above immigration rules. For each rule, we ask which partition is stable. A partition is said to be stable whenever either no individual wants to migrate to another jurisdiction, or such migration would be prohibited by the immigration rule. For example, when the rule is admission by majority vote, a partition is stable if, when an individual wants to migrate to jurisdiction A, a majority of members of A oppose it. When the rule is admission by unanimous consent, a partition is stable if, when an individual wants to migrate to jurisdiction A, at least one of the previous residents in A opposes it.[3] In the threshold rule, low-demand immigrants have a right of free mobility, but an immigrant is subject to veto by any resident if his demand for public services is higher than that of other residents. This latter rule seems close to the ones observed in North America, where immigrants must show professional qualifications, or otherwise demonstrate that they will not burden the public coffers.

We show the following relationships among the admissions rules.

First, a partition that is stable under the most permissive immigration rule, *i.e.* free mobility, is also stable under any other immigration rule. This is because under the most permissive rule an individual never considers moving to another jurisdiction.

Second, the partitions that result from admission by majority vote and from free mobility are respectively unique, and they coincide. Thus the restriction that a majority of previous residents must approve of an immigrant has no restrictiveness relative to free mobility. This is a robust result with an intuition that goes beyond the model presented here. The reasoning is as follows. An (infinitesimal) immigrant changes the median voter in a way that pleases half the population—the half whose demand for public services is

2. For an early application of the core concept to jurisdiction formation, formulated as clubs, see Pauly (1967, 1970).

3. This idea differs from the concept of ''core'' in that coordinated defections are not allowed. We maintain the noncooperative idea of individual action in all four migration rules.

on the same side of the median as the immigrant's.[4] In addition, the entire population benefits from sharing the cost of public services with more people. Thus, at least half the population support the admission of any immigrant, and the only relevant criterion is the free mobility criterion: No individual wants to migrate, ignoring the question of whether he would be welcome.

Third, although free mobility and admission by majority vote give the same unique partition, the more exclusive immigration rules of unanimous consent and admission below a threshold can lead to many partitions.

The welfare effects of the four admission rules are hard to disentangle. Migration presumably improves the utility of the migrant, but whether it improves social welfare is ambiguous. Uncompensated externalities arise in the host jurisdiction because there are no side payments to account for the external effects of migration on public goods and taxes. We present a positive and normative theory to compare the four rules.

In Section 2 we present a model with heterogeneous tastes that allows us to address our question. Although the model is simple, it captures the key feature of interest: that agents have different preferences for public goods, which, together with equal cost sharing, creates a disagglomerative force.

Because some immigration rules (in particular the third and fourth) can lead to many different partitions, it is delicate to compare admission rules from either (i) a welfare point of view or (ii) a positive point of view. On the welfare side, free mobility and admission by majority vote provide lower average utility than some partitions that are equilibria under admission by unanimous consent, and the reverse holds for others. However, among the partitions that can arise with the four rules, the one that maximizes average utility is only stable under the very strong exclusion rule, admission by unanimous consent. On the positive side, we investigate whether a given admission rule (and equilibrium partition) is stable in the sense that citizens would vote against changing it. This depends on their beliefs about the ensuing equilibrium partition if the rule is changed. We define two notions of (rule)-stability, and show what rules are stable. This investigation is in the spirit of recent contributors on constitutional political economy, in particular, Buchanan (1991), who includes both normative and positive analyses of rule-making. The main point is to recognize that individual choices are always preceded by institutional rules.

In Section 3 we characterize free-mobility equilibrium, adding to what is known already in the literature. In Section 4 we introduce the notion of equilibrium with admission by majority vote and show that it coincides with free-mobility equilibrium. In Section 5 we characterize equilibrium with admission by unanimous consent and equilibrium with admission below a threshold. In Section 6 we give positive and normative theories of exclusion.

2. MODEL

We assume that individuals have heterogeneous preferences for a public good. The public good is provided locally, and benefits only the residents of the jurisdiction. Individuals

4. Strictly speaking, there is an indeterminacy for those citizens who are very close to (even if on the same side as) the median voter as to whether the change of median voter is favourable to them. However, the effect even if negative is negligible relative to the cost sharing effect, therefore leading to the same conclusion. To see this, observe that a marginal change of the median voter results in a marginal change in the choice of public good. A marginal change in the amount of public good has a negligible effect on the median voter's payoff because by definition of the median voter the original amount of public good maximizes his payoff (and thus the marginal effect must be zero). In contrast, the cost sharing effect is of order 1.

form jurisdictions knowing that afterwards they will vote on the public services and share the costs equally. We ask which partitions of the population into jurisdictions are stable under various immigration rules.

The model of public goods is stylized in that it assumes that spillovers between jurisdictions are nil, and that the cost of the public good does not increase with the number of users, that is, the public good within jurisidictions is ‘‘pure’’. These assumptions will typically not be satisfied in an exact sense, even for public goods like broadcasting (which may have spillovers) and parks (where cost depends on usage, at least to some extent). The model follows the literature in assuming that these effects are small relative to other effects. (But see our (1993) paper for a comparison of at least some of the equilibrium concepts in a congestion model.)

Preferences and costs. We suppose that consumers have different taste parameters $\theta \in (\theta_0, \theta^0)$, for a public good, and that the taste parameter is distributed uniformly with measure one on each unit interval. We assume $0 < \theta_0 < \theta^0 < \infty$ unless stated otherwise. The preferences of a consumer of type θ can be represented $\theta z - t$, where z is the public good he consumes and t is his tax. The public good can be interpreted as a quality parameter, which increases the plausibility of the assumption that there are no congestion costs. For simplicity we take the cost of z as z^2. (The precursor to this paper showed how the result extends to more general cost functions.) The restriction to such a cost function makes preferences single peaked within jurisdictions, and thus allows us to focus on existence issues in partitioning the population, rather than existence issues in the internal governance of jurisdictions.

Jurisdictions. A *jurisdiction*, say A, is a finite union of intervals in (θ_0, θ^0). We adopt the convention that each interval is closed on the left and open on the right, except the highest interval, which is also closed on the right. A *partition* of (θ_0, θ^0) is a collection of jurisdictions $\{A_i\}_{i=1}^k$ such that $\bigcup_{i=1}^k A_i = (\theta_0, \theta^0)$ and $A_i \cap A_j = \varnothing$ for all A_i, A_j in the collection. We make no *a priori* restriction about the total number of jurisdictions and k may either be finite or infinite. We let $\mathcal{P}$ represent the set of all partitions.

Internal governance. In order to analyse the partitioning of the population into jurisdictions, we need to know the public goods and taxes that will ensue in each jurisdiction after it is formed. Individuals in each jurisdiction, say A, vote on the level of public goods (or equivalently on expenditures) knowing that costs will be shared equally. By equal cost sharing we mean that if z is the amount of public good in jurisdiction A, each individual $\theta \in A$ pays the same tax $t = z^2/|A|$, where $|A|$ is the measure of A (or the ‘‘number’’ of individuals in jurisdiction A in more intuitive terms). The voting outcome for public goods is well defined in each jurisdiction because consumers’ preferences on expenditures are single peaked. We let $z(A)$ refer to the public goods chosen by the median voter in a jurisdiction A, namely $z(A) \equiv \frac{1}{2}\theta^M(A)|A|$, where $\theta^M(A)$ is the median voter in jurisdiction A. (This follows from the observation that the most preferred level of public good of individual θ in jurisdiction A is $\frac{1}{2}\theta|A|$.) For a jurisdiction A we let $U(\cdot, A): (\theta_0, \theta^0) \to \mathbb{R}$ refer to the utility of individual θ if he belongs to jurisdiction A, namely $U(\theta, A) \equiv \theta z(A) - z(A)^2/|A|$. For an arbitrary partition $P \in \mathcal{P}$, we define $u(\cdot; P): (\theta_0, \theta^0) \to \mathbb{R}_+$ as follows

$$u(\theta; P) = U(\theta, A_i) \qquad \text{if} \theta \in A_i \quad \text{and} \quad A_i \in P.$$

We will use

$$U(\theta, A) = \theta z(A) - \frac{z(A)^2}{|A|} = \tfrac{1}{2}\theta^M(A)|A|[\theta - \tfrac{1}{2}\theta^M(A)], \quad (1)$$

and, taking θ fixed,

$$dU(\theta, A) = \tfrac{1}{2}|A|[\theta - \theta^M(A)]d\theta^M(A) + \tfrac{1}{2}\theta^M(A)[\theta - \tfrac{1}{2}\theta^M(A)]d|A|. \quad (2)$$

If each jurisdiction contains a single interval, say $A_i = (\theta_{i-1}, \theta_i)$ for each $i = 1, \ldots, k$, the ''number'' (measure) of members of jurisdiction i is $|\theta_i - \theta_{i-1}|$, and the median voter in the jurisdiction is $\theta^M = (\theta_{i-1} + \theta_i)/2$. Thus the public good provided in jurisdiction A_i is $z(\theta_{i-1}, \theta_i) = \frac{1}{4}(\theta_i^2 - \theta_{i-1}^2)$. By continuity, we assume that an individual consumes no public goods and receives zero utility if he occupies a jurisdiction alone. (In the sequel, we will make the assumption that there always exist unoccupied jurisdictions to which an individual can move.)

The model has been chosen so that the constitutional rules for internal governance lead to provisions of public goods within interval jurisdictions that maximize average utility (and would be internally efficient even if transfers of private goods were allowed). The internal optimum allows us to focus on the inefficiency that comes from partitioning. It follows from the form of the utility function, the uniform distribution of θ, and the fact that jurisdictions are intervals.

3. FREE MOBILITY EQUILIBRIUM

A partition into jurisdictions is a free-mobility equilibrium if no ''individual'' wants to migrate to another jurisdiction. Residents in a jurisdiction therefore have no right of exclusion, since an individual can immigrate whether or not the previous residents wish to accept him. It is also possible to migrate to an unoccupied or ''singleton'' jurisdiction, and consume no public good. An individual must be enticed, and not compelled, to belong to some jurisdiction in society. This assumption, which we use in each equilibrium concept, sets a bound on how much a resident can be exploited for the common good. By making the number of jurisdictions endogenous, it allows us to investigate how ''disagglomerative'' is the internal equal-sharing rule under the various admissions rules.

Definition.[5] A *free mobility equilibrium* (FME) is a partition $P = \{A_i\}_{i=1}^k$ of the population (θ_0, θ^0) such that there exists $b > 0$ such that for all $i = 1, \ldots, k$,

1. for every interval $B \subset (\theta_0, \theta^0) \backslash A_i$, with $0 < |B| < b$, there exists an interval $C \subset B$ with $|C| > 0$ such that $U(\theta, A_i \cup B) < u(\theta; P)$ for all $\theta \in C$.
2. $u(\theta; P) \geqq 0$ for all $\theta \in (\theta_0, \theta^0)$.

We refer to the set of partitions that are free-mobility equilibria as $\mathcal{P}$ (FME). (With the preferences described it will be a singleton.)

In the definition, the coalition B represents a migrant ''individual'', since the parameter b can be chosen arbitrarily small. Thus, although we use the continuum for convenience in using the calculus, we capture the notion that individuals are not really infinitesimal, and a migrant affects the level of public spending in his new jurisdiction. For a given partition P of the population, the notion of free mobility is that B wants to

5. In the precursor to this paper, the definition included a stability condition. That condition is implicit here, since we consider migration by non-infinitesimal intervals of agents.

move to A_i if no $\theta \in B$ redues his utility by joining A_i. The partition P is a free mobility equilibrium if no small B wants to move (condition 1), and if every individual receives at least as much utility as in a singleton coalition alone, namely zero (condition 2). Proposition 1 characterizes FME, and equilibrium utilities are graphed in Figure 1.

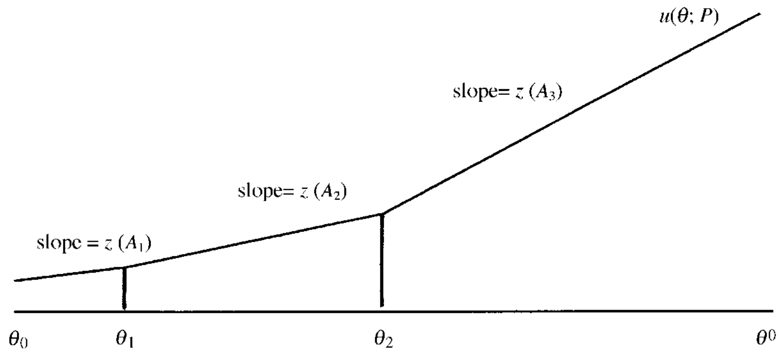

FIGURE 1
Utilities in free mobility equilibrium

Proposition 1. (*Characterization of free-mobility equilibrium*.)

1. $P \in \mathcal{P}$ (FME) *if and only if P satisfies* (a), (b), (c) *below*.
2. *If* $P \in \mathcal{P}$ (FME), *the measure of the highest-θ jurisdiction is more than half the measure of the population.*
3. $\mathcal{P}$ (FME) *contains a unique partition with a finite number of jurisdictions, but the number becomes unbounded as* $\theta^0 \to \infty$.
   (a) *Every jurisdiction* A_i *is an interval, say* (θ_{i-1}, θ_i).
   (b) *For* $i = 1, \ldots, k-1$, $U(\theta_i, A_i) = U(\theta_i, A_{i+1})$.
   In addition, $\theta_{i+1}/\theta_i = f(\theta_i/\theta_{i-1})$, *where f is defined by*

$$f(\lambda) = (1 + \lambda + \sqrt{13\lambda^2 + 6\lambda - 3})/2\lambda$$

   (c) $\theta_i/\theta_{i-1} \leqq 3$ for $i = 1, \ldots, k$.

The intuition behind Proposition 1 is as follows. (a) Utility functions have a single crossing property, namely, if θ' and θ'' prefer jurisdiction A to B so does any $\theta \in (\theta', \theta'')$. From this it follows that jurisdictions must be intervals. (b) Taking the limit as b goes to zero in the definition of FME imposes that the boundary point θ_i must be indifferent between joining jurisdiction A_i or A_{i+1} (otherwise locally every individual would strictly prefer either one or the other jurisdiction contradicting the premise that locally there are individuals in both A_i and A_{i+1}). For the functional forms considered here and for fixed θ_{i-1} and θ_{i+1} such that $\theta_{i+1} \geqq 3\theta_{i-1}$, there are two values θ_i that solve $U(\theta_i, (\theta_{i-1}, \theta_i)) = U(\theta_i, (\theta_i, \theta_{i+1}))$. These are depicted in Figure 4 in the Appendix. However, only one of them satisfies the stability requirement that is implicit in considering migrant sets B of small measure in the definition of FME. Some manipulations lead to the expression $\theta_{i+1}/\theta_i = f(\theta_i/\theta_{i-1})$. (c) Follows from the observation that when the jurisdiction is too large, individual θ_{i-1} of A_i would prefer to move to a singleton jurisdiction.

The number of jurisdictions in a free-mobility equilibrium is endogenous and turns out to be unique. If the number were too small, some jurisdictions would be "large", and the low-θ individuals in those jurisdictions would prefer to be alone. If the number were too large, (some) individuals would migrate such as to achieve economies of scale in providing public goods, thus reducing the number of jurisdictions.

We have described free-mobility equilibrium as a benchmark against which to compare equilibria with alternative admission rules. Free mobility equilibrium has been well studied in slightly different models (see Westhoff (1977), Epple *et al*. (1984, 1993)).

4. ADMISSION BY MAJORITY VOTE

The idea behind free mobility is that a jurisdiction cannot prevent immigration, and it cannot expel residents. While the right to expel residents is rare (perhaps unknown) in constitutions, the right to prevent immigration is common. One might even say that the right to prevent immigration defines a jurisdiction as a nation.

The admission rule in the following equilibrium concept is that an immigrant will be admitted if (i) he wants to come, and (ii) a majority of the jurisdiction votes to accept him, anticipating the effect this will have on the level of public good and local taxes.

Definition. An *equilibrium with admission by majority vote* (MVE) is a partition $P \equiv \{A_i\}_{i=1}^{k}$ such that there exists $b > 0$ such that for all $i = 1, \ldots, k$,

1. If condition 1 of FME does not hold for some interval $B \subset (\theta_0, \theta^0) \backslash A_i$, then there exists $C \subset A_i$ such that $|C| \geqq \frac{1}{2}|A_i|$ and $u(\theta; P) > U(\theta, A_i \cup B)$ for $\theta \in C \subset A_i$.
2. $u(\theta; P) \geqq 0$ for all $\theta \in (\theta_0, \theta^0)$.

We refer to these equilibrium partitions as $\mathcal{P}$ (MVE).

Condition 1 says that in equilibrium no agent (small B) that wants to immigrate would be accepted by a majority of the previous residents. Condition 2 says that every agent must receive nonnegative utility in equilibrium because he has the option to emigrate to a singleton jurisdiction and receive zero utility.

Lemma 1. $P \in \mathcal{P}$ (MVE) *if and only if* (a), (b) *and* (c) *listed in Proposition* 1 *are satisfied*.

Proposition 2. $\mathcal{P}$ (MVE) $=$ $\mathcal{P}$ (FME)

The ''if'' part of the Lemma implies that $\mathcal{P}$ (FME)$\subset \mathcal{P}$ (MVE): If $P \in \mathcal{P}$ (FME), then by Proposition 1, (a)–(c) are satisfied, and hence $P \in \mathcal{P}$ (MVE). The reason for the inclusion is clear: If no individual (small B) wishes to join another jurisdiction, then it is irrelevant whether a majority of the jurisdiction would vote to accept him.

The ''only if'' part of the lemma, which implies that $\mathcal{P}$ (MVE)$\subset \mathcal{P}$ (FME), is less immediate. In FME, residents of a jurisdiction have no power to exclude potential immigrants. In MVE, immigration can be prevented if a majority vote against it. Consequently there could conceivably be partitions that are stable in the sense of MVE that are not stable in the sense of FME, but this turns out not to be true for the intuitive reason given in the Introduction, which is developed formally in Claim 1.

Proof of Lemma 1. (Only If). We use the following claim, which says that there is a majority in any jurisdiction A_i who would vote for the annexation of any small coalition B.

Claim 1. *Let* $P \equiv \{A_i\}_{i=1}^{k}$ *be a partition such that* $u(\cdot; P) \geqq 0$. *For each* A_i *in the partition and each* $\theta \notin A_i$, *there is a small neighbourhood B containing* θ *and* $C \subset A_i$, $|C| \geqq \frac{1}{2}|A_i|$, *such that* $U(\theta, B \cup A_i) > u(\theta; P)$ *for every* $\theta \in C$.

Proof. The annexation of any small interval B to A_i will increase its size, and either $d\theta^M(A_i) = \frac{1}{2}d|A_i|$ or $d\theta^M(A_i) = -\frac{1}{2}d|A_i|$. The result follows by equation (2). The first term is nonnegative either for $\theta \geqq \theta^M(A_i)$ or for $\theta \leqq \theta^M(A_i)$ (half the members of A_i in both cases), and the last term is positive by equation (1) for all $\theta \in A_i$, using $U(\theta, A_i) \geqq 0$ for $\theta \in A_i$.[6] ||

Claim 1 implies that if $P \in \mathcal{P}$ (MVE), then $u(\cdot; P)$ satisfies (4). But the proof of Proposition 1(1) (Only If) shows that if no coalition wishes to migrate, then (a) and (b) hold. And since coalitions are intervals by (a), it then follows from equation (4) and condition 2 of MVE that (c) holds. ||

5. ADMISSION BY UNANIMOUS CONSENT

5.1. *The explicit vote approach*

We have shown above that requiring a majority to approve every immigrant has no effect. We now consider a more extreme admission rule in which all, rather than only a majority, of the previous residents must vote for the immigrant. Every previous resident has veto power.

Definition. An *equilibrium with admission by unanimous consent* (UCE) is a partition $P \equiv \{A_i\}_{i=1}^{k}$ of (θ_0, θ^0) such that there exists $b > 0$ such that for all $i = 1, \ldots, k$,

1. For every interval $B \subset (\theta_0, \theta^0) \backslash A_i$ with $b > |B| > 0$ there exists an interval $C \subset B \cup A_i$ with $|C| > 0$ such that $U(\theta, A_i \cup B) < u(\theta; P)$ for all $\theta \in C$.
2. $u(\theta; P) \geqq 0$ for all $\theta \in (\theta_0, \theta^0)$.

We let $\mathcal{P}$ (UCE) denote the set of partitions that are equilibria with unanimous consent.

By condition 1, no agent (small B) could be annexed to A_i and make all the agents, both in A_i and in the annexed subset B, better off. By condition 2, every agent is better off in equilibrium than in a ''singleton'' coalition with no public goods.

There is no reason that jurisdictions in $\mathcal{P}$ (UCE) must be intervals; however the following characterization applies in the case that they are, and Figure 2 shows the utilities achieved in such a $P \in \mathcal{P}$ (UCE) with interval jurisdictions.

Proposition 3. (*Characterization of* UCE *with interval jurisdictions*.)

(1) $P \in \mathcal{P}$ (UCE) *and each* $A_i \in P$ *is a single interval, say* (θ_{i-1}, θ_i), *then* (a), (b), *and* (c) *are satisfied*.

(2) *Consider a partition P such that each* $A_i \in P$ *is a single interval, say* (θ_{i-1}, θ_i). *If the partition P satisfies* (a), (b) *and* (d), *then* $P \in \mathcal{P}$ (UCE).

(a) $\theta_i / \theta_{i-1} \leqq 3$ *for* $i = 1, \ldots, k$.

(b) $U(\theta_i, A_i) \geqq U(\theta_i, A_{i+1})$, $i = 1, \ldots, k-1$.

6. The proof is written for the case where θ^M is continuously differentiable. However, it may happen for some sets A_i that θ^M be locally discontinuous. For such cases, it can be shown that exactly half of A_i is happy to welcome B, thus resulting in the same conclusion.

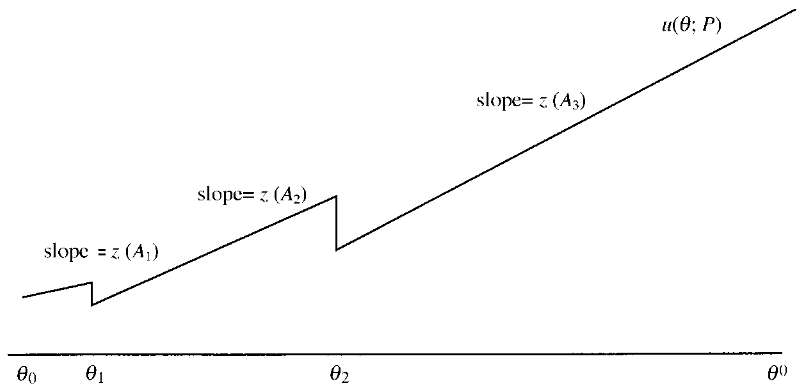

FIGURE 2
Utilities in UCE and ET

(c) *For* $i = 1, \ldots, k-1$, if $\theta_i/\theta_{i-1} < 1 + 2\sqrt{3}$ *then* $U(\theta_i, A_i) = U(\theta_i, A_{i+1})$.
(d) *For* $i = 1, \ldots, k-1$, $\theta_i/\theta_{i-1} > 1 + 2/\sqrt{3}$.

The above proposition shows that a partition in P (UCE) need not coincide with the free-mobility equilibrium partition. Thus, the admission rules are not equivalent.[7] Specifically, condition (a) expresses the fact that if a jurisdiction is too large, then the low-θ individuals would prefer to form a jurisdiction on their own, contradicting condition 2 of UCE. Condition (c) expresses the fact that if the jurisdiction is too small, then the low-θ-members would favour an expansion at the upper boundary because benefits of sharing the costs of public goods outweigh the disadvantage that the level will be higher than they prefer. If condition (b) held with inequality, they would admit members at the upper boundary.

The above proposition characterizes UCE partitions in which jurisdictions are intervals. However, UCE might alternatively group individuals inefficiently in a way that does not minimize the intrajurisdiction variance in taste. Figure 3 shows a UCE partition with

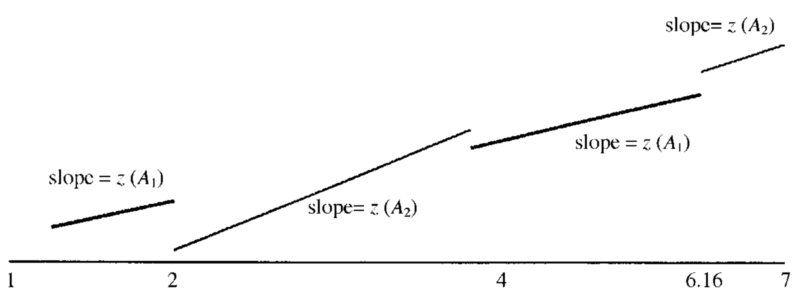

FIGURE 3
UCE with non-interval jurisdictions

7. This should be contrasted with Greenberg and Weber (1986), Guesnerie and Oddou (1981), and Konishi (1996) who obtain that a partition in the core is necessarily a free mobility partition. The reason for their finding is that they assume the level of public good can be chosen at the time of the coalition formation as opposed to after the coalition formation through a voting mechanism as it is the case in our model. As a result, in their framework if some individual is willing to join another group he is always welcome, since the level of public good can always be kept constant (and thus only the positive cost sharing effect arises).

two jurisdictions, each of which is the union of two intervals. It can be checked that the partition is a UCE. High-θ agents in jurisdiction 1, which is $(1,2)\cap(4,6.16)$, would be willing to join jurisdiction 2, which is $(2,4)\cup(6.16,7)$, but low-θ agents in jurisdiction 2 would not agree to accept them. One can see this from equation (2), which is negative if $dU(\theta, A_2)/d\theta^M(A_2)=\frac{1}{2}$ and $dU(\theta, A_2)/d|A_2|)=1$ when $\theta^M(A_i)/\theta_1>1+1/\sqrt{3}$. Similarly low-$\theta$ agents in jurisdiction 2 would like to joint jurisdiction 1, but low-θ agents in jurisdiction 1 would not agree to accept them for the analogous reason.

5.2. *Admission below a threshold*

The definition of UCE assumes that there is an explicit vote about the candidate immigrant, and if anyone objects, immigration is denied. However two problems with such a vote are that it is administratively costly, and it might be politically unacceptable. We therefore consider an indirect rule with similar consequences.

As can be seen in Figure 2, the main effect of the strong exclusion rule in UCE is to exclude immigrants who would increase the public services. Low-θ individuals in jurisdiction 2 would like to join jurisdiction 1, but the low-θ individuals in jurisdiction 1 will veto them because they will increase the demand for public goods. Since the low-demand residents do not want to subsidize higher public goods than they desire, they have an incentive to exclude high-demand individuals. Instead of having an explicit vote about each immigrant, we now assume that exclusion takes the administrative form that in order to be eligible for unapproved immigration in jurisdiction A_i, the candidate should have a taste parameter θ (demand for public services) no greater than the highest demand θ of any resident in the jurisdiction. Only applicants with higher demand can be vetoed. Of course such a rule can only be implemented if the taste parameter is correlated with something observable like wealth, profession, or family structure. In fact such screening variables are commonplace, particularly in North America.

For an arbitrary partition $P\equiv\{A_i\}_{i=1}^k$, let $\{\theta_i\}_{i=1}^k$ satisfy for each i, $\theta_i=\sup\{\theta\in A_i\}$.

Definition. An *equilibrium with free admission below a threshold* (TE) is a partition $P\equiv\{A_i\}_{i=1}^k$ of (θ_0,θ^0) such that for some $b>0$:

1. For every[8] $B\leqq\theta_i$, if $0<|B|<b$ and $B\cap A_i=\varnothing$, then $U(\theta, A_i\cup B)<u(\theta;P)$ for some $\theta\in B$.
2. For every $B>\theta_i$ with $0<|B|<b$, if $U(\theta, B\cup A_i)\geqq u(\theta;P)$ for all $\theta\in B$, then there exists $C\subset A_i$ with $|C|>0$ such that $U(\theta, B\cup A_i)<u(\theta;P)$, $\theta\in C$.
3. $u(\theta;P)\geqq 0$ for all $\theta\in(\theta_0,\theta^0)$.

Condition 1 says that in equilibrium no individual with $\theta\leqq\theta_i$ wants to move to jurisdiction A_i. Free mobility is permitted below the threshold θ_i. Condition 2 says that if an individual with $\theta>\theta_i$ wants to move to A_i, the previous members would not agree unanimously to admit him. (Agreeing to admit such individuals is equivalent to unanimously deciding to relax the threshold.) Condition 3 is the usual requirement that an individual does not prefer to form a singleton jurisdiction.

The following lemma is proved in the Appendix. We let $\mathcal{P}$ (TE) denote the partitions that are equilibria with free admission below a threshold, and are also *robust* in the sense that, if $P\in\mathcal{P}$ (TE), then any sufficiently small perturbation of the boundaries of P yields

8. We use the notation $X<z$ (resp. $\leqq z$) to mean that for every $x\in X$, $x<z$ (resp. $\leqq z$).

another partition in $\mathcal{P}$ (TE). This robustness condition allows us to characterize all the equilibria. It means that small perturbations of the assignment of individuals to jurisdictions would not alter their stability with respect to the rule. An interpretation is that the assignment must be robust to a "tremble" in where indifferent citizens locate.

Lemma 2. *$P \in \mathcal{P}$ (TE) if and only if each $A_i \in P$ is a single interval, say (θ_{i-1}, θ_i), and* (a)–(c) *hold*:

(a) $\theta_i/\theta_{i-1} \leqq 3$ for $i = 1, \ldots, k$.
(b) $U(\theta_i, A_i) > U(\theta_i, A_{i+1})$, $i = 1, \ldots, k-1$.
(c) For $i = 1, \ldots, k-1$, $\theta_i/\theta_{i-1} > 1 + 2/\sqrt{3}$.

Proposition 4. *$\mathcal{P}$ (TE) is a subset of $\mathcal{P}$ (UCE) with interval jurisdictions.*

The proposition follows immediately from the lemma, using Proposition 3. In fact, $\mathcal{P}$ (TE) contains all the partitions in $\mathcal{P}$ (UCE) that have interval jurisdictions, except the one where condition (b) of Proposition 3 is an equality. (This is due to the robustness condition of TE.)

A $P \in$ (TE) is represented in Figure 2. Free mobility below the threshold ensures that a citizen with demand lower than the threshold of any jurisdiction cannot be treated worse in equilibrium than if he immigrated to the jurisdiction. It follows from this (using Lemma 3 in the Appendix) that if some jurisdiction were not an interval, then there would be two jurisdictions with the same public goods and costs. But then there would be some individual (small B) with demand above the median in both jurisdictions. By immigrating to the other jurisdiction, he would raise the median and make himself better off.

As a prelude to our welfare comparisons in the next section, we now prove a proposition relating the number of jurisdictions in TE with the number in FME or MVE.

Proposition 5. (*Number of jurisdictions in* TE.) *Given (θ_0, θ^0), suppose a partition $P \in \mathcal{P}$ (TE) has k jurisdictions, and that $P^{FM} \in \mathcal{P}$ (FME) has k^{FM} jurisdictions. Then $k^{FM} \geqq k - 1$.*

6. A POSITIVE THEORY OF EXCLUSION

We now ask which admission rule will be preferred and/or adopted. We assume that voting is the political process that determines the admission rule, as well as determining the provision of public services within jurisdictions.

We will think of the problem as follows. There is a current immigration rule and a current partition that is an equilibrium under that rule. We ask whether citizens want to overturn the current rule and partition in favour of another rule. However the following problem arises: For the rules UCE and TE, the equilibrium partition is not unique, and therefore citizens cannot reliably predict how well off they will be if they choose such a rule. To address this problem, we will distinguish two notions of (rule)-stability that differ according to the citizens' beliefs about the ensuing equilibrium partition under an alternative immigration rule. We say that the current rule and partition are stable against an alternative in a weak sense if there is *some* equilibrium partition for that rule that would be rejected by a majority. We say that the current rule and partition are stable against an alternative rule in a strong sense if *every* equilibrium partition for that rule would be rejected by a majority.[9]

Formally, for two partitions P, P', let $B(P', P)$ be the set of θ who prefer P' to P:

$$B(P', P) = \{\theta \in (\theta_0, \theta^0) \mid u(\theta; P') > u(\theta; P)\}.$$

We define the set of immigration rules $IR^* = \{\text{FME, MVE, UCE, TE}\}$. An *immigration rule profile* is a pair (P, IR) such that $IR \in IR^*$ and $P \in \mathcal{P}(IR)$.

Definition. An immigration rule profile (P, IR) is *weakly* stable if for all $IR' \in IR^*$ there exists $P' \in \mathcal{P}(IR')$ such that $|B(P', P)| < \frac{1}{2}(\theta_0, \theta^0)$.

Definition. An immigration rule profile (P, IR) is *strongly stable* if for all $IR' \in IR^*$ and all $P' \in \mathcal{P}(IR')$ it holds that $|B(P', P)| < \frac{1}{2}(\theta_0, \theta^0)$.

Proposition 6. (UCE *is strongly stable*.) *The only strongly stable immigration rule profile is* (P, UCE) *where* $A_k = (\frac{1}{3}\theta_0, \theta^0)$, $A_k \in P$. *This immigration rule profile is also weakly stable.*

Thus it may be hard to destabilize the most exclusionary rule, at least for some equilibrium partitions. But the next proposition shows that each of the four admission rules may be weakly stable for at least one partition. Thus, it may explain why various forms of admission rules arise. More precisely, each of the four admissions rules appears to be weakly stable in concert with the free-mobility partition, which is an equilibrium for each of them. The free-mobility partition provides smaller average utility than the ''best'' stable partition under exclusion by unanimous consent.

Proposition 7. *The immigration rule profile* (P, IR), $P \in \mathcal{P}(\text{FME})$, *is weakly stable for every immigration rule* $IR \in IR^*$.

Proof. This follows directly from the definition, noticing that $P \in \mathcal{P}(IR)$, all $IR \in IR^*$.[10] ||

Instead of the positive approach to immigration policy proposed so far, we might alternatively be interested in the welfare impacts of admission rules. Our measure of welfare will be the average of all citizens' utilities. The welfare criterion coincides with the common objective of every citizen if the rule is chosen from ''behind the veil of ignorance'', *i.e*. before the citizens know the realization of their taste parameter.

The following proposition considers the case of a large economy in which the domain (θ_0, θ^0) is large. To compare welfare, we calculate the average loss in utility due to the fact that the population is partitioned, rather than all citizens being in one jurisdiction, as they ''should'' be, given that the public goods are pure. We show that the loss is smaller if this partitioning is done according to a higher growth rate of boundary points in the partition. This will enable us to compare the welfare of different equilibrium partitions

9. Compare with the stability criteria of Alesina and Spolaore (1997) and Le Breton and Weber (2000). In those papers, stability refers to the partitions themselves, rather than the constitutional rule under which the partition arises.

10. For the ''admission below a threshold'' rule it only belongs to the closure of it because of the robustness requirement. However, by perturbing the partition in a way that increases the highest theta group, we obtain the desired property.

(hence different admissions rules), since the boundary points under different rules have different growth rates.

For a large domain, the boundary points $\{\theta_i\}$ of an FME (or MVE) partition grow at an almost constant rate, namely, the limit λ^* deduced in Lemma 8 (Appendix). Thus, for sufficiently large i, it almost holds that $\theta_{i+1} = \lambda^* \theta_i$. For the two admission rules UCE and TE, we consider those partitions which also have constant growth rates (with every jurisdiction being a single interval). According to conditions (b) of Propositions 3 and 4, the growth rates in those partitions can be larger, and must be no smaller, than in FME.

The following expression represents the ratio of total utility with growth rate λ to the total utility with everyone in one jurisdiction. This ratio is less than one, and the distance from one measures the inefficiency. The ratio is given for a fixed number of jurisdictions k, assuming for simplicity that $\theta_0 = 1$, and letting θ^0 adjust to accommodate the growth rate λ and k. Thus both the numerator and denominator of the ratio displayed below represent the sum of utilities for citizens with θ in a domain $(1, \lambda^k)$, although the numerator assumes the citizens are in k jurisdictions and the denominator assumes they are in one jurisdiction. The ratio displayed below uses the facts that the sum of utilities in an interval (θ_{j-1}, θ_j) is $(\theta_j^2 - \theta_{j-1}^2)^2/16$ and $\theta_j = \lambda\theta_{j-1}$. For fixed λ, as k grows, the limiting ratio measures the limiting welfare loss in a large population with growth rate λ

$$\sum_{j=1}^{k-1}\left(\frac{(\lambda^{2j+2}-\lambda^{2j})^2}{16}\right)\left(\frac{16}{(\lambda^{2k}-1)^2}\right) \to \frac{(\lambda^2-1)}{(\lambda^2+1)} < 1. \tag{3}$$

Proposition 8. (TE *and a selection from* UCE *provide higher average utility than* FME *and* MVE *in large economies*.) *Let* $P \in \mathcal{P}$ (UCE) *or* $P \in \mathcal{P}$ (TE) *satisfy* (i) *every jurisdiction* $A_j = (\theta_{j-1}, \theta_j)$ *in* P *is a single interval, and* (ii) $\theta_j = \lambda\theta_{j-1}$ *for some* λ. *Then when the domain* (θ_0, θ^0) *is sufficiently large,* P *yields higher welfare* (*a higher value of* (3)) *than* $P' \in \mathcal{P}$ (FME) $= \mathcal{P}$ (MVE).

Proof. Condition (b) in Propositions 3 and 4 implies that λ is no smaller than λ^*. The rest follows because (3) increases with λ. ||

7. CONCLUSION

We have introduced three new equilibrium concepts that seem to reflect the rules of immigration embodied in constitutions. They differ in the ability of previous residents to exclude immigrants. Free mobility grants no rights of exclusion, while admission by majority vote prevents immigration unless a majority agree, and admission by unanimous consent prevents immigration unless everyone agrees. Admission below a threshold permits free mobility below the threshold, but requires unanimous consent above the threshold.

Two principles unify the admissions rules we have considered. First, individuals should always be allowed to defect to singleton jurisdictions. This requirement can be interpreted as setting a lower bound on how much an agent can be exploited. Second, immigration should always be permitted when there is unanimous consent in the host jurisdiction. The four admission rules satisfy these conditions, but impose additional restrictions that differ.

Of course the best rule to govern migration should depend on how goods are allocated *within* jurisdictions. We have assumed that within jurisdictions the public expenditures are chosen by majority vote and the burden shared equally. One justification for equal cost-sharing is administrative simplicity. Another reason for equal sharing, and also for voting on public goods, is equity. When governments have limited ability to transfer income, redistribution can be accomplished through equal sharing of costs for public goods which have unequal benefits.

Rules of admission affect different citizens differently. Since we wish to interpret the rules of admission as mandates of the constitution, we need a theory for how they arise. We have investigated both a positive theory and a normative theory.

In the positive theory, citizens can vote to overturn an existing rule of admission, together with an equilibrium partition. A problem is that there may be many equilibrium partitions consistent with a new admission rule, in particular, admission by unanimous consent. For that rule, some of the equilibrium partitions might be quite inefficient, even though one of them provides the highest possible average utility. To address this problem, we defined two notions of (rule)-stability which are forward-looking with respect to the resulting equilibrium partition if the rule is changed. If citizens are pessimists in the sense that they predict the worse equilibrium partition, then any admission rule can be stable. If they are optimists in the sense that they all predict a partition that is very good for a majority of them, then the most exclusionary rule, admission by unanimous consent, is the only stable one.

To place our investigation in context, we conclude with a few remarks on how jurisdiction formation has been conceptualized in other branches of the literature on local public goods. If there are no restrictions on either side payments or on free mobility, then a natural equilibrium concept is the core with no constraints on side payments. With pure public goods as here (no crowding costs), the core could be large, and all citizens would be in one jurisdiction (Muench (1972)), and with crowding costs the core will typically be empty because it will typically be impossible to partition the consumers into groups of the ''optimal'' size (Pauly (1967, 1970)). The latter problem carries over to price-taking equilibria (see Scotchmer (1999) for a discussion of the existence problem.

Whether or not side payments are restricted, concepts of jurisdiction formation differ in how a jurisdiction will be governed once it is formed. In the concept of equilibrium in this paper, jurisdictions do not have ''managers''. Once citizens are grouped, members of a jurisdiction vote on public goods and share its costs equally. One can think of these institutional arrangements as dictated by a constitution. In other equilibrium concepts, a manager commits to public goods and taxes or prices before the group has formed, knowing that the public goods and taxes will attract a certain clientele. This occurs in both the price-taking model and in Nash equilibrium in prices (Scotchmer (1985*a*, *b*), Wildasin (1988)). Alternatively, jurisdictions may establish their fiscal policies in an explicit negotiation with each other in order to avoid competition (Jehiel (1997)). In addition there is an enormous literature on fiscal competition that does not consider either entry of new jurisdictions or migration among them.

APPENDIX

Lemma 3. *Let the partition* $P=\{A_i\}_{i=1}^{k}$ *be such for every* $B\subset(\theta_0,\theta^0)$ *with* $0<|B|$ *and* $B\cap A_i=\emptyset$, $U(\theta, A_i\cup B)<u(\theta;P)$ *for some* $\theta\in B$. *Then* $u(\cdot;P)$ *satisfies the following, and is therefore continuous, convex, and increasing*

$$u(\theta;P)=\max\{U(\theta,A_i)\,|\,A_i\in P\}. \tag{4}$$

Proof. Suppose that equation (4) does not hold; that is, for some A_i, A_j and $\theta\in A_j$, $u(\theta;P)=U(\theta,A_j)<U(\theta,A_i)$. Let ε be small, and in particular, small enough so that $B=(\theta-\varepsilon,\theta+\varepsilon)\subset A_j$. Then $u(\theta;P)=U(\theta,A_j)<U(\theta,A_i\cup B)$ for all $\theta\in B$, a contradiction. Hence (4) holds, and therefore $u(\cdot;P)$ is continuous. Since

each $U(\theta, A_i)$ is increasing in θ, it follows that $u(\cdot; P)$ is increasing, and $u(\cdot; P)$ is convex because it is the upper envelope of linear functions. ||

Lemma 4. *Suppose that P is a partition with interval jurisdictions, that* $3 \geqq \theta_i/\theta_{i-1}$ *for each i, and that* $U(\theta_i, A_i) = U(\theta_i, A_{i+1})$ *and* $|A_i| = |A_{i+1}|$. *Let* $B = (\theta_i, \theta_i + b)$, *where* $b < |A_i|$. *Then* $U(\theta, A_i \cup B) > U(\theta, A_{i+1})$ *for all* $\theta \in B$.

Proof. Let $|A_i| = |A_{i+1}| = a$ and $\theta_b = \theta_i + b$. Then $\theta^M(A_i) \equiv \theta_i^M = \theta_i - \frac{1}{2}a$, $\theta^M(A_{i+1}) \equiv \theta_{i+1}^M = \theta_i + \frac{1}{2}a$, and $\theta^M(B \cup A_i) = \theta^M(A_i) + \frac{1}{2}b$. Then

$$U(\theta_b, B \cup A_i) - U(\theta_b, A_{i+1}) = \tfrac{1}{2}b[\tfrac{3}{4}(\theta_i - \tfrac{1}{2}a)a - a(\theta_i + \tfrac{1}{2}a) + \tfrac{1}{4}\theta_i(2\theta_i + a) + \tfrac{1}{8}b((2\theta_i + a + 3b) + 6b(\theta_i + \tfrac{1}{2}b))]$$
$$\geqq \tfrac{1}{2}b[\tfrac{3}{4}(\theta_i - \tfrac{1}{2}a)a - a(\theta_i + \tfrac{1}{2}a) + \tfrac{1}{4}\theta_i(2\theta_i + a)].$$

The last expression is positive if $4.1 = \frac{1}{3}(7 + 2\sqrt{7}) > \theta_i/\theta_{i-1} > \frac{1}{3}(7 - 2\sqrt{7}) = 0{\cdot}57$. But $\theta_i/\theta_{i-1} > 1$ by definition, and $3 \geqq \theta_i/\theta_{i-1}$. Therefore the expression is always positive. It is similarly easy to show that $U(\theta_i, B \cup A_i) - U(\theta_i, A_{i+1}) > 0$, and it follows that this relationship holds for all $\theta \in (\theta_i, \theta_i + b)$. ||

Lemma 5. *Suppose that P satisfies* (a), (b) *and* (c) *in Proposition* 1 *and* $A_i \in P$. *Then for every* $B \subset (\theta_i, \theta^0)$ *such that* $|B| < |A_i|$, *there exists* $\theta \in B$ *for which* $U(\theta, A_i \cup B) < u(\theta; P)$.

Proof. First, if a coalition $B = (\theta_i, \theta_i + b)$ would not want to migrate to A_i, then no other (interval) coalition of size b in (θ_i, θ^0) would want to migrate to A_i. $U(\theta, A_i \cup B)$ has the same value for all B with same size, since it depends only $|A_i \cup B|$ and $\theta^M(A_i \cup B)$ which are the same for all B with the same size, provided $B < |A_i|$. But $U(\theta, A_i \cup B)$ increases linearly with θ. Since $u(\cdot; P)$ has nondecreasing slope, if $U(\theta_i + b, A_i \cup B) - u(\theta_i + b; P) < 0$ then $U(\theta' + b, A_i \cup B') - u(\theta' + b; P) < 0$ for $B' = (\theta', \theta' + b)$ and $\theta_{i+1} > \theta' > \theta_i$.

Let $\theta_b = \theta_i + b$. We will use the simplified notation $\theta_i^M \equiv \theta^M(A_i)$, $\theta_B^M \equiv \theta^M(A_i \cup B)$. We show that $U(\theta_b, A_i \cup B) < U(\theta_b, A_{i+1})$.

$$U(\theta_b, A_i \cup B) = \tfrac{1}{2}(b + |A_i|)\theta_B^M(\theta_b - \tfrac{1}{2}\theta_B^M)$$
$$\tfrac{1}{2}|A_i|\theta_B^M(\theta_i + b - \tfrac{1}{2}\theta_B^M) + \tfrac{1}{2}b\theta_B^M(\theta_i + b - \tfrac{1}{2}\theta_B^M)$$
$$= \tfrac{1}{2}|A_i|(\theta_i^M + \tfrac{1}{2}b)(\theta_i + b - \tfrac{1}{2}\theta_i^M - \tfrac{1}{4}b) + \tfrac{1}{2}b\theta_B^M(\theta_i + b - \tfrac{1}{2}\theta_B^M)$$
$$= U(\theta_i, A_i) + \tfrac{3}{8}|A_i|b\theta_i^M + \tfrac{1}{2}b[(\tfrac{1}{2}|A_i| + \theta_B^M)(\theta_i - \tfrac{1}{2}\theta_i^M + \tfrac{3}{4}b)].$$

$U(\theta_b, A_{i+1}) = U(\theta_i, A_{i+1}) + \frac{1}{2}b|A_{i+1}|\theta_{i+1}^M$. Hence, using $U(\theta_i, A_i) = U(\theta_i, A_{i+1})$, which follows from condition (b),

$$U(\theta_b, B \cup A_i) - U(\theta_b, A_{i+1}) = \tfrac{1}{8}b[3\theta_i^M|A_i| - 4\theta_{i+1}^M|A_{i+1}| + 4(\theta_i + \tfrac{1}{2}b)(\theta_i - \tfrac{1}{2}\theta_i^M + \tfrac{3}{4}b)].$$

We want to show that this expression is negative. Since $U(\theta_i, A_i) \geqq 0$, it follows from equation (1) that $(\theta_i - \frac{1}{2}\theta_i^M) \geqq 0$. Using in addition that $\theta_i^M = \theta_i - \frac{1}{2}|A_i|$, $\theta_{i-1}^M = \theta_i + \frac{1}{2}|A_{i+1}|$ and $(\theta_i - \frac{1}{2}\theta_i^M) = \frac{1}{4}(2\theta_i + |A_i|)$, we have

$$3\theta_i^M|A_i| - 4\theta_{i+1}^M|A_{i+1}| + 4(\theta_i + \tfrac{1}{2}b)(\theta_i - \tfrac{1}{2}\theta_i^M + \tfrac{3}{4}b) \leqq 3\theta_i^M|A_i| - 4\theta_{i+1}^M|A_{i+1}| + \theta_i(2\theta_i + |A_i|)$$
$$= 4\theta_i(|A_i| - |A_{i+1}|) - \tfrac{3}{2}|A_i|^2 - 2(|A_{i+1}|^2 - \theta_i^2) < 0.$$

Each term is negative, since (using (b)) $|A_i| < |A_{i+1}|$, $(|A_{i+1}|^2 - \theta_i^2) = \theta_{i+1}(\theta_{i+1} - 2\theta_i)$ and

$$\frac{\theta_{i+1}}{\theta_i} = f\left(\frac{\theta_i}{\theta_{i-1}}\right) > 2. \quad ||$$

Lemma 6. *Suppose that* $P \equiv \{A_i\}_{i=1}^k$ *is a partition for which*

(i) *for each* $i = 1, \ldots, k$, A_i *is a single interval* (θ_{i-1}, θ_i),
(ii) *for* $i = 2, \ldots, k$, $\theta_i/\theta_{i-1} > 2$,
(iii) *equation* (4) *holds.*

Then for every coalition $B \subset (\theta_0, \theta_{i-1})$ *such that* $|B| < |A_i|$ *there exists* $\theta \in B$ *such that* $U(\theta, A_i \cup B) < u(\theta; P)$.

Proof. Let $b = |B|$. Let $\theta^M(A_i) = \theta_i^M$ for each i. Define $\tilde{b}$ such that $\theta_B^M \equiv \theta^M(B \cup A_i) = \theta_i^M - \frac{1}{2}\tilde{b}$. Let A_j be such that $B \cap A_j \neq \emptyset$. For some $\theta \in B \cap A_j$ we will show that $U(\theta, B \cup A_i) - u(\theta; P) < 0$. We have

$U(\theta, B\cup A_i) - u(\theta; P) = [U(\theta, B\cup A_i) - U(\theta, A_i)] + [U(\theta, A_i) - u(\theta; P)]$. The first term is

$$\begin{aligned} U(\theta, B\cup A_i) - U(\theta, A_i) &= \tfrac{1}{2}(b+|A_i|)\theta_B^M(\theta - \tfrac{1}{2}\theta_B^M) - u(\theta, A_i) \\ &= \tfrac{1}{2}|A_i|\theta_B^M(\theta - \tfrac{1}{2}\theta_B^M) + \tfrac{1}{2}b\theta_B^M(\theta - \tfrac{1}{2}\theta_B^M) - U(\theta, A_i) \\ &= \tfrac{1}{2}|A_i|(\theta_i^M - \tfrac{1}{2}\tilde{b})(\theta - \tfrac{1}{2}\theta_B^M) + \tfrac{1}{2}b\theta_B^M(\theta - \tfrac{1}{2}\theta_B^M) - U(\theta, A_i) \\ &= \tfrac{1}{2}|A_i|\theta_i^M(\theta - \tfrac{1}{2}\theta_B^M) - \tfrac{1}{4}|A_i|\tilde{b}(\theta - \tfrac{1}{2}\theta_B^M) + \tfrac{1}{2}b\theta_B^M(\theta - \tfrac{1}{2}\theta_B^M) - U(\theta, A_i) \\ &= \tfrac{1}{2}|A_i|\theta_i^M(\theta - \tfrac{1}{2}\theta_i^M + \tfrac{1}{4}\tilde{b}) + (\tfrac{1}{2}b\theta_B^M - \tfrac{1}{4}|A_i|\tilde{b})(\theta - \tfrac{1}{2}\theta_B^M) - U(\theta, A_i) \\ &= U(\theta, A_i) + \tfrac{1}{8}|A_i|\tilde{b}\theta_i^M + (\tfrac{1}{2}b\theta_B^M - \tfrac{1}{4}|A_i|\tilde{b})(\theta - \tfrac{1}{2}\theta_B^M) - U(\theta, A_i). \end{aligned} \tag{5}$$

We now specialize this expression for the case that $B\subset(\theta_0, \theta_{i-1})$ and $|B| < |A_i|$ so that $b = \tilde{b}$. Let $\theta = \inf\{\tilde{\theta}\in B\cup A_i\}$, and assume without loss of generality that $\theta\in A_j \equiv (\theta_{j-1}, \theta_j)$, where $j < i$. Then $\theta \leqq \theta_{i-1} - b$. The last expression is

$$= \tfrac{1}{8}b[|A_i|\,\theta_i^M + (2\theta_B^M - |A_i|)(2\theta - \theta_B^M)].$$

By equation (4), $u(\theta; P) = U(\theta, A_j) \geqq U(\theta, A_r)$ for all $r = 1, \dots, k$, and using $\theta \leqq \theta_{i-1} - b$,

$$\begin{aligned} U(\theta, A_i) - u(\theta; P) &= U(\theta, A_i) - U(\theta, A_j) \leqq U(\theta, A_i) - U(\theta, A_{i-1}) \\ &= [U(\theta_{i-1}, A_i) - \tfrac{1}{2}z(A_i)(\theta_{i-1} - \theta)] - [U(\theta_{i-1}, A_{i-1}) - \tfrac{1}{2}z(A_{i-1})(\theta_{i-1} - \theta)] \\ &= -(\theta_{i-1} - \theta)(z(A_i) - z(A_{i-1})) \leqq -b(z(A_i) - z(A_{i-1})). \end{aligned}$$

Hence

$$\frac{8}{b}[U(\theta, A_i\cup B) - u(\theta; P)] \leqq |A_i|\theta_i^M + (2\theta_i^M - |A_i| - b)(2\theta - \theta_i^M + \tfrac{1}{2}b) - 8(z(A_i) - z(A_{i-1})).$$

By equation (1), $U(\theta, A_i\cup B) < 0 \leqq u(\theta; P)$ unless $\theta_{i-1} - b \geqq \frac{1}{3}\theta_i$. The above expression is increasing in b for $0 < b < \theta_{i-1} - \frac{1}{3}\theta_i$, hence it is enough to show that it is negative at $b = \theta_{i-1} - \frac{1}{2}\theta_i$. Using $z(A_i) = \frac{1}{4}(\theta_i^2 - \theta_{i-1}^2) = \frac{1}{2}|A_i|\theta_i^M$ for all i, and $\theta < \theta_{i-1}$, the last expression is

$$\begin{aligned} &\leqq 2z(A_i) + \tfrac{1}{2}(2\theta_{i-1} - b)(3\theta_{i-1} - \theta_i + b) - 8z(A_i) + 8z(A_{i-1}) \\ &= -6z(A_i) + 8z(A_{i-1}) + 2\theta_{i-1}^2 - \tfrac{4}{9}\theta_i^2 \\ &\leqq -6\theta_i^2 + 18\theta_{i-1}^2 < 0, \end{aligned}$$

where the result follows because

$$\left(\frac{\theta_i}{\theta_{i-1}}\right)^2 = f\left(\frac{\theta_{i-1}}{\theta_{i-2}}\right)^2 > 2^2 = 4. \quad ||$$

Lemma 7. *Suppose that P is a partition with interval jurisdictions, that $U(\theta_{i-1}, A_{i-1}) = U(\theta_{i-1}, A_i)$, that $|A_i| = |A_{i-1}|$, and that $\theta_i/\theta_{i-1} \leqq 3$ for all i. Let $\theta_b = \theta_{i-1} - b$ and $B = (\theta_b, \theta_{i-1})$. Then $U(\theta, A_i\cup B) > U(\theta, A_{i-1})$ for all $\theta\in B$.*

Proof. Let $a = |A_{i-1}| = |A_i|$, and notice that $\theta_{i-1}^M = \theta_i^M - a$, $\theta^M(A_i\cup B) \equiv \theta_B^M = \theta_i^M - \frac{1}{2}b$. We use equation (5) derived in Lemma 6, namely

$$U(\theta_b, A_i\cup B) = U(\theta_b, A_i) + \tfrac{1}{8}ab\theta_i^M + (\tfrac{1}{2}b\theta_B^M - \tfrac{1}{4}ab)(\theta_b - \tfrac{1}{2}\theta_B^M). \tag{6}$$

In addition

$$U(\theta_b, A_{i-1}) = \tfrac{1}{2}a\theta_{i-1}^M(\theta_b - \tfrac{1}{2}\theta_{i-1}^M) = U(\theta_b, A_i) + \tfrac{1}{2}a^2b.$$

Thus

$$U(\theta_b, A_i\cup B) - U(\theta_b, A_{i-1}) = \tfrac{1}{8}ab\theta_i^M + (\tfrac{1}{2}b\theta_B^M - \tfrac{1}{4}ab)(\theta_b - \tfrac{1}{2}\theta_B^M) - \tfrac{1}{2}a^2b.$$

Substituting appropriately and ignoring the b^2 terms, this is equal to

$$= \tfrac{1}{2}b(-\tfrac{7}{4}a^2 + \theta_{i-1}^2) = \tfrac{1}{8}b(-7\theta_{i-1}^2 - 3\theta_i^2 + 14\theta_i\theta_{i-1}),$$

which is positive if $\theta_i/\theta_{i-1} < 1 + 2/\sqrt{7}$. But if the latter inequality did not hold, then $(\theta_{i-2} + 2a)/(\theta_{i-2} + a) > 1 + 2/\sqrt{7}$, which implies that $(1 - 2/\sqrt{7})a > (2/\sqrt{7})\theta_{i-2}$. But in addition $(\theta_{i-2} + a)/\theta_{i-2} \leqq 3$, which implies that $a \leqq 2\theta_{i-2}$. These inequalities contradict each other. $||$

In the following lemma, the notation $f^{j}(\cdot)$ means j compositions of the function f, *e.g.* $f^{2}(\lambda) = f \circ f(\lambda) = f(f(\lambda))$.

Lemma 8. *The function f defined by Proposition* 1(b) *has the following properties*:

1. $\theta_{i+1}/\theta_i = f(\theta_i/\theta_{i-1})$ *if and only if* $U(\theta_i, (\theta_{i-1}, \theta_i)) = U(\theta_i, (\theta_{i-1}, \theta_i))$ *and* $\theta_i < \frac{1}{2}(\theta_{i-1} + \theta_{i+1})$.
2. *f is decreasing and continuous, and* $|f'(\lambda)| < 1$. *Starting from any* λ, *the sequence* $\{f^{i}(\lambda)\}_i = \cdots$ *converges to* λ^*, *and the values in the sequence alternate between values larger than* λ^* *and values smaller than* λ^*.
3. *The value of* $\lambda f(\lambda) f^{2}(\lambda) \ldots f^{k}(\lambda) \equiv \lambda \Pi_{i=2}^{k} f^{i-1}(\lambda)$ *is increasing with* λ *for all k*.

Proof. To verify 1 and 2 is simply a matter of algebraic manipulation. We prove 3. One can see from the expression for f that $\lambda f(\lambda)$ increases with λ. In addition, since $f'(\lambda) < 0$, $f \circ f(\lambda)$ increases with λ. It follows that for any even j, $f^{j}(\lambda)$ increases with λ. Since $\lambda \Pi_{i=2}^{k+1} f^{i-1}(\lambda) = [\lambda \Pi_{i=2}^{k} f^{i-1}(\lambda)][f^{k}(\lambda)]$, and since $f^{k}(\lambda)$ increases with λ when k is even, it is enough to show that $\lambda \Pi_{i=2}^{k} f^{i-1}$ increases with λ when k is even. But because k is even, $\lambda \Pi_{i=2}^{k} f^{i-1}$ can be expressed as

$$[\lambda f(\lambda)][f^{2}(\lambda) f(f^{2}(\lambda))][f^{4}(\lambda) f(f^{4}(\lambda))], \ldots, [f^{k-2}(\lambda) f(f^{k-2}(\lambda))].$$

Each of these terms is increasing in λ. By putting $f(f(\lambda))$ on the left-hand side of f and substituting $f^{j}(\lambda)$ for λ on the right-hand side, and then cross-multiplying, one can see that since $f^{j}(\lambda)$ increases with λ (since j is even), the product $[f^{j}(\lambda) f(f^{j}(\lambda))]$ increases with λ. ||

Proof of Proposition 1(1). (Only If). Suppose that $P = \{A_i\}_{i-1}^{k}$ is an FME. We must show that it satisfies (a), (b) and (c). Using equation (1), (c) holds because $U(\theta, A_j) \geqq 0$ for all j and all $\theta \in A_j$.

We now show (a). A_i is by definition a union of intervals. Let B be an interval that fills the space between two intervals in A_i. If $\theta \in A_j$, then the slope of $u(\cdot; P)$ at θ is the slope of $U(\theta, A_j)$, which is $z(A_j)$. Lemma 3 applies to FME utilities. By convexity of $u(\cdot; P)$, if B contains some members of jurisdiction j (that is, $A_j \cap B \neq \emptyset$), then $z(A_j) = z(A_i)$. If $z(A_j) > z(A_i)$, then the slope of $u(\cdot; P)$ on the interval $A_j \cap B$, is larger than the slope on both its left and right, and if $z(A_j) < z(A_i)$, it is smaller. Both contradict convexity. In addition $U(\theta, A_j) = U(\theta, A_i)$ for all $\theta \in B$, hence for all $\theta \in (\theta_0, \theta^0)$. Otherwise equation (4) would be violated.

There are two cases, $\theta^{M}(A_j) \geqq \theta^{M}(A_i)$ and $\theta^{M}(A_j) \leqq \theta^{M}(A_i)$. We only need to argue for one of these cases. Suppose that $\theta^{M}(A_j) \geqq \theta^{M}(A_i)$. There exists a small $B \subset A_j$ such that $\theta > \theta^{M}(A_i)$ and $U(\theta, A_i \cup B) > u(\theta; P) = U(\theta, A_j) = U(\theta, A_i)$ for $\theta \in B$, which contradicts condition 1 of FME. The latter follows from equation (2) because $|A_i|$ and $\theta^{M}(A_i)$ are increased in proportions $d\theta^{M}(A_i) = \frac{1}{2} \mathrm{d}|A_i|$, and for $\theta \in B$, $\theta > \theta^{M}(A_i)$. We conclude that each A_i must be an interval.

We now show (b), assuming that each A_i is an interval. We label the boundary points $\{\theta_i\}_{i=1}^{k}$ such that $A_i = (\theta_{i-1}, \theta_i)$, $i = 1, \ldots, k$. It follows from continuity of $u(\cdot; P)$ (due to equation (4)) that $U(\theta_i, A_i) = U(\theta_i, A_{i+1})$ for each i. To show the second part of the statement we characterize the dividing point θ_i in an interval $(\theta_{i-1}, \theta_{i+1})$ as the point where $U(\theta, (\theta_i, \theta_{i+1})) - U(\theta, \theta_{i-1}, \theta_i)) = 0$. See Figure 4. One can verify that if $\theta_{i+1} < 3\theta_{i-1}$, then there is only one zero for the difference, namely the midpoint, but otherwise, as shown in Figure 4, there are two zeroes, with the smaller one described by f. That is, (b) is satisfied if and only if $U(\theta_i, (\theta_{i-1}, \theta_i)) = U(\theta_i, (\theta_i, \theta_{i+1}))$ and $\theta_i < \frac{1}{2}(\theta_{i-1} + \theta_{i+1})$. Lemma 4 excludes that $|A_i| = |A_{i+1}|$, and (b) follows. ||

Proof of 1(1). (If). Lemma 5 shows that if (a), (b) and (c) hold, then for every $B \subset (\theta_i, \theta^0)$ there exists $\theta \in B$ such that $U(\theta, B \cup A_i) < u(\theta; P)$. Lemma 6 shows the same thing for small $B \subset (\theta_0, \theta_{i-1})$, since condition (ii) of Lemma 6 follows from (b). ||

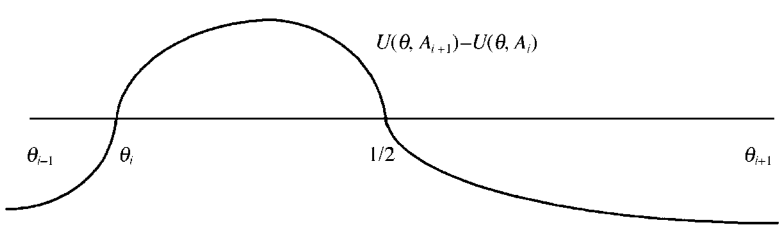

FIGURE 4

Division into two jurisdictions

Proposition 1(2) is proved by Jehiel and Scotchmer (1996). It follows from Figure 1, which shows that each jurisdiction is more than twice as large as the one below it. This property aggregates so that the highest-θ jurisdiction is more than half the population.

Proof of Proposition 1(3). The number of jurisdictions cannot be infinite when θ^0 is finite because then all but a finite number of $\lambda_i = \theta_i/\theta_{i-1}$ would be very close to one, which contradicts the property of f that every second growth rate must be larger than $\lambda^* > 1$. On the other hand, since $\theta_i/\theta_{i-1} \leqq 3$ for every i the number of jurisdictions must become unbounded as θ^0 becomes unbounded.

We prove existence using Figure 5. Define $\theta^j(\lambda_1) \equiv \theta_0\lambda_1\Pi_{i=2}^j f^{i-1}(\lambda_1)$, so that $\theta^j(\lambda_1) = \theta_j$ if $\theta_1 = \lambda_1\theta_0$. The key features of Figure 5 are that since $f(1) = 3$, we have $\theta^{k-2}(3) = \theta^{k-1}(1) < \theta^{k-1}(3) = \theta^k(1) < \theta^k(3)$, and that each $\theta^k(\lambda_1)$ is monotone. The initial growth rate λ_1 must satisfy $1 < \lambda_1 \leqq 3$ by Proposition 1. (If not then condition 2 of the definition of FME would not be satisfied.) An arbitrary initial growth rate determines the entire partition according to the functions $\theta^j(\lambda_1)$. For an arbitrary initial growth rate the last boundary point will typically not coincide with θ^0, but Figure 5 shows that there exists an initial growth rate such that $\theta^0 = \theta^k(\lambda_1)$ for some $k = k^{FM}$, and further, that it is unique. ||

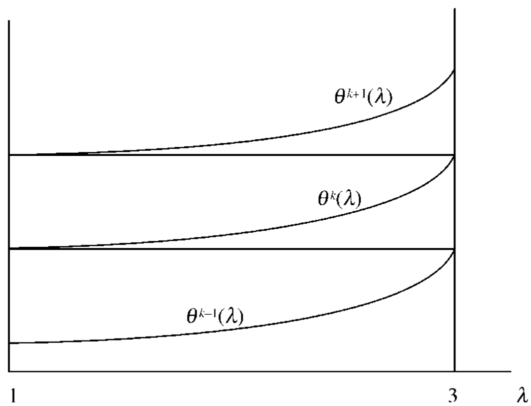

FIGURE 5

Proof of Proposition 3(1). We must show that if one of the conditions (a), (b) or (c) does not hold, the partition is not a UCE.

Suppose first that (a) does not hold. Then condition 2 of UCE is violated, using equation (1).

Claim 1. *Suppose first that for some i, $A_i = (\theta_{i-1}, \theta_i)$ and $\theta_i/\theta_{i-1} < 1 + 2/\sqrt{3}$. Then there exists $b > 0$ such that if $B = (\theta_{i-1} - b, \theta_{i-1})$, $U(\theta, A_i \cup B) > U(\theta, A_i)$ for all $\theta \in A_i$.*

Proof of Claim. It is enough that $dU(\theta, A_i)$ is negative for all $\theta \in A_i$, when the lower boundary θ_{i-1} is lowered. Since $\partial\theta^M(A_i)/\partial\theta_{i-1} = \frac{1}{2}$, and $\partial|A_i|/\partial\theta_{i-1} = -1$, $dU(\theta, A_i)$ is decreasing in θ, and hence it is enough that the derivative is negative for θ_{i-1}. This follows from equation (2). ||

Now suppose that condition (b) does not hold so that $U(\theta_i, A_{i-1}) < U(\theta_i, A_i)$ for some i. Then for B as described in the above claim, $U(\theta, B \cup A_i) > u(\theta; P)$ for all $\theta \in B \cup A_i$, contradicting condition 1.

Now suppose that (c) does not hold (but (b) holds), that is, for some $i = 1, \ldots, k-1$, $\theta_i/\theta_{i-1} < 1 + 2/\sqrt{3}$ and $U(\theta_i, A_i) > U(\theta_i, A_{i+1})$. Then if $B = (\theta_i, \theta_i + \delta)$ for some small δ, $U(\theta, B \cup A_i) \geqq u(\theta; P)$ for all $\theta \in B \cup A_i$ with strict inequality for at least $\theta \in (\theta_i, \theta_i + \delta)$. This follows because $d\theta^M(A_i)/d\theta_i = \frac{1}{2}$ when $d|A_i|/d\theta_i = 1$, and when $\theta_i/\theta_{i-1} < 1 + 2/\sqrt{3}$ $dU(\theta, A_i)/d\theta_i > 0$ for all $\theta \in A_i$. Since the derivative is increasing in θ, it is enough to show it is positive for θ_{i-1}. $dU(\theta_{i-1}, A_i)/\mathrm{d}\theta_i = -3\theta^M(A_i)^2 + 6\theta_{i-1}\theta^M(A_i) - 2\theta_{i-1}^2$. Since $\theta^M(A_i) \geqq \theta_{i-1}$, there is one root and the derivative is positive if $\theta^M(A_i)/\theta_{i-1} < (1 + 1/\sqrt{3})$, which holds if and only if $\theta_i/\theta_{i-1} < 1 + 2/\sqrt{3}$. ||

Proof of Proposition 3(2). We must show that if a partition satisfies (a), (b) and (d), then conditions 1 and 2 of the definition of UCE are satisfied.

First, if (a) holds, then condition 2 is satisfied. The more difficult part is to show condition 1, assuming that (b) and (d) hold as well.

For given A_i we first consider $B\subset(\theta_i, \theta^0)$. We must dismiss the possibility that $U(\theta, B\cup A_i)\geqq u(\theta; P)$ for all $\theta\in B\cup A_i$ with strict inequality for some subset $C\subset B\cup A_i$ with positive measure. We shall assume that $|B|\leqq|A_i|$. Otherwise it follows from (d) that $\theta_i/\theta_{i-1}>3$, so that $U(\theta_{i-1}, B\cup A_i)<0\leqq u(\theta_{i-1}; P)$.

We shall simplify notation by letting $b+a=|B|+|A_i|$. We let $\theta_i^M=\theta^M(A_i)$, $\theta_{i-1}^M=\theta^M(A_{i-1})$, and $\theta_B^M=\theta^M(B\cup A_i)$. When $B\subset(\theta_i, \theta^0)$ and $|B|<|A_i|$, then $\theta^M(B\cup A_i)=\theta^M(A_i)+\frac{1}{2}b$.

$$
\begin{aligned}
U(\theta_{i-1}, B\cup A_i) &= \tfrac{1}{2}(b+a)\theta_B^M(\theta_{i-1}-\tfrac{1}{2}\theta_B^M)\\
&= \tfrac{1}{2}a\theta_B^M(\theta_{i-1}-\tfrac{1}{2}\theta_B^M)+\tfrac{1}{2}b\theta_B^M(\theta_{i-1}-\tfrac{1}{2}\theta_B^M)\\
&= \tfrac{1}{2}a(\theta_i^M+\tfrac{1}{2}b)(\theta_{i-1}-\tfrac{1}{2}\theta_B^M)+\tfrac{1}{2}b\theta_B^M(\theta_{i-1}-\tfrac{1}{2}\theta_B^M)\\
&= U(\theta_{i-1}, A_i)-\tfrac{1}{4}ab\theta_B^M+\tfrac{1}{8}b[(a+2\theta_B^M)(3\theta_{i-1}-\theta_i-b)].
\end{aligned}
$$

Multiplying the last two terms by $8/b$, $-2a\theta_B^M+(a+2\theta_B^M)(3\theta_{i-1}-\theta_i-b)$

$$
\begin{aligned}
&= -2a\theta_i^M-ab+(a+2\theta_i^M)(3\theta_{i-1}-\theta_i)+b(3\theta_{i-1}-\theta_i)-b(a+2\theta_B^M)\\
&= -2a\theta_i^M+2\theta_i(3\theta_{i-1}-\theta_i)-b[4(\theta_i-\theta_{i-1})+b]\\
&\leq -2a\theta_i^M+2\theta_i(3\theta_{i-1}-\theta_i),
\end{aligned}
$$

which is negative if condition (d) holds.

Turning to $B\subset(\theta_0, \theta_{i-1})$, by Lemma 6, for every $B\subset(\theta_0, \theta_{i-1})$, there exists $\theta\in B$ such that $U(\theta, B\cup A_i)<u(\theta; P)$. ||

Proof of Lemma 4. We first prove that TE jurisdictions must be intervals. Suppose that $\{A_i\}_{i=1}^k$ is a TE and that a particular A_i is not an interval. Then there exists θ such that for some arbitrarily small ε: $(\theta-\varepsilon, \theta)\subset A_i$, $(\theta, \theta+\varepsilon)\subset A_j$ for some $j\neq i$ and $A_i\cap(\theta+\varepsilon, \theta^0)\neq\varnothing$. Since $(\theta-\varepsilon, \theta+\varepsilon)<\max A_i$, there is free mobility between A_i and A_j, which implies that $U(\theta, A_i)=U(\theta, A_j)$. Since θ is at a boundary between A_i and A_j, a small perturbation of $\{A_i\}_{i=1}^k$ would perturb A_i, A_j, θ, to A_i', A_j', θ' so that $U(\theta', A_i')\neq U(\theta', A_j')$. By the above argument the perturbed partition $\{A_i'\}_{i=1}^k$ is not a TE, and therefore by condition 4, $\{A_i\}_{i=1}^k$ is not a TE.

We now argue that (a) holds. Since TE jurisdictions are intervals, $A_i=(\theta_{i-1}, \theta_i)$, $i=1, \ldots, k$. If $\theta_i/\theta_{i-1}>3$ for some i, then a low-θ member would migrate to a singleton jurisdiction (see (1)), in violation of condition 3. Also, reasoning as in Proposition 3(1)(c), if $\theta_i/\theta_{i-1}<1+2/\sqrt{3}$ for any $i\leqq k-1$ then $U(\theta_i, A_i)=U(\theta_i, A_{i+1})$ is not robust to a small perturbation of the boundary points. Thus $\theta_i/\theta_{i-1}\geqq 1+2/\sqrt{3}$ for all $i\leqq k-1$, and using condition 4 again, $\theta_i/\theta_{i-1}>1+2/\sqrt{3}$ for all $i\leqq k-1$. Similarly, the reasoning behind Proposition 3(1)(b) implies (c), that $U(\theta_i, A_i)>U(\theta_i, A_{i+1})$ for all $i=1, \ldots, k-1$. The latter, together with $\theta_{i+1}/\theta_i>1+2/\sqrt{3}$, implies (b), that $\theta_{i+1}/\theta_i>f(\theta_i/\theta_{i-1})$ as can be seen from Figure 4. Thus, (a), (b) and (c) are necessary if $\{A_i\}_{i=1}^k$ is a TE. The sufficiency of (a), (b) and (c) is shown in a similar fashion as in Proposition 3. ||

Proof of Proposition 5. We will characterize the maximum number of jurisdictions in TE. We do this by considering the dual problem, which is to find the smallest possible θ^0 such that a partition into k interval jurisdictions is a TE. Define the sequence $(\lambda_i)_{i=1,\ldots,k}$ by $\lambda_i=\theta_i/\theta_{i-1}$, corresponding to a sequence of boundary points $\{\theta_i\}_{i=1}^k$, where $\theta^k=\theta^0$ and for each $j=1, \ldots, k$, $\theta_j=\theta_0\Pi_{i=1}^j\lambda_i$.

By (b) in Proposition 4, $\lambda_{i+1}\geqq f(\lambda_i)$ for each $i=1, \ldots, k-1$. Referring to Figure 4, by condition (c), the dividing point $\theta_i\in(\theta_{i-1}, \theta_{i+1})$ is either in the interval to the left of θ^{FM} or above the midpoint. If above the midpoint, condition (b) is violated.

To solve the dual problem we characterize the minimum of $\Pi_{i=1}^k\lambda_i$ subject to $\lambda_i\geqq f(\lambda_{i-1})$, $i=1, \ldots, k-1$ and the definitional constraint $\lambda_i\geqq 1$. Since $\lambda_{i+1}\geqq f(\lambda_i)>1$ for $i\geqq 1$, the constraint $\lambda_i\geqq 1$ can only bind for λ_1, and $\lambda_1=1$ is the limiting case where the first coalition is empty. By induction the solution to the minimization problem is $\lambda_1=1$ and $\lambda_i=f(\lambda_{i-1})$ for $i=1, \ldots, k$, as follows. Conditional on the first $(\lambda_1, \ldots, \lambda_{k-1})$, $\theta^0=\theta_0\Pi_{i=1}^k\lambda_i$ is minimized by minimizing λ_k; *i.e.* by setting $\lambda_k=f(\lambda_{k-1})$. Conditional on the first $(\lambda_1, \ldots, \lambda_{k-2})$, and accepting that $\lambda_k=f(\lambda_{k-1})$, $\theta_0\lambda_1, \ldots, \lambda_k$ is minimized by minimizing $\lambda_{k-1}f(\lambda_{k-1})$. By Lemma 8 in the Appendix, this product increases with λ_{k-1}, and hence is minimized by choosing $\lambda_{k-1}=f(\lambda_{k-2})$. The result follows by induction.

If θ^0 is given in advance, then for some k we will have that $\theta^{k-1}(1)\leqq\theta^0<\theta^k(1)$. (See Figure 5). Then k is an upper bound on the number of jurisdictions in P of (θ_0, θ^0). For any larger k, say $k+1$, we have that for all $\lambda_1\geqq 1$, $\theta^{k+1}(\lambda_1)>\theta^k(\lambda_1)\geqq\theta^k(1)>\theta^0$. Thus it is impossible to satisfy the necessary condition (a) of Proposition 4 with larger k.

We now compare the maximum number of jurisdictions in P with the number in P^{FM}, say k^{FM}. Suppose that $\theta^{k-1}(1)\leqq\theta^0<\theta^k(1)$ for some k. We argued in the above paragraph that k is an upper bound for the number of jurisdictions in P. By monotonicity (see Figure 5), there exists $\lambda_1 > 1$ such that $\theta^{k-1}(\lambda_1) = \theta^0$, so that $k^{FM} = k-1$. The result follows. ||

Proof of Proposition 6. We show that there is no other partition that is strictly preferred by a majority. Let $\hat{\mathcal{P}} = \mathcal{P}\,(\text{FME}) \cup \mathcal{P}\,(\text{MVE}) \cup \mathcal{P}\,(\text{UCE}) \cup \mathcal{P}\,(\text{TE})$. We show that for every partition $P' \in \hat{\mathcal{P}}$, $|B(P', P)| < \frac{1}{2}(\theta^0 - \theta_0)$. It is enough to show that for every $\theta \geqq \frac{1}{2}\theta^0$, there is no $P' \in \hat{\mathcal{P}}$ such that $u(\theta, P') > u(\theta, P)$.

Claim 2. *Let B be a jurisdiction in $P' \in \hat{\mathcal{P}}$. Let $\underline{\theta} = \inf B$. Then $|B| \leqq 2\underline{\theta}$.*

Proof. Fixing $\underline{\theta}$ and B, if $|B| > 2\underline{\theta}$, it follows from (1) that $U(\underline{\theta}, B) \leqq U(\underline{\theta}, (\underline{\theta}, \underline{\theta} + |B|)) < 0$. Thus, if $|B| > 2\underline{\theta}$, then $U(\underline{\theta}, B) < 0$, so $P' \notin \hat{\mathcal{P}}$. ||

Consider $\theta \geqq \frac{1}{2}\theta^0$. Then $\theta \in (\frac{1}{3}\theta^0, \theta^0)$. We will argue that $u(\theta, P) \geqq u(\theta, P')$ for all $P' \in \hat{\mathcal{P}}$. If this is not true, there exists a coalition $B \in P'$ such that $U(\theta, (\frac{1}{3}\theta^0, \theta^0)) < U(\theta, B)$, and $\theta \in B$. Since the only aspects of a jurisdiction that an agent cares about are its size and median voter, there is an interval jurisdiction, say $\tilde{B}$, such that $\inf B \leqq \inf \tilde{B}$, $|B| = |\tilde{B}|$, and $\theta \in \tilde{B}$, and $U(\theta, (\frac{1}{3}\theta^0, \theta^0)) < U(\theta, \tilde{B}) = U(\theta, B)$. Let $\tilde{B} = (y, x)$ for some $y, x \in [\theta_0, \theta^0]$. By Claim 2, $y \geqq \frac{1}{3}x$, and since, for fixed θ, $U(\theta, \tilde{B})$ increases if $\tilde{B}$ is enlarged at its lower boundary (see (2) with $d\theta^M = -\frac{1}{2}d|A|$), we can assume that $\tilde{B} = (\frac{1}{3}x, x)$ for some x. Thus $U(\theta, \frac{1}{3}(\theta^0, \theta^0)) < U(\theta, (\frac{1}{3}x, x))$. But since $U(\theta, (\frac{1}{3}x, x))$ increases with x for $\theta > \frac{1}{2}\theta^0 \geqq \frac{1}{2}x$, it follows that $U(\theta, (\frac{1}{3}\theta^0, \theta^0)) < U(\theta, (\frac{1}{3}\theta^0, \theta^o))$, which is a contradiction.

But since $|\{\theta \in (\theta_0, \theta^0) \mid \theta \geqq \frac{1}{2}\theta^0\}| \geqq \frac{1}{2}$, this proves that if there exists a partition $P \in \mathcal{P}\,(\text{UCE})$ such that the highest-θ group is $(\frac{1}{3}\theta^0, \theta^0)$, such a partition must be strongly stable.

We must find $P \in \mathcal{P}\,(\text{UCE})$ that satisfies $A_k = (\frac{1}{3}\theta^0, \theta^0)$, and show that there is no $P' \in \hat{\mathcal{P}} \setminus \mathcal{P}\,(\text{UCE})$ with that property.

We know that for any interval, in particular, $(\theta_0, \frac{1}{3}\theta^0)$, FME exists. Let $\{\tilde{A}_i\}_{i=1}^{k-1}$ be an FME for $(\theta_0, \frac{1}{3}\theta^0)$. Let the partition $\{A_i\}_{i=1}^{k}$ be defined by $A_i = \tilde{A}_i$ for $i = 1, \ldots, k-1$, and $A_k = (\frac{1}{3}\theta^0, \theta^0)$. Then the partition $\{A_i\}_{i=1}^{k}$ satisfies conditions (a), (b) and (c) in Proposition 3, and is therefore in $\mathcal{P}\,(\text{UCE})$.

The robustness requirement, condition 4 of TE, implies that there is no partition in $\mathcal{P}\,(\text{TE})$ such that $\theta_k/\theta_{k-1} = 3$, hence the partition is not in $\mathcal{P}\,(\text{TE})$.[11] The partition is also not in $\mathcal{P}\,(\text{FME}) = \mathcal{P}\,(\text{MVE})$, since $\theta_k/\theta_{k-1} = 3 > f(\theta_{k-1}/\theta_{k-2})$. ||

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11. TE is eliminated only as an artifact of the robustness requirement. The closure of the partitions in TE includes a partition such that the highest-θ jurisdiction is $(\frac{1}{3}\theta^0, \theta^0)$. If we include the closure, TE would be constitutionally stable as well.

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