

Manipulative Auction Design*

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Abstract

This paper considers an auction design framework in which bidders get partial feedback about the distribution of bids submitted in earlier auctions: either bidders are asymmetric but past bids are disclosed in an anonymous way or several auction formats are being used and the distribution of bids but not the associated formats are disclosed. I employ the analogy-based expectation equilibrium (Jehiel, 2005) to model such situations. First-price auction in which past bids are disclosed in an anonymous way generates more revenues than the second-price auction while achieving an efficient outcome in the asymmetric private values two-bidder case with independent distributions. Besides, by using several auction formats with coarse feedback a designer can always extract more revenues than in Myerson's optimal auction, and yet less revenues than in the full information case whenever bidders enjoy ex-post quitting rights and the assignment and payment rules are monotonic in bids. These results suggest an important role of feedback disclosure as a novel instrument in mechanism design.

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1 Introduction

Standard equilibrium approaches of games with incomplete information (à la Harsanyi) assume that players know the distributions of signals held by other players as well as these players' strategies as a function of their signals (see Harsanyi (1995)). Yet, this requires a lot of knowledge that need not be easily accessible to players. Modern approaches to equilibrium rely on learning to justify this knowledge (see Fudenberg and Levine (1998) for an overview of the literature on learning in games). But, it is in general questionable that enough information feedback is available to the players at the learning stage for convergence to equilibrium to be reasonably expected.

For the sake of illustration, consider a series of first-price auctions of similar objects (say, iphones) involving each time new bidders of observable characteristics $i = 1, 2, \dots, n$ (say, bidders of different age). Assume that every bidder knows his own valuation for the object but not those of other bidders, and assume that the distribution of valuation depends on the observable characteristic i . In many contexts, it seems unlikely that bidders would *a priori* know these distributions and how they depend on i . In such cases, bidders would look at the bids submitted in earlier similar auctions so as to form a judgement as to what the distribution of others' bids is likely to be in the current auction of interest.¹ In many practical auction designs such as those used on ebay or treasury auctions, bidders have access only to the aggregate distribution of bids in past auctions without being informed of the characteristics of the bidders of the corresponding bids. It is then dubious that bidders would be able to play a best-response to the actual distribution of bids of the other bidders because there is no way a bidder can assess the distribution of bids conditional on the observable characteristic based on the feedback he receives. Instead, bidders are more likely to play a best-response to the conjecture that all bidders - no matter what their characteristic is - bid according to the aggregate distribution of bids that mix the distribution of bids of all bidders. In the long run, assuming convergence of the overall process, bidders are not playing a Nash equilibrium but an analogy-based expectation equilibrium with bidder-anonymous analogy partition (see Jehiel (2005) for the first exposition of this concept and Section 2 for an application to the private value

¹Typically, on ebay, one has access to the history of bids made in auctions of similar objects run in the last month.

auction setup considered here).²

As an alternative example, consider promotions in an organization. Each promotion takes the form of a contest between several employees who each make a proposal for the job task specification in case of promotion (i.e., what they will effectively do for the organization if promoted). Each employee has some private interest for the promotion (depending on how he values power, social status, responsibilities...), which is known to him only. Besides, while the criterion for the current promotion (based on the proposals) is typically known to the contestants, the criterion may typically change from one promotion to the next.³ Nash equilibrium would require that contestants know the distribution of others' proposals given the criterion applying to the current promotion. Assuming that contestants form their expectations by looking at past promotions, this would require that employees have access to the joint distribution of proposals and corresponding criterion. Yet, if employees only have access to the distribution of proposals (and not of the corresponding criterion), contestants will not be able to play a Nash equilibrium. Instead, I propose that contestants play a best-response (given the current criterion) to the conjecture that the distribution of proposals is the same irrespective of the criterion (and that it corresponds to the aggregate distribution of proposals they have access to). This is again an analogy-based expectation equilibrium with appropriately chosen analogy partitions.

In this paper, I generalize the above two examples. I consider one-object private values auction environments in which the valuations are independently distributed across bidders, bidders receive coarse feedback about the distribution of previous bids, and every single bidder participates in just one auction. Specifically, I consider situations in which, as in the first-price auction example, only the aggregate distribution of bids with no reference to the characteristics of bidders is disclosed, and also situations in which, as in the promotion example, different auction formats are being used (this is the analog of different criteria being used) and only the aggregate distribution of bids across the

²In the language of econometrics, the model is not identifiable. Most applied econometricians would particularize the model making an extra symmetry assumption (see Athey and Haile (2006)). In a sense, this paper is assuming that bidders follow the same line as the econometricians, even though this does not boil down to assuming that the true underlying problem is symmetric, as we shall see.

³In the formal model to be described in Section 2, the private interest should be identified with the valuation, and the proposal (even though generally multi-dimensional) should be identified with the bid (maybe as in scoring auctions).

various auction formats is being disclosed. Any combination of such coarse disclosures is also allowed. The equilibrium concept that is used to describe the interaction of bidders with such coarse feedback disclosure - the analogy-based expectation equilibrium - requires that bidders play a best-response to the aggregate distribution of bids, as given by the feedback they receive.

I explore whether and when it is the case that the designer is better off when bidders play an analogy-based expectation equilibrium rather than a Nash equilibrium and how the answer is affected by the specific forms of feedback disclosure and auction rules. Addressing such questions opens new avenues in mechanism design. As it turns out, providing coarse feedback about past behaviors - as considered in this paper - can enhance the designer's objective, which suggests the relevance of an instrument not previously considered in mechanism design.⁴

To highlight the potential role of feedback disclosure in mechanism design, I assume that (in addition to the format(s)) the designer is free to choose which kind of feedback (within the class specified above) to disclose to bidders. In the promotion example, choosing which feedback to disclose sounds natural for the designer given that the feedback is in principle under the control of the organization. In the auction example, this is an idealization that would fit well if a single seller had many similar objects to auction off over time. My framework in the auction case can more usefully be interpreted as providing insights as to what kind of feedback policy an auction house such as ebay should adopt so as to increase the revenues of sellers (which in turn determine the revenues of the auction house through the fees).

I first assume that the main objective of the designer is welfare maximization whereas the auxiliary goal is revenue (as is the case in many government auctions). The main question I address with this objective in mind is: Can the designer do better than using a second-price auction (or equivalently an ascending-price auction) by using a coarse feedback device?

In the classic setup (relying on Nash equilibrium), the so called revenue equivalence theorem implies that the designer can do no better. This is so because the second-price auction induces an efficient outcome and any efficient mechanism that respects the

⁴For a related investigation in the context of symmetric first-price auctions with affiliated signals, see the independent work of Esponda (2008a), which is discussed in subsection 5.1.

participation constraints of bidders must achieve a revenue no greater than that of the second-price auction (see, for example, Milgrom (2004) for an exposition of the revenue equivalence theorem).

In my setup, I show that the designer can sometimes do better, thereby illustrating a failure of the revenue equivalence theorem when the solution concept is the analogy-based expectation equilibrium. Specifically, in the case of two bidders with asymmetric distributions of valuations, I show that the first-price auction in which the designer provides as feedback the aggregate distribution of bids with no reference to the characteristic of the bidders (as considered above) always induces an efficient outcome and always generates an expected revenue that is strictly larger than that of the second-price auction no matter what the distributions of valuations are. I also provide conditions ensuring that such a result holds true when there are more than two bidders.⁵

I next consider the case in which the designer is only interested in revenues and I assume that bidders can always veto the transaction ex-post (thereby limiting the scope for manipulation). The main question I am interested in is: Can the designer generate more revenues than in the classic optimal auction characterized by Myerson (1981) and Riley and Samuelson (1981)?

I show that this is always so and that the best revenue so obtained lies strictly below the full information maximal revenue. In other words, a clever use of coarse feedback coupled with a judicious choice of format(s) may reduce the informational rent left to bidders even though some rent must be left to bidders.⁶

The rest of the paper is organized as follows. In Section 2 the model is introduced together with the analogy-based expectation equilibrium and the mechanism design problem. In Section 3, the first-price auction with bidder-anonymous feedback partition is analyzed. In Section 4, it is shown how one can generate more revenues than in Myer-

⁵Such clear-cut revenue comparisons (at least in the two-bidder case) should be contrasted with the ambiguous revenue ranking between the first-price auction and the second-price auction obtained with Nash equilibrium when bidders are asymmetric (see Maskin and Riley (2000)).

⁶The auction design and feedback policy used to prove that one can go strictly beyond Myerson's optimal auction revenue require the use of several (in fact two) auction formats and that bidders be only informed of the aggregate distribution of bids over the various formats. Such auction designs do not clearly resemble existing ones, and more work is required to map this theoretical result to practical auction design. Such a construction may possibly shed light on why transparency need not be optimal in promotion contexts.

son's optimal auction. Section 5 offers a discussion including how the paper relates to the literature, what would happen if the transaction need not be approved ex post or if the designer could use shill bidders. Section 6 concludes. All proofs are in the Appendix.

2 Basic definitions

An object is to be auctioned off and there are n bidders $i = 1, \dots, n$. Each bidder i knows his own valuation v_i for the object, but not that of the other bidders $j \neq i$. The distribution of valuations are independent across bidders. The valuation v_i is drawn from a distribution with support $[c, d]$ and (continuous) density $f_i(\cdot)$ where $f_i(v) > 0$ for all $v \in [c, d]$ and $d > c \geq 0$. Bidders have quasi-linear preferences and they are risk neutral. That is, if a bidder with valuation v expects to win the object with probability p and expects to make (an expected) transfer t to the seller, his expected utility is $pv - t$. The seller's valuation is v_s .

The object is auctioned off through possibly different auction formats M_k , $k \in K = \{1, \dots, r\}$ where format M_k is used with probability λ_k . Each auction format M_k (which together with λ_k is chosen by the designer) is restricted to take the following form:

- Bidders $i = 1, \dots, n$ simultaneously submit a bid $b_i \in [0, d]$.
- Based on the profile of bids $b = (b_i)_{i=1}^{i=n}$, with probability $\varphi_i^k(b)$ where $\sum_i \varphi_i^k(b) \leq 1$, bidder i is offered to get the object in exchange for a payment $\tau_i^k(b)$ to the seller. The transaction takes place if after being informed of $\tau_i^k(b)$ bidder i approves the terms of the contract.
- For every i, k , $\varphi_i^k(b)$ is a non-decreasing function of b_i and a non-increasing function of b_j , $j \neq i$; $\tau_i^k(b)$ is a non-decreasing functions of b_i and of b_j , $j \neq i$. Moreover, if $b_i < v_s$, then $\varphi_i^k(b) = 0$.

The auction formats considered in this paper require that the bidders approve the terms of the contract ex-post. That is, assuming that bidder i with valuation v_i is declared the winner with a tentative transaction price $\tau_i^k(b)$, there is effectively a transaction (at

price $\tau_i^k(b)$) only if $v_i > \tau_i^k(b)$ and otherwise there is no transaction and the seller keeps the object.⁷

The monotonicity assumptions made on $\varphi_i^k(b)$ and $\tau_i^k(b)$ are in line with the auction interpretation: As a bidder increases his bid, his probability of winning (weakly) increases and so does the price this bidder must pay. As a competing bidder increases his bid, the probability of winning (weakly) decreases and the price paid in case of transaction (weakly) increases. Moreover, the requirement that if $b_i < v_s$, $\varphi_i^k(b) = 0$ is meant to represent the idea that the valuation v_s of the seller is the minimum starting point of the auction (as the seller would refuse to sell at a price lower than v_s).

It should be mentioned that the first-price auction, and the second-price auction with reserve price no smaller than v_s both belong to the above class of auctions, and that it is always possible to pick an auction in this class that achieves Myerson's optimal revenue no matter what the densities $f_i(\cdot)$ are. Moreover, the requirement that bidders should approve the terms of the contract ex-post ensures that a bidder no matter what bidding strategy he employs and no matter what he expects the distribution of other bids to be is better off participating in the auction rather than staying outside.

I have in mind situations in which bidders have no prior idea about what the densities of valuations $f_i(\cdot)$, $i \in I$, are and in which every individual bidder participates in just one auction. In order to form their expectations about the distribution of other bids, bidders rely on the feedback about past play that is made available by the designer.

Specifically, when auction format M_k prevails, bidder i is assumed to be informed of the functions $\varphi_i^k(b)$ and $\tau_i^k(b)$ that apply to him in this format (but he need not be informed of other characteristics of M_k , see more on this below). Thus, if bidder i with valuation v_i bids b_i and expects the bid profile $b_{-i} = (b_j)_{j \neq i}$ to be distributed according to the random variable \tilde{b}_{-i} , his perceived expected utility in M_k is:

$$u_i^k(v_i, b_i; \tilde{b}_{-i}) = \underset{\tilde{b}_{-i}}{E} [\varphi_i^k(b_i, b_{-i}) \max(v_i - \tau_i^k(b_i, b_{-i}), 0)]$$

A strategy of bidder i is a family of bid functions $\beta_i = (\beta_i^k)_k$, one for each auction

⁷I will also assume that there is a tiny (rather than exactly zero) cost to cancelling out the transaction. This will allow me to rule out crazy behaviors from bidders with valuations $v < v_s$.

format M_k where $\beta_i^k(v_i)$ denotes bidder i 's bid in format M_k when i 's valuation is v_i .⁸

Nash equilibrium would require that for each k and v_i , bidder i plays a best-response to the *actual* distribution of bids of bidders $j \neq i$ in M_k . That is,

$$\beta_i^k(v_i) \in \arg \max_{b_i} u_i^k(v_i, b_i; \beta_{-i}^k)$$

where (with some slight abuse of notation) β_{-i}^k stands for the random variable of bids $(\beta_j^k(v_j))_{j \neq i}$ as generated by the densities $(f_j(\cdot))_{j \neq i}$.

As already highlighted, bidders are not assumed to know (or have access to) β_j^k for every j and k . Instead, each bidder i receives partial feedback about the distribution of bids observed in past auctions. They play a best-response to this feedback (in a sense to be defined next) and a steady state is assumed to have been reached.

Specifically, I consider the following class of partial feedback. Endow each bidder i with a partition \mathbf{P}_i of the set $\{(j, k), j \in I \text{ and } k \in K\}$ referred to as the analogy partition of bidder i . A typical element of \mathbf{P}_i is denoted by α_i and referred to as an analogy class of bidder i . The element of \mathbf{P}_i containing (j, k) is denoted by $\alpha_i(j, k)$. The interpretation of \mathbf{P}_i is that bidder i gets only informed of the empirical distribution of bids of bidders engaged in past auctions where all bids b_j submitted in M_k with $(j, k) \in \alpha_i$ are treated alike (i.e., they are not distinguished).

I further assume that a steady state (in which new sets of bidders with newly drawn valuations arrive each time) has been reached so that the empirical distributions of previous bids correspond to the actual distributions. When making his choice of strategy in auction format M_k , bidder i is thus assumed to know only (in addition to $\varphi_i^k(b)$ and $\tau_i^k(b)$) the aggregate distribution of bids in every α_i . He is further assumed to play a best response to the conjecture that bidder j in format M_k bids according to the aggregate distribution of bids in $\alpha_i(j, k)$, the analogy class to which (j, k) belongs.

Formally, let $\mathbf{A} = (M_k, \lambda_k, \mathbf{P}_i)_{i \in I, k \in K}$ denote an auction design. The solution concept is defined as:

Definition 1 *An analogy-based expectation equilibrium of $\mathbf{A} = (M_k, \lambda_k, \mathbf{P}_i)_{i \in I, k \in K}$ is a*

⁸Strictly speaking, allowing for mixed strategies $\beta_i^k(v_i)$ should be a distribution over bids. Yet, for my purpose, considering pure strategies is enough.

strategy profile $\beta = (\beta_i)_{i \in I}$ such that for every k and v_i ,

$$\beta_i^k(v_i) \in \arg \max_{b_i} u_i^k(v_i, b_i; \bar{\beta}_{-i}^k)$$

where $\bar{\beta}_{-i}^k = (\bar{\beta}_j^k)_{j \neq i}$, and $\bar{\beta}_j^k$ is the aggregate distribution of bids in $\alpha_i(j, k)$. That is, $\bar{\beta}_j^k$ is the distribution of bids that assigns weight $\lambda_{k'}/\sum_{(j'', k'') \in \alpha_i(j, k)} \lambda_{k''}$ to the distribution $\beta_{j'}^{k'}(v_{j'})$ as generated by the density $f_{j'}(\cdot)$ for every $(j', k') \in \alpha_i(j, k)$, and the distributions $\bar{\beta}_j^k$, $j \neq i$ are perceived by bidder i to be independent of each other.

Remarks. 1) It should be mentioned that the feedback received by bidders is about the distribution of individual bids and not about the distribution of bid profiles.⁹ 2) The analogy-based expectation equilibrium has been first introduced in Jehiel (2005) for extensive form games and Jehiel and Koessler (2008) for static games of incomplete information. It will be further interpreted in the context of private value auctions in subsection 2.2, and it will be related to other approaches in the literature in particular the self-confirming equilibrium in subsection 5.1.

Various objectives for the designer will be considered. The first objective will be a lexicographic criterion with welfare ranked first and revenues ranked second. The second objective will be the seller's expected utility as measured by the revenue she gets when there is a transaction and her valuation v_s when there is no transaction. In all cases, the designer is assumed to be risk neutral. She assesses an auction design $\mathbf{A} = (M_k, \lambda_k, \mathbf{P}_i)_{i \in I, k \in K}$ according to the expected value of her objective assuming that bidders behave according to an analogy-based expectation equilibrium of \mathbf{A} .¹⁰

⁹Accordingly, every bidder i treats every bidder j 's distribution of bids, $j \neq i$, as being independent of each other. This fits in well when bidders do not have access (or pay attention to) whether the past bids they observe were submitted at the same or different times.

¹⁰As in Myerson (1981), one also implicitly assumes that the designer can choose the analogy-based expectation equilibrium she likes best. Yet, which analogy-based expectation equilibrium is played turns out to be inessential for the main results provided weakly dominated strategies are never played.

2.1 Examples of analogy partitions and auction designs

The class of analogy partitions considered in Definition 1 is quite large, as it allows both for the bundling of bidders and the bundling of formats in any possible way. For the possibility results showing that the designer can do strictly better than using the finest analogy partition, I will either consider the bundling of bidders (Propositions 1 and 2) or the bundling of formats (Proposition 3) but not a combination of the two. The more general formulation allowed by Definition 1 is useful for the impossibility result (Proposition 4) expressing that the full information benchmark is a strict upperbound on what the designer can hope to achieve whatever the manipulation.

Specifically, the following classes of auction designs with public feedback (all \mathbf{P}_i are the same across i) will play a central role in the analysis of Propositions 1, 2 and 3.

1) *Bidder-anonymous analogy partition*: In this case, there is only one auction format, and the feedback is about the aggregate distribution of bids across all bidders. That is, $K = \{1\}$, and for all $i \in I$, $\mathbf{P}_i = \left\{ \bigcup_{j \in I} \{(j, 1)\} \right\}$. For example, the object could be sold through a first-price auction, and bidders would receive feedback about the aggregate distribution of bids with no reference to the characteristics of the bidders who generated the various bids. This is the situation studied in Section 3.

2) *Format-anonymous analogy partition*: In this case, bidders know the aggregate distribution of bids across the different auction formats M_k , $k \in K$, but they differentiate the distribution of bids for the various bidders $i \in I$. That is, for all $i \in I$, $\mathbf{P}_i = \left\{ \bigcup_{k \in K} \{(j, k)\} \right\}_{j \in I}$. This situation corresponds to the contest application mentioned in introduction, and it will be considered in Section 4.

2.2 Interpretation

The interpretation of an analogy-based expectation equilibrium is that it stands for the limiting outcome of a learning process in which 1) at every stage there is a new auction and new bidders of observable characteristics $i \in I$;¹¹ 2) auction format M_k is used with

¹¹The profile of bidders' characteristics is assumed to stay the same throughout the process (see the discussion section for some elaboration on the case in which the number of bidders may vary stochastically).

frequency λ_k and 3) a bidder with characteristic i receives as feedback the aggregate empirical distribution of past bids in every α_i .¹² If behaviors stabilize in such a learning process, it must be to a feedback equilibrium provided bidders consider the simplest theory that is consistent with the feedback they receive.

The mechanism design perspective considered in the paper corresponds to the idealization that a single designer can optimize on the auction formats M_k , their frequencies λ_k and the analogy partitions \mathbf{P}_i provided to bidders, and that behaviors have stabilized to a corresponding analogy-based expectation equilibrium of \mathbf{A} . As mentioned in introduction, such a view is appropriate in situations in which a single seller repeatedly sells similar objects or in situations in which a single organization repeatedly organizes contests for promotion. It may also be useful to understand the incentives of an auction house such as ebay which is obviously interested in increasing the sellers' revenues (through the fees these generate) and which can control both the formats and the feedback about past auctions that is disclosed to bidders.

The approach developed in this paper has a non-Bayesian element in the sense that upon learning the coarse feedback the designer reports to them, bidders do not update their belief about the distribution of others' bids based on some (possibly subjective) prior. Instead, bidders are assumed to consider the simplest theory consistent with the feedback they receive: They play a best-response to the conjecture that the distribution of bids is uniformly the same over the various (j, k) that are bundled in the same analogy class.¹³ I believe this is a natural assumption in many practical situations of interest in which 1) subjects would have no other data than the feedback they receive to form their prior and¹⁴ 2) the functions φ_j^k and τ_j^k that govern bidder j 's incentive in format M_k are either the same across all (j, k) that belong to the same analogy class or they are not known to subjects with characteristic i , $i \neq j$.¹⁵ In such situations, it would seem

¹²That is, a bidder with characteristic i is informed of the aggregate empirical distribution of past bids $\{b_j^k, (j, k) \in \alpha_i\}$ with no reference to which (j, k) generated the bid.

¹³Technically speaking, the analogy-based expectation equilibrium can be viewed as a refinement of some variant of the self-confirming equilibrium in which bidders adopt the "simplest" conjecture as just described, see elaborations in subsection 5.1.

¹⁴This in particular requires that bidders have no prior knowledge about the densities f_i not even about the frequencies λ_k .

¹⁵In the first-price auction with bidder-anonymous analogy partition the former property applies (ϕ_j^k and τ_j^k are the same across all bidders). And it may be argued that in the contest for promotion example

rather hard (in fact impossible) for subjects to understand how the distribution of bids vary across different (j, k) that belong to the same analogy class simply based on the aggregate empirical distribution they are informed about: assuming that the distribution is the same across these various (j, k) seems focal, and I propose it gives good account for bidders' mode of thinking in such situations.

2.3 Preliminaries

A few preliminary observations follow. First, by picking a single auction format M and by adopting the finest analogy partition, the designer can always replicate the revenue generated in M assuming behaviors are governed by Nash equilibrium. Thus, if the designer seeks to maximize revenues, she can always achieve a revenue at least as large as Myerson (1981)'s optimal revenue. The question is whether she can achieve larger revenues.

Second, consider an auction format M in which bidder i has a dominant strategy. Then in any auction design including format M , an analogy-based expectation equilibrium requires that bidder i plays his dominant strategy in M (remember that bidders are informed of the allocation rule and the payment rule that applies to them in the format they are in). This is a straightforward observation, since bidder i will find his strategy best no matter what his expectation about the distribution of others' bids is, and thus no matter how the auction design is further specified.

Third, one of the auction designs that will be studied falls in the following class. There is one auction format M , which respects the anonymity of bidders. That is, consider two bid profiles b and b' obtained by permuting the bids of players i and j , then $\varphi_i(b) = \varphi_j(b')$ and $\tau_i(b) = \tau_j(b')$ and for all $m \neq i, j$, $\varphi_m(b) = \varphi_m(b')$, $\tau_m(b) = \tau_m(b')$. Consider the *anonymous-bidder analogy partition* defined above, and call \mathbf{A} the corresponding auction design. One can relate the analogy-based expectation equilibria of \mathbf{A} to the Nash Bayes equilibria of the game $\Gamma^{ba}(\mathbf{A})$ defined by the auction format M in which, for each i , the distribution of bidder i 's valuation has the average density $\bar{f}(v_i) = \sum_{i \in j} f_j(v_i)/n$ instead of

mentioned in introduction the latter property is often met given that the criteria used for different promotions are often not very transparent to outsiders.

$f_i(v_i)$.

Claim 1: A symmetric strategy profile is an analogy-based expectation equilibrium of \mathbf{A} if and only if it is a Bayes Nash equilibrium of $\Gamma^{ba}(\mathbf{A})$.

Fourth, another class of auction designs \mathbf{A} considered below is such that the various auction formats M_k in \mathbf{A} satisfy: $\varphi_i^k(b_i, b_{-i}) = \varphi_i(b_i, b_{-i})$ for all $k \in K$ (for example in all formats the object is allocated to the player who submitted the highest bid). When the *anonymous-format analogy partition* prevails, one can relate the analogy-based expectation equilibria of such auction designs \mathbf{A} to the Nash Bayes equilibria of the following game referred to as $\Gamma^{fa}(\mathbf{A})$:

Game $\Gamma^{fa}(\mathbf{A})$: Each bidder i (simultaneously) submits a bid b_i ; the object is assigned to bidder i with probability $\varphi_i(b_i, b_{-i})$; prior to bidding, bidder i is privately informed of his valuation v_i drawn from $f_i(\cdot)$ and of his method of payment k defined by $\tau_i^k(b_i, b_{-i})$; the methods of payment k are identically and independently drawn across bidders and every bidder i is subject to the method of payment k with probability λ_k .¹⁶

Claim 2: Suppose that the format-anonymous analogy partitions prevail and that in all auction formats M_k of \mathbf{A} , $\varphi_i^k(b_i, b_{-i}) = \varphi_i(b_i, b_{-i})$ for all $k \in K$ and $i \in I$. Then a strategy profile β is an analogy-based expectation equilibrium of \mathbf{A} if and only if it is a Bayes Nash equilibrium of $\Gamma^{fa}(\mathbf{A})$.

3 Efficiency and revenues

Assume the designer's valuation v_s is 0 and that the designer is interested both in efficiency and revenues, and suppose that the primary objective of the designer is efficiency while revenue is only the secondary objective. In the standard Myerson's paradigm, the so called revenue equivalence result holds. That is, if two mechanisms result in the same allocation rule and the expected payment made by any bidder i with minimal valuation $v_i = c$ is 0 then both mechanisms must yield the same revenues. Since an efficient outcome can be achieved by a second-price auction *SPA*, the standard approach (i.e. relying on Nash equilibrium) concludes that the designer can do no better than using a *SPA*.

¹⁶Compared to the true auction design, the difference is that the methods of payments are independently distributed across bidders in Γ^{fa} whereas they are (perfectly) correlated in Γ .

I now observe that, within the framework introduced in Section 2, the designer can sometimes achieve strictly larger revenues (than that obtained through the *SPA*) while still preserving efficiency, thereby illustrating a failure of the allocation equivalence in a manipulative auction design setup. Besides, this gain in revenues is achieved by using a fairly standard auction format (with the bidder-anonymous analogy partition).

Proposition 1 *Consider a two-bidder $i = 1, 2$ auction setup with asymmetric distributions $(F_1(\cdot) \neq F_2(\cdot))$ on a set of strictly positive measure). There is a unique analogy-based expectation equilibrium of the first price auction with bidder-anonymous analogy partition. Moreover, this analogy-based expectation equilibrium induces an efficient outcome and it generates a strictly higher revenue than the second-price auction. The revenue gain is*

$$\int_c^d \frac{1}{4} (F_1(v) - F_2(v))^2 dv + \int_c^d \frac{1}{4} \frac{d\beta(v)}{dv} (F_1(v) - F_2(v))^2 dv > 0$$

where $\beta(v) = \int_c^v x \bar{f}(x) dx / \bar{F}(v)$, $\bar{f}(x) = \frac{f_1(x) + f_2(x)}{2}$ and $\bar{F}(v) = \int_c^v \bar{f}(x) dx$.¹⁷

What is the intuition for the above result? First, observe that the use of the bidder-anonymous analogy partition leads the bidders (whatever their characteristic) to best-respond to the same distribution of bids, which given the anonymous character of the first-price auction, ensures efficiency. Second, the use of the bidder-anonymous analogy partition leads the bidders to feel that they are in competition with a fictitious bidder who has a distribution of valuations that is the average distribution between the distributions of the various bidders (this essentially follows from claim 1 above). In the two bidder case, the price level in the second-price auction is determined by the lowest valuation, hence by the weak bidder. Roughly, the manipulation generated by the bidder-anonymous analogy partition enhances revenues because it makes the strong bidder feels the weak bidder is less weak than he really is.

When there are more than two bidders, the first-price auction with anonymous-bidder analogy partition remains efficient, but the revenue comparison with the second-price

¹⁷ $\beta(\cdot)$ is the equilibrium bid function in a symmetric two-bidder FPA with density of valuations \bar{f} . As such, $\beta(\cdot)$ is an increasing function.

auction can go either way depending on the form of the asymmetry of the distributions.¹⁸ The following result in which R^{SPA} denotes the expected revenue generated in the second-price auction with densities of valuations $f_i(\cdot)$, $i \in I$, and \bar{R} the expected revenue generated in the fictitious second-price auction with symmetric densities of valuations $\bar{f}_i(v) \equiv \bar{f}(v) = \frac{f_1(v)+f_2(v)+\dots+f_n(v)}{n}$ provides a generalization of Proposition 1 to the n bidder case.

Proposition 2 *Consider an n -bidder setup and assume that $\bar{R} \geq R^{SPA}$. There is a unique analogy-based expectation equilibrium of the first price auction with bidder-anonymous analogy partition. Moreover, this analogy-based expectation equilibrium induces an efficient outcome and it generates a strictly higher revenue than the second-price auction.*

Proposition 2 is proven by noting that 1) the revenues in the second-price auction or in the first-price auction with symmetric densities coincide (this is the standard revenue equivalence result) and that 2) the expected revenue in the first-price auction with bidder-anonymous analogy partition is strictly above the expected revenue in the first-price auction with symmetric densities of valuations $\bar{f}(v) = \frac{f_1(v)+f_2(v)+\dots+f_n(v)}{n}$ where the latter result follows because i) bidders employ the same strategy in both cases (by claim 1) and ii) the distribution of highest valuation in the asymmetric case first-order stochastically dominates the distribution of highest valuation in the symmetric case. While $\bar{R} \geq R^{SPA}$ always holds in the two-bidder case (thereby explaining why Proposition 1 holds with no restriction on $f_1(\cdot)$ and $f_2(\cdot)$), this need not be so in the more than two bidder case.^{19/20}

¹⁸To see that revenues can go either way, consider first a situation with two bidders whose distribution of valuations is concentrated around d and a third bidder whose distribution of valuation is concentrated around c . It is readily verified that the first-price auction with bidder anonymous feedback partition generates less revenues than the second-price auction (which achieves a revenue approximately equal to d). Consider next a situation with one bidder whose distribution of valuations is concentrated around d while other bidders have a distribution of valuations concentrated around c , the first-price auction with bidder anonymous feedback partition generates more revenues than the second-price auction (which generates a revenue very close to c). Thus the revenue comparison can go either way.

¹⁹Intuitively, one would expect to have $\bar{R} \geq R^{SPA}$ when there is one stronger bidder who is facing symmetrically weak bidders, but the required notion of strong bidder does not boil down to the first order stochastic dominance relation in the distributions of valuations.

²⁰It should also be noted that when the distributions of valuations are nearly the same across bidders (say the cumulative functions differ up to ε), then the revenues of the two auction designs differ according to a smaller magnitude (of order ε^2).

When the distributions are very asymmetric, the difference of revenues can be quite substantial. For

The above insights (Propositions 1 and 2) are distinct even though related to Myerson's insight about how to increase revenues in asymmetric auctions. An important implication of Myerson's analysis is that in the asymmetric case, competition between bidders should be biased in favor of weak bidders in order to increase revenues. The net effect of such biased auctions is that some inefficiencies are induced letting the weak bidder sometimes win the object. As Proposition 1 (and to some extent Proposition 2) illustrates, the use of the bidder-anonymous feedback partitions allows in some cases to symmetrize (a bit) the competition without sacrificing on efficiency. Of course, this is achieved by moving away from the Nash equilibrium paradigm, which the use of partial feedback permits.²¹

Comment. In some applications, the distribution of winning bids as opposed to the aggregate distribution of all bids is available to bidders. From this feedback, bidders can compute a best-response strategy based on the assumption that all bidders bid according to the same distribution (this might be argued to be the simplest conjecture in this case). Applied to the asymmetric first-price auction format, the analog of Claim 1 would reveal that in this case bidders would bid as if the distribution of valuations of each bidder were $F(v) = \left(\prod_{i=1}^n F_i(v) \right)^{\frac{1}{n}}$.²² It is interesting to note that in this case, no matter how many bidders are around, revenues would be higher in the first-price auction (in which only the winning bids are observed) than in the second price auction.²³

example, in the two-bidder case considered in Proposition 1, assume that the distribution of valuations of one bidder is concentrated around d whereas the distribution of valuations of the other bidder is concentrated around c . In this case, the first-price auction with anonymous bidder feedback partition provides a revenue of $\frac{3c+d}{4}$, which should be compared with the revenue c of the second-price auction. Clearly, as d gets large relative to c , the revenue gain, $\frac{d-c}{4}$, can be quite substantial in such asymmetric setups.

²¹There is some experimental evidence that in asymmetric first-price auctions, the observed bidding strategy is less asymmetric than Nash equilibrium would require (see Güth et al (2005)). Such an experimental finding is consistent with the view that bidders when considering which bid to submit may look at previous bids without paying attention to the strength of the bidder who submitted the bid (which was modeled here through the apparatus of the bidder-anonymous analogy partition).

²²Indeed, symmetry implies that all bidders bid according to the same increasing bid function. Thus, only the bid of the highest valuation bidder would be observed, thereby providing the wished result.

²³The analog of step 2 in the proof of Proposition 1 is Theorem 1 in Cantillon (2008), and step 3 holds with an equality because the distribution of the highest valuation is the same with (F_1, \dots, F_n) and (F, \dots, F) .

4 Optimal Auctions

Assume now that the designer seeks to maximize her expected payoff as measured by the revenue she gets when there is transaction and her valuation v_s when there is no transaction. Our first main observation is:²⁴

Proposition 3 *The largest expected payoff that the designer can achieve in a manipulative auction design is strictly larger than her expected payoff in Myerson's optimal auction (denoted by R^M).*

The intuition for Proposition 3 is as follows. Myerson's optimal auction can always be implemented in such a way that every bidder has a (weakly) dominant strategy and ex-post quitting rights of bidders are fulfilled (think of the second-price auction with well chosen reserve price in the symmetric regular case). One can now think of an auction design in which this auction format - call it MD - is mixed with a little bit of first-price auction with v_s reserve price (FPA), and bidders get only to know the aggregate distribution of bids over the two auction formats. In format MD , the strategies are the same as in the standard case (because bidders have a weakly dominant strategy in MD). In format FPA , bidders play a best-response to the aggregate distribution of bids over the two formats. For many choices of MD , this construction need not induce an expected payoff to the seller that is higher than R^M .²⁵ But, there are many variants of MD in which submitted bids are first monotonically transformed before the original format is being applied. For a suitable choice of such a variant, the construction leads bidders in FPA to bid very aggressively because they are led to think that by shading too much their bid the chance of winning in FPA gets too small. In the limit, a bidder with valuation v may be induced to bid very close to v whenever $v > v_s$. Given that such bidding strategies in FPA induce an expected payoff to the seller that is close to the full

²⁴By inspecting the proof of Proposition 3, one can see that all analogy-based expectation equilibria (not employing weakly dominated strategies) of the auction design considered there are such that the designer obtains higher revenues than in Myerson's optimal auction. Thus, the conclusion of Proposition 3 would hold under the stronger full implementation requirement (provided one restricts oneself to equilibria not employing weakly dominated strategies).

²⁵It can be checked, for example, that in the case of uniform distributions, a mix of second-price auctions and first-price auctions would have no effect on revenues.

information optimal expected payoff $R^F = E(\max_i(v_i, v_s))$, and given that $R^F > R^M$, the result of Proposition 3 follows.

Proposition 3 establishes that the designer can do better than using Myerson's optimal auction (with fine analogy partitions), but how much can she gain? Clearly, the best that she can hope to get in auction designs with ex-post quitting rights cannot exceed the maximal full information expected payoff $R^F = E(\max_i(v_i, v_s))$. This trivially follows from the observation that a winner of the auction would always object if he were asked to pay more than his valuation. As it turns out, the designer's best expected payoff in our manipulative design setup lies strictly below R^F .

Proposition 4 *The best expected payoff that the designer can achieve in a manipulative auction design with ex-post quitting rights is strictly smaller than the full information expected payoff R^F if $c < v_s < d$.*

In the proof of Proposition 3 some (Myerson-optimal) mechanism MD implementable in (weakly) dominant strategy was mixed with a little bit of first-price auction with reserve price v_s , FPA , and the seller's expected payoff obtained in FPA was shown to be close to R^F . However, such a construction requires that the frequency with which FPA is used is set sufficiently small. As one increases the frequency of FPA , the manipulation loses its force, and, of course, in the limit as the designer almost always picks FPA , one gets the standard seller's expected payoff generated in the first-price auction with reserve price v_s , which following Myerson's analysis cannot be larger than R^M .

What Proposition 4 establishes is that within the class of mechanisms under study one can never reach R^F whatever the manipulation. This observation is of particular interest because if the designer could freely choose the belief of bidders (with no constraint), she could get a payoff arbitrarily close to R^F (see the arguments surrounding Proposition 7). Thus, Proposition 4 establishes that there is some limitation to manipulation imposed by the requirement that the feedback should be correct (even if coarse).

To get an intuition for Proposition 4, think of a symmetric scenario in which the auction design \mathbf{A} uses the format-anonymous feedback partition. To get close to R^F , it would be required that, in all auction formats M_k used in \mathbf{A} , every bidder with valuation $v > v_s$ pays a price close to his valuation when he wins. This implies that every bidder

should perceive to make almost zero profit.²⁶ Yet, a bidder with valuation $v \simeq d$ can always consider submitting a bid $\beta^k(\frac{d+v_s}{2})$ in format M_k and he should expect there to win and make a payment no more than $\frac{d+v_s}{2}$, thereby making a net gain of no less than $\frac{d-v_s}{2} > 0$, whenever other bids b_j are smaller than $\beta^k(\frac{d+v_s}{2})$.²⁷

Now, it is not *a priori* clear what perceived probability bidder i attaches in format M_k to the event that all $b_j, j \neq i$ are smaller than $\beta^k(\frac{d+v_s}{2})$. But, one can always rank the formats M_k by increasing order of $\beta^k(\frac{d+v_s}{2})$. For those formats M_{k^*} such that $\beta^{k^*}(\frac{d+v_s}{2})$ is above the median of $\beta^k(\frac{d+v_s}{2})$ (in the distribution in which $\beta^k(\frac{d+v_s}{2})$ has probability λ_k), it is clear that in the format-anonymous analogy partition, this perceived probability is no less than $\frac{1}{2} \Pr(\max_{j \neq i} v_j < \frac{d+v_s}{2})$.

A contradiction is obtained given that, in format M_{k^*} , a bidder with valuation $v \simeq d$ cannot perceive his payoff from following $\beta^{k^*}(d)$ (this should be approximately 0) to be strictly smaller than his payoff from following $\beta^{k^*}(\frac{d+v_s}{2})$ (this has been shown to be no less than $\frac{1}{2} \frac{d-v_s}{2} \Pr(\max_{j \neq i} v_j < \frac{d+v_s}{2})$, which is strictly positive).

Comments. 1) From the proof of Proposition 4, it is not clear whether if $v_s \leq c$, a similar conclusion arises because it is not then *a priori* guaranteed that for the full information payoff to arise, a bidder should necessarily perceive to make negligible profit. It should be noted though that with the additional requirement that in every format M_k the payment made by the winner should lie in between the largest and the second largest bid, then the same conclusion as in Proposition 4 would arise even if $v_s < c$ because the rules of the formats would imply that it is weakly dominated to bid above one's own valuation (and this property can easily be used to establish that bidders cannot perceive to make significant gains if R^F is to be achieved). 2) If the designer were allowed to commit to offering positive payments to losers and if the payments from the winner were not assumed to be monotonic in bids, then the designer could get a revenue close to R^F

²⁶The perceived payoff might *a priori* differ from the actual one due to the manipulative character of the design. To establish this, I make use of $v_s > c$ and of the monotonicity of $\tau^k(b_i, b_{-i})$, but I suspect this to hold much more generally.

²⁷This is because in format M_k , the actual bid of bidder i with valuation $v < \frac{d+v_s}{2}$ is $\beta^k(v)$ and a bidder with valuation $\frac{d+v_s}{2}$ should win and pay a price approximately equal to his valuation whenever he meets bidders with lower valuations.

while still preserving the ex-post participation constraints of bidders.²⁸ The restriction on mechanisms (i.e., not allowing positive payments to losers and imposing that payments from winners be monotonic in bids) can then be thought of as resulting from the regulatory desire to protect bidders from manipulation.

5 Discussion

5.1 Related literature

This paper is related to several strands of literature. First, the concept used in this paper follows Jehiel (2005), Jehiel and Koessler (2008) and Ettinger and Jehiel (2010) which defined the analogy-based expectation equilibrium for extensive form games of complete information, static games of incomplete information and multi-stage games of incomplete information, respectively. The analogy partitions considered in this paper are slightly less general than those considered in Jehiel and Koessler (2008) except for the fact that bidder i 's own bids can be lumped together with others' bids in bidder i 's analogy partition.²⁹ The main novelty of the approach pursued here is that the feedback partitions are viewed as a choice made by the designer. That is, they are not exogenously given as in Jehiel

²⁸To see this, consider a symmetric two-bidder scenario in which bidders' valuations are identically distributed on (c, d) . Consider an auction design with format-anonymous analogy partition and two formats \underline{M} and \overline{M} used in equal proportion. In format \underline{M} , the equilibrium bids will lie in $[0, d]$; in format \overline{M} , the equilibrium bids will lie in $\{0\} \cup [d, 2d]$. In each format, a bidder with negative valuation bids 0 in equilibrium. In both \underline{M} and \overline{M} , the bidder with highest bid wins the auction if this bid is strictly positive. In \underline{M} , if $b_i \in (0, d)$ for $i = 1, 2$, the winner pays his own bid and the loser receives no transfer. In \overline{M} , if $b_i \in \{0\} \cup [d, 2d]$ for $i = 1, 2$, the winner i^* pays $b_{i^*} - d$ and the loser receives no transfer. The idea is to augment the transfers in \underline{M} and \overline{M} to cover all bid profile configurations even for bid realizations that will never occur in the respective formats. So in \underline{M} , a (losing) bidder submitting $b_i \in (0, d)$ will be offered a promise of transfer $\underline{h}(b_i)$ if $b_j \in (d, 2d)$ and in \overline{M} , a (winning) bidder submitting $b_i \in (d, 2d)$ will be offered a transfer $\overline{h}(b_i)$ if $b_j \in (0, d)$. By suitable choices of \underline{h} and \overline{h} , one can ensure that for $v_i > 0$ bidding $\underline{\beta}(v_i) = v_i$ in \underline{M} and bidding $\overline{\beta}(v_i) = v_i + d$ in \overline{M} is an analogy-based expectation equilibrium. [For example, in the uniform distribution case, $\underline{h}(b) = \frac{b^2}{2d} - \frac{bc}{d}$ and $\overline{h}(b) = \frac{(b-d)^2}{2d} - \frac{(b-d)c}{d}$. These functions are determined so that the expected perceived transfers correspond to those that would be made in a SPA with 0 reserve price.] With such bidding strategies, the expected revenues generated in each format are R^F , and thus the designer gets R^F in expectation.

²⁹As already mentioned such an extension is particularly adapted if each individual bidder i participates in just one auction.

(2005) or Jehiel and Koessler (2008).³⁰

The analogy-based expectation equilibrium has a close relationship to some variant of the self-confirming equilibrium in which bidder i would receive as signal the aggregate distribution of bids in α_i for the various $\alpha_i \in P_i$ (see Fudenberg and Levine (1998) for a general presentation of the self-confirming equilibrium).

Specifically, suppose at the end of an auction round described by the format M_k , the profile of bids and the profile of valuations (b, v, k) , subjects of population i receive as feedback various signals $y_{s_i}^z(b, v, k) \in [0, d]$, for $z = 1 \dots n(s_i, k)$ and $s_i \in S_i$ where $n(s_i, k)$ denotes the number of signals of type s_i received when the format is M_k . If bidders follow the strategy β and formats are drawn according to λ (see Section 2),

$$\Pr_{\beta, \lambda}(y_{s_i} \in X) = \left(\sum_k \lambda_k \sum_{z=1}^{n(s_i, k)} \Pr_{\beta}(y_{s_i}^z \in X) \right) / \left(\sum_k \lambda_k n(s_i, k) \right)$$

is the empirical frequency with which a signal y_{s_i} falls in X for every $X \subseteq [0, d]$. A self-confirming equilibrium allows bidders i to entertain subjective views about λ and β , but it requires that bidders play best-responses given these subjective views and that the empirical frequencies as computed from the subjective views coincide with the correct empirical frequencies of y_{s_i} as generated by λ and β . Formally,

Definition 2 *A strategy profile β is a self-confirming equilibrium given y if and only if for every bidder i , there exist conjectures $\tilde{b}^i = (\tilde{b}_i^{i, k}, \tilde{b}_{-i}^{i, k})_k$, $\tilde{\lambda}^i = (\tilde{\lambda}_k^i)_k$ such that for every k and every $v_i \in (c, d)$*

$$\beta_i^k(v_i) = \arg \max_{b_i} u_i^k(v_i, b_i; \tilde{b}_{-i}^{i, k}) \quad (1)$$

and for every $s_i \in S_i$ and $X \subseteq [0, d]$,

$$\Pr_{\beta, \lambda}(y_{s_i} \in X) = \left(\sum_k \tilde{\lambda}_k^i \sum_{z=1}^{n(s_i, k)} \Pr_{\tilde{b}^i}(y_{s_i}^z \in X) \right) / \left(\sum_k \tilde{\lambda}_k^i n(s_i, k) \right) \quad (2)$$

³⁰By assuming that the designer can control the analogy classes of the bidders, this paper takes the more rational interpretation of the concept, that is, assuming that players make the maximal use of the information provided to them. In Jehiel (2005), it is alternatively suggested that an analogy-based expectation equilibrium might arise because players may not pay attention to all of the details of their past experiences, which amounts to a bounded rationality interpretation of the concept.

Compared to the usual definition of self-confirming equilibrium, it is somehow non-standard that several realizations of signals of type s_i would be observed in a given auction round. It is also non-standard to require the consistency of the conjectures only with respect to the marginal distributions of y_{s_i} (as expressed in (2)) as opposed to the joint distribution over all signals received by player i . These differences are in part motivated by the fact that I have in mind situations in which an individual player plays only once whereas the literature that has introduced the self-confirming equilibrium had in mind situations in which players would play many times.³¹

Equipped with this notion of self-confirming equilibrium, by identifying s_i with an analogy class α_i and identifying the various $y_{s_i}^r(b, v, k, t)$ with the various b_j such that $(j, k) \in \alpha_i$, it is readily verified that an analogy-based expectation equilibrium (Definition 1) is a self-confirming equilibrium given this signal structure.

As was anticipated in Subsection 2.2, an analogy-based expectation equilibrium is a selection of self-confirming equilibrium,³² the one in which the theory or conjectures adopted by bidders is the simplest one in the sense that their theory has the coarsest measurability property while being consistent with the feedback.³³ More precisely, the analogy-based expectation equilibrium by stipulating that player i best-responds to the conjecture that for every $(j, k) \in \alpha_i$ the distribution of bids of bidder j in format M_k coincides with the aggregate distribution of bids in α_i is such that player i 's theory about the play of bidder j in M_k is the same across all $(j, k) \in \alpha_i$. That is, identifying the state space of theories with $\{(j, k), j \in I, k \in K\} \times \Delta_r$ where Δ_r is the r -dimensional simplex (representing probability distributions over the various formats M_k $k = 1, \dots, r$) player i 's theory in an analogy-based expectation equilibrium is measurable with respect to his analogy partition, and any theory that is consistent with the feedback would have to be

³¹Requiring the consistency of the marginal distributions only corresponds to the idea that players do not keep track of when previous realizations of signals occurred.

³²Dekel et al. (2004) show that in private value settings such as the one in this paper, a self-confirming equilibrium coincides with a Nash equilibrium whenever players can observe the actions of the players. An analogy-based expectation equilibrium in my setup may differ from Nash equilibrium because bidders do not observe other bidders' actions separately in each format M_k they may participate in.

³³While such a selection should be the subject of empirical test, the experiment in Huck et al. (2010) gives some support to this selection, even though not in an auction setup.

at least as fine.³⁴

The literature on self-confirming equilibrium has not considered the feedback by the players as a design issue with the notable exception of Esponda (2008a). Esponda considers first-price auctions in which the *same* bidders get involved over sequences of auctions, and get information about the joint distribution of highest bids (and possibly second-highest bids) and their own valuation and bid. In a symmetric first-price auction with private and affiliated values he shows that *symmetric* self-confirming equilibria (of the static auction) generate at least as much revenues as the Nash equilibrium.

The main common aspect of the two papers is that the solution concept that we use is not the Nash equilibrium but some concept related to the self-confirming equilibrium and that it is applied to a mechanism design question. Yet, there are several important differences between Esponda (2008a) and this paper that I now discuss.

First, by allowing for a large class of auction formats, I am able to place my analysis in the context of the optimal auction design literature (Myerson, 1981), which would not have been possible if I were to confine myself to first-price auctions. Second, motivated by some applications such as ebay, I have considered here a case in which fresh bidders arrive each time, which requires amending the usual definition of self-confirming equilibrium (see Definition 2). More importantly, I consider a refinement of self-confirming equilibrium based on simplicity considerations. In general, when one looks at all self-confirming equilibria, it is hard to make clear cut comparative statics regarding the effect of the feedback (in particular, because the Nash equilibrium is always a self-confirming equilibrium whatever the feedback). For example, in the symmetric context studied by Esponda (2008a), in order to obtain clear-cut comparative statics, he had to restrict himself to *symmetric* self-confirming equilibria, as it is not true that his result would hold if asymmetric self-confirming equilibria were considered. But, in general one has to provide some motivation for the chosen restriction.³⁵ In the research started in Jehiel (2005) and

³⁴Observe that this implies that no theory about λ need be specified in an analogy-based expectation equilibrium, which is not so in most other self-confirming equilibria (as soon as the conjecture of the distribution of bids are not the same for (j, k) and (j', k') in the same analogy class, some theory about $\lambda_k/\lambda_{k'}$ is required to check the consistency condition).

³⁵Given that there are several symmetric possible conjectures in Esponda's setup, it is not clear by which process the bidders should be coordinated on a symmetric self-confirming equilibrium, thereby making the restriction to symmetric self-confirming equilibrium questionable.

that this paper follows, I have proposed to look at the selection in which players adopt the simplest theory consistent with the feedback they receive (in a sense made precise above), and one might argue that the simplicity criterion is a common theme of the entire literature on bounded rationality (see, for example, Simon (1955)), thereby suggesting some motivation for the chosen selection. Equipped with such a selection, I was able to obtain clear cut comparisons because essentially the chosen selection allowed me to have a comparable predictive power whatever the feedback (which is not so if one works with the entire set of self-confirming equilibria).

Some other equilibrium approaches that move away from Nash equilibrium (and thus permits erroneous expectations) have been proposed in the recent past. These include the cursed equilibrium of Eyster and Rabin (2005) and the behavioral equilibrium of Esponda (2008b). These approaches shed new light on the winner's curse and on the possibility of trade in adverse selection problems. Yet, in private values setting such as the one considered here, these approaches coincide with Nash equilibrium and as such are not closely related to the present study (see Jehiel and Koessler (2008) for a discussion of the link between the cursed equilibrium and the analogy-based expectation equilibrium in general Bayesian games).

There is also a strand of literature concerned with learning in mechanism design. This strand includes among others Cabrales (1999) and Cabrales and Serrano (2007) (see the latter for a more comprehensive review of that strand of literature). A typical question addressed by this literature is about equilibrium selection when there are several Nash equilibria and whether the choice of mechanism may induce good convergence properties of the corresponding learning process. The approach pursued here is complementary to this strand. It offers a different perspective by suggesting how the use of coarse feedback may result in convergence to non-Nash equilibria, i.e. analogy-based expectation equilibria. The approach pursued here also assumes to start with that a steady state has been reached. In line with the literature just mentioned, it would be of interest to study the convergence properties of the learning process that was suggested to motivate the present approach.

Finally, there have been various approaches to study how a mechanism designer should deal with various behavioral biases assumed on agents. These include among others the work of Eliaz (2002) who assumes that a fixed number of agents may have a crazy beha-

avior, the work of Eliaz and Spiegler (2007) who assume that agents may have erroneous subjective beliefs and the work of Crawford et al. (2009) who assume that bidders behave according to the level k-mode of thinking.³⁶ Beyond the obvious observation that the bias considered in this paper is of a different nature than the ones considered by these authors, I believe the current approach differs from these other approaches in an important way. Somehow the bias in the expectation formation that appears in an analogy-based expectation equilibrium is induced by the choice of analogy partition made by the designer. It is thus as if the cognitive limitations of the bidders were endogenously created by the designer rather than being there to start with.

5.2 Complete information

In the above analysis, some uncertainty about bidders' valuations was assumed. When each bidder i 's valuation can take a single value v_i , the designer can extract a revenue equal to $R^F = \max_i(v_i, v_s)$ in the classic rationality setup.

In the above setup with ex-post quitting rights, no manipulation can allow the designer to extract more than R^F given that a bidder will never accept to pay more than his valuation if he wins the object (and he will never accept to pay anything if he does not win the object). Thus, within the setup introduced in Section 2, some private information is required for manipulation to be of effective use to the designer.

5.3 Interim participation constraints

In the above analysis, auction formats with ex-post quitting rights were considered. If participation constraints are only required at the interim stage before bidders know the outcome of the auction and if relatedly the designer can also require payments from losers, then the designer can generate much larger revenues if bidders play according to an analogy-based expectation equilibrium.³⁷

³⁶In a related vein, Matsushima (2008) considers an implementation problem when agents rely on two levels of eliminations of dominated strategies.

³⁷An analogy-based expectation equilibrium can be defined in a similar way as in Definition 1 for such auction designs without ex-post quitting rights.

Proposition 5 *Suppose there are at least two bidders and that bidders cannot withdraw from the auction later on. Then by a suitable choice of auction design the designer can make arbitrary large amounts of money.*

The idea of the proof which is detailed in the appendix is as follows. By choosing several formats and by using a format-anonymous analogy partition for say bidder 1, the designer can mislead bidder 1 in his understanding of the distribution of bids of other bidders $i \neq 1$. She can then propose a bet to bidder 1 whose monetary outcome is contingent on the realization of b_i , $i \neq 1$, in such a way that the bet sounds profitable from the viewpoints of both bidder 1 and the designer. By increasing the stakes of the bet, bidder 1 will still agree on the terms of the bet given our assumption of risk neutrality, which translates into potentially arbitrarily large revenues for the designer.

The above argument bears strong resemblance with the observation that with subjective prior beliefs the logic of the no trade theorem breaks down.³⁸ Of course, here since the designer is assumed to know the correct distributions of bids, one makes the further inference that it is the designer (and not the bidder) who benefits from the bet. Another key difference with the literature on subjective priors is that the erroneous perception of the bidders is viewed here as resulting from the feedback manipulation of the designer and not from the subjective character of bidders' prior beliefs.

Comment. To the extent that bidders know that the designer is more informed than they are about the distributions of bids, one might argue that in the context of the above manipulation bidder 1 might be suspicious, thereby deciding to stay outside the auction room rather than playing according to an analogy-based expectation equilibrium.³⁹ This is to be contrasted with auction designs with ex-post quitting rights as considered in the main part of this paper in which staying outside the auction room is always a bad idea (nothing worse can happen by participating). In the class of auction designs with ex-post quitting rights, it is not clear what else (i.e. other than playing according to an

³⁸See however Morris (1994) for an exploration of when the no trade theorem continues to hold in the subjective prior paradigm.

³⁹Alternatively, if one has in mind that there is a risk that bidder 1 would play according to an analogy-based expectation equilibrium even for large stakes, one may think of the ex-post quitting rights scenario considered in the main part of the paper as a regulatory constraint imposed on designers to better protect bidders from manipulation.

analogy-based expectation equilibrium) a player could do.⁴⁰

5.4 Further restrictions on the set of mechanisms

Restrictions on mechanisms beyond those made in Section 2 can be considered. A question of interest is whether with such extra restrictions the results of Propositions 3 and 4 still hold. For example, consider the case in which in addition to the assumptions made in Section 2, the payment $\tau_i^k(b)$ made by the winner is required to be in the convex hull generated by the submitted bids (one might argue that such a feature is satisfied by all auctions in the real world so that the use of other auctions might trigger the suspicion of bidders). Then clearly, the result of Proposition 4 would *a fortiori* hold, since I am restricting the domain on which the designer can choose her auction design. Whether the result of Proposition 3 still holds should be the subject of a more systematic investigation. I provide a special case in which it would hold and I suspect it might hold much more generally.

Proposition 6 *Suppose there are two bidders whose valuations are uniformly distributed on $[c, c + 1]$ with $c > v_s + 1$. The seller can do strictly better than in Myerson's optimal auction in a manipulative auction design with ex-post quitting rights in which the price paid by the winner is constrained to be in between the bids submitted by the two bidders.*

5.5 Shill bidding

In the above analysis, the only players in the auction were the bidders $i \in I$. It might be argued that the designer could also employ shill bidders in addition to the real bidders $i \in I$. In the standard paradigm (i.e. relying on Nash equilibrium), this does not help the designer obtain a better outcome (as results from Myerson's analysis), but in a manipulative mechanism design setup it does, as I now illustrate.

⁴⁰Recently, Lehrer (2007) has proposed a selection of self-confirming equilibrium based on the most pessimistic conjecture (rather than the simplest conjecture as in this paper). Such an approach would lead bidders not to accept bets as considered in the proof of Proposition 5. But, it would also lead bidders not to take part in any auction of the sort analyzed throughout this paper as long as feedback is partial and there is a slight cost to participate in auctions. I find the latter conclusion unrealistic.

Assume that the designer can costlessly hire shill bidders who have no intrinsic value for the object.⁴¹ Specifically, consider the same setup as in Section 2 except that the designer can also add to the set $I = \{1, \dots, n\}$ of actual bidders any set $S = \{n + 1, \dots, n + m\}$ of shill bidders (with 0 valuations). When the seller sells to a shill bidder, I assume her payoff is v_s (this is equivalent to the seller keeping the object and not making any payment). Otherwise, the seller's payoff is as in Section 4. That is, her payoff is the revenue if she sells to a bidder $i \in I$ and v_s if she does not sell.

The following proposition shows that in a manipulative auction design with ex-post quitting rights, the seller can get a payoff close to the full information payoff R^F whenever she can freely hire shill bidders, which should be contrasted with the finding of Proposition 4.

Proposition 7 *Suppose the cost of hiring shill bidders is zero. Then the designer can get a revenue close to R^F in the optimal manipulative auction design.*

The proof of Proposition 7 follows the logic used to prove Proposition 3. By inviting m shill bidders, the designer can make them bid as she wishes, say each according to a distribution of bids $g(\cdot)$ with support on (v_s, d) . Consider now a variant of the first-price auction with reserve price v_s defined as follows. Only a bidder $i^* \in I$ can win the auction and he wins if $b_{i^*} = \max_{i \in I} b_i$ and $b_{i^*} \geq v_s$. Otherwise, the seller keeps the object. The shill bidders never win the auction (and thus never make any payment), but they are requested by the seller to bid according to a distribution of bids with density $g(\cdot)$.⁴²

Consider now the bidder-anonymous analogy partition in the above auction format in which every bidder gets only to know the aggregate distribution of bids that mixes the bids of bidders $i \in I$ and the bids of shill bidders. It is readily verified that, in an analogy-based expectation equilibrium, as m grows to infinity, bidders $i \in I$ will submit a bid that is approximately a best-response to the conjecture that bidder $i \in I = \{1, \dots, n\}$ bids according to $g(\cdot)$ (because in the aggregate distribution of bids that mixes the bids of all bidders, the distribution of bids of shill bidders overcrowds the distribution of bids

⁴¹One may argue that in practice the cost of hiring shill bidders is high to the extent that shill bidding is illegal. The following proposition helps understand why it may be a good idea to make shill bidding illegal.

⁴²Shill bidders are indifferent as to how they bid given the auction rule, but I assume that they follow the request of the designer. They could easily (and cheaply) be incentivized to do so.

of bidders $i \in I$). That is, each bidder $i \in I$ with valuation $v > 0$ will submit a bid close to $\beta(v) \in \arg \max_{b \geq v_s} (v - b)G^n(b)$ where $G(\cdot)$ is the cumulative of $g(\cdot)$. By considering a cumulative $G(\cdot)$ of the form $G(v) = \left(\frac{v-v_s}{d-v_s}\right)^q$ with q large enough, one easily obtains that $\beta(v)$ gets close to v , thereby providing a proof of Proposition 7 (see more details in the proof of Proposition 3).

5.6 Random number of bidders

In the above analysis, the set of bidder I was deterministic (this was also to simplify the comparison with Myerson's optimal auction paper). How are the results affected if the set of bidders I is stochastic?

For concreteness, consider a symmetric regular case in which the valuation of every bidder i is drawn from the same distribution with density $f(\cdot)$ and $v \rightarrow v - \frac{1-F(v)}{f(v)}$ is increasing. When bidders are risk neutral as assumed in this paper, the best revenue in the classic rationality setup is achieved by having a regular auction (say a second price or first-price) auction with reserve price R set such that $R - \frac{1-F(R)}{f(R)} = v_s$. This is so because such a format would achieve the best revenue even if the number of bidders were known to the designer and no matter what this number is (see McAfee and McMillan (1987) for the treatment of risk aversion when the number of bidders is stochastic).

Can the designer achieve larger revenues in a manipulative auction design setup when the number of bidders is stochastic? The answer is yes and this is shown in the same way as Proposition 3 was proven, that is, by mixing a little bit of first-price auction with reserve price v_s with a well chosen auction format that is strategically equivalent to the second-price auction with reserve price R , and by considering the format-anonymous analogy partition. Similarly, Proposition 4 extends to the stochastic number of bidder case.

In a vein similar to that of Propositions 1 and 2, one might also be interested in scenarios in which the number of bidders would vary from one auction to the next and bidders would observe the number of competitors they face in the current auction. If a second-price auction is considered, observing the number of competitors has no effect on the optimal strategy, but if a first-price auction is considered such an observation will affect the optimal strategy (as it is indicative of how the chance of winning depends on the

bid shading). In the latter case, one may also consider the effect of providing as feedback the distribution of past bids without telling how many bidders were present in the auction room when the bid was submitted. How does the revenue generated in such a first-price auction design compare to the revenue of the second-price auction? Addressing such a question is left for future research.

5.7 Cheating on feedback

An important assumption made throughout the paper is that the feedback reported by the designer must be correct. There are important reasons why I believe this is a natural assumption. First, in most countries it would be illegal to report false pieces of information (this should be contrasted with the kind of manipulation considered in this paper in which every released information is correct even if partial). Second, even if there is no legislation about the correctness of feedback, it is likely to be in the interest of sellers to report truthful feedback, as otherwise if bidders realize feedback may be erroneous there is no reason why bidders would trust the feedback that is transmitted to them. Along this line, it may be argued that it is in the interest of an auction house such as ebay to being committed to never report false feedback to bidders (and it seems clear that no one would dispute the correctness of the feedback provided by ebay to bidders).

5.8 Other forms of feedback

Even if the class of feedback considered in this paper is quite large, some forms of feedback are not covered. A key difficulty should be addressed though if one wishes to consider more general classes of feedback. That is, one need to define an appropriate/focal notion of best-response to the feedback received by the bidder, thereby leading to an appropriate notion of equilibrium. While this may be defined in some cases beyond the class of feedback considered here (see the discussion at the end of Section 3), I believe this would be often problematic for general forms of feedback as the feedback would not easily translate into a focal conjecture about other players' strategies. And, as already mentioned, considering the whole set of self-confirming equilibria is unlikely to give sharp predictions in general.

5.9 The replacement assumption

To motivate the analogy-based expectation equilibrium, I have assumed that new bidders participate in each auction (see subsection 2.2). It would clearly be of interest to cover also situations in which each individual bidder remains active longer. When bidders remain active arbitrarily long, convergence to Nash equilibrium should be expected in the private values setup considered here given that in each format M_k bidders could learn from their own past observations the distribution of other bidders' bids.⁴³ However, if each individual bidder does not remain active for ever, then some outcome in between the Nash equilibrium and the feedback equilibrium (as defined in this paper) should be expected. More work is required to model such intermediate cases in a satisfactory way.

6 Conclusion

I believe the above abstract setup is useful to understand a number of applications. The idea that bidders would form their beliefs about others' bidding strategies by looking at the history of past bids should sound familiar to anyone who has considered buying or selling on ebay (under the item "completed listings" one has access to the history of previous bids in auctions of similar objects that took place within a month).⁴⁴ The feedback provided on ebay is partial in the sense that one has never access to the characteristics (such as gender, age etc) of the bidders and the same feedback appears whether or not a buyout option prevailed, as long as the option was not exerted (technically speaking, whether or not there is a buyout option should be interpreted as corresponding to different auction formats).

What this paper has emphasized is the use of feedback policy as a new instrument in mechanism design. My main interest was in understanding the effect of the feedback policy on efficiency and revenues.

The main contribution of this paper has been to show that there is a role for a strategic use of feedback disclosure in mechanism design. On the one hand, first-price auctions with

⁴³This is implicitly assuming that players make the best use of their observations. If however, their attention is limited they may still be playing an analogy-based expectation equilibrium in this case.

⁴⁴See Reynolds and Wooders (2008) for an analysis of auctions with buyout options. I am grateful to John Wooders for introducing me to the various formats and feedback available on ebay.

bidder-anonymous analogy partition generate more revenues than second-price auctions when there are two bidders or more generally when the revenue of the second-price auction with symmetrized distributions exceeds the revenue in the second price auction (with asymmetric distributions). Thus, in such cases, providing coarse feedback in first-price auctions may be thought of as a new way of promoting more competition in asymmetric auctions that avoids the cost of reducing efficiency. On the other hand, the insight that with coarse feedback one can generate more revenues than in Myerson's optimal auction is suggestive that the lack of transparency that is often observed in promotion-like contests may be desirable for organizations. More work is required to understand more generally how much can be gained with the use of coarse feedback in mechanism design and how one should regulate such markets in order to protect consumers.⁴⁵

⁴⁵For example, there is an active debate as to whether the Swoopo auction site (which currently includes bids in different currencies as well as bidding fees) should be regulated. In line with the insights developed in this paper, it might make sense to impose that a common currency be used (as otherwise there is a risk that bids made in different currencies be confused when looking at past auctions).

Appendix

Proof of Claim 1: Consider a symmetric analogy-based expectation equilibrium β of \mathbf{A} (where $\beta(v)$ refers to the equilibrium bid of any bidder with valuation v).⁴⁶ By definition, bidder i plays a best-response to the distribution of bids of other bidders that has assigns density $\frac{\sum_{j \in I} f_j(v)}{n}$ to the bid $\beta(v)$. But, this is the definition of a Bayes Nash equilibrium of $\Gamma^{ba}(\mathbf{A})$. The converse part is also immediate. **Q. E. D.**

Proof of Claim 2: Consider an equilibrium β of $\Gamma^{fa}(\mathbf{A})$. In $\Gamma^{fa}(\mathbf{A})$, bidder i whatever his payment method expects every other bidder $j \in I$ to be facing the payment method k' with probability $\lambda_{k'}$, hence to be playing according to strategy $\beta_j^{k'}(\cdot)$ with probability $\lambda_{k'}$. Thus, in $\Gamma^{fa}(\mathbf{A})$, when the payment method is k , bidder i plays a best-response $\beta_i^k(v_i) \in \arg \max_{b_i} u_i^k(v_i, b_i; \bar{\beta}_{-i}^k)$ where $\bar{\beta}_j^k = \sum_{k'} \lambda_{k'} \beta_j^{k'}$ and $\beta_j^{k'}$ is the distribution of bids of bidder j when j has the method of payment k' . But, this corresponds exactly to the definition of an analogy-based expectation equilibrium of \mathbf{A} . The converse part is also immediate. **Q. E. D.**

Proof of Proposition 1:

Step 1: Consider the first-price auction with bidder-anonymous analogy partition. There exists a unique analogy-based expectation equilibrium defined as follows: for $i = 1, 2$, $\beta_i(v) = \beta(v) = \frac{\int_c^v x \bar{f}(x) dx}{\bar{F}(v)}$ where $\bar{f}(x) = \frac{f_1(x) + f_2(x)}{2}$ and $\bar{F}(v) = \frac{F_1(v) + F_2(v)}{2}$. Bidders never quit ex-post and the outcome is always efficient, i.e. the bidder who values the good most gets the object.

Proof of step 1. Consider an analogy-based expectation equilibrium $\beta_i(\cdot)$ for $i = 1, 2$. Standard incentive compatibility considerations imply that $\beta_i(\cdot)$ must be a non-decreasing function of the valuation (as otherwise a higher valuation type of bidder i would perceive to win the object with a probability strictly lower than a lower valuation type, which is ruled out by incentive compatibility). Thus, the bid functions $\beta_i(\cdot)$ must be continuous almost everywhere.

⁴⁶The anonymity properties of M_1 ensure the symmetry (across bidders) of the best-response correspondence.

Suppose we have a non-symmetric equilibrium (that is not equivalent almost everywhere to a symmetric equilibrium). This implies that for a positive measure of v , $\beta_1(v) \neq \beta_2(v)$ and both $\beta_1(v)$ and $\beta_2(v)$ are best-responses for a bidder with valuation v to the aggregate distribution of bids. There must then be a neighborhood of v within which a positive measure of v has this property. Yet, this implies that we can make another selection of the best-response correspondence that violates the monotonicity of $\beta_i(\cdot)$, thereby showing a contradiction.⁴⁷

The rest of the argument follows from Claim 1 (see Section 4). Indeed, any symmetric analogy-based expectation equilibrium must be a Nash Bayes equilibrium of the *FPA* with symmetric bidders and density $\bar{f}(v)$ and vice versa. Given the analysis of the *FPA* with symmetric bidders, we may conclude as desired. (The fact that bidders never exert their ex-post quitting rights follows from the rules of *FPA*. No bidder finds it optimal to bid above his valuation and thus when he wins a bidder finds it optimal to accept the deal.) **Q. E. D.**

Call R the revenue generated in the first price auction with bidder-anonymous analogy partition. Call R^{SPA} the revenue generated in the second-price auction. Finally, call \bar{R} the expected revenue generated in the second-price auction with symmetric bidders and density of valuations $\bar{f}(v) = \frac{f_1(v)+f_2(v)}{2}$. These revenues write (the identity between the last two expressions can be obtained as a consequence of the allocation equivalence):

$$\begin{aligned} R &= \int_c^d \beta(v) [f_1(v)F_2(v) + f_2(v)F_1(v)] dv \\ R^{SPA} &= \int_c^d v f_1(v) [1 - F_2(v)] dv + \int_c^d v f_2(v) [1 - F_1(v)] dv \end{aligned}$$

⁴⁷Suppose $\beta_1(v) < \beta_2(v)$. By continuity $\beta_1(v + \varepsilon) < \beta_2(v)$ and $\beta_2(v + \varepsilon) > \beta_1(v)$. The definition of an analogy-based expectation equilibrium implies that $b_1(v) = \beta_2(v)$ and $b_1(v + \varepsilon) = \beta_1(v + \varepsilon)$ with all other bids unchanged should also be part of an equilibrium. But, such bids would violate the incentive compatibility conditions and as a result cannot maximize (over bids) the corresponding expected payoffs of bidder 1 with valuations v and $v + \varepsilon$.

$$\begin{aligned}\bar{R} &= 2 \int_c^d v \bar{f}(v) [1 - \bar{F}(v)] dv \\ \bar{R} &= 2 \int_c^d \beta(v) \bar{f}(v) \bar{F}(v) dv\end{aligned}$$

Step 2: $\bar{R} - R^{SPA} = \int_c^d \frac{1}{4} (F_1(v) - F_2(v))^2 dv$

Proof of step 2. Using the first expression of \bar{R} , we have that $\bar{R} - R^{SPA}$ can be written as

$$\begin{aligned}& \int_c^d v \left[-\frac{1}{2} (F_1(v) + F_2(v)) (f_1(v) + f_2(v)) + f_1(v)F_2(v) + f_2(v)F_1(v) \right] dv \\ &= \int_c^d -\frac{v}{2} (f_1(v) - f_2(v)) (F_1(v) - F_2(v)) dv \\ &= \int_c^d \frac{1}{4} (F_1(v) - F_2(v))^2 dv\end{aligned}$$

where the last equality is obtained by integration by parts (noting that $F_1(v) - F_2(v) = 0$ for $v = c$ and d). **Q.E.D.**

Step 3: $R - \bar{R} = \int_c^d \frac{1}{4} \frac{d\beta(v)}{dv} (F_1(v) - F_2(v))^2 dv$

Proof of step 3. Using the second expression of \bar{R} , we have that $R - \bar{R}$ can be written as

$$\begin{aligned}& \int_c^d \beta(v) \left[f_1(v)F_2(v) + f_2(v)F_1(v) - 2 \frac{f_1(v) + f_2(v)}{2} \cdot \frac{F_1(v) + F_2(v)}{2} \right] dv \\ &= \int_c^d -\frac{1}{2} \beta(v) (f_1(v) - f_2(v)) (F_1(v) - F_2(v)) dv \\ &= \int_c^d \frac{1}{4} \frac{d\beta(v)}{dv} (F_1(v) - F_2(v))^2 dv\end{aligned}$$

where the last equality is obtained by integration by parts (noting that $F_1(v) - F_2(v) = 0$ for $v = c$ and d). **Q. E. D.**

Observe that $\frac{d\beta(v)}{dv} > 0$ for all v . Hence, Proposition 1 follows from steps 1, 2, 3. **Q. E. D.**

Proof of Proposition 2:

We proceed in the same way as for Proposition 1 and define R the revenue generated in the first price auction with bidder anonymous feedback partition, R^{SPA} the revenue generated in the second-price auction and \bar{R} the expected revenue generated in the second-price auction with symmetric bidders and density of valuations $\bar{f}(v) = \frac{f_1(v)+\dots+f_n(v)}{n}$. We also denote by $\beta(v)$ the equilibrium bidding strategy in the first price-auction in which the densities are $\bar{f}(v) = \frac{f_1(v)+\dots+f_n(v)}{n}$, by

$$H(v) = F_1(v)\dots F_n(v)$$

and

$$H_{sym}(v) = \bar{F}(v)^n$$

where $F_i(v)$ is the cumulative of v according to $f_i(\cdot)$ and $\bar{F}(v)$ is the cumulative of v according to $\bar{f}(\cdot)$.

Revenue equivalence and claim 1 imply that

$$\begin{aligned} R &= \int_c^d \beta(v)H'(v)dv = \beta(d) - \int_c^d \beta'(v)H(v)dv \\ \bar{R} &= \int_c^d \beta(v)H'_{sym}(v)dv = \beta(d) - \int_c^d \beta'(v)H_{sym}(v)dv \end{aligned}$$

Thus,

$$R - \bar{R} = \int_c^d \beta'(v)[H_{sym}(v) - H(v)]dv$$

Given that $H_{sym}(v) \geq H(v)$ due to the concavity of $x \rightarrow \ln x$ and given that $\beta(\cdot)$ is increasing, we conclude that $R \geq \bar{R}$. Proposition 2 follows. **Q. E. D.**

Proof of Proposition 3:

We start with the following observation:

Step 1: Myerson's optimal auction can be implemented while satisfying the ex-post quitting rights of the bidders in a direct truthful mechanism in which reporting the truth is a weakly dominant strategy for every bidder.

Proof. This is easily shown by simple adaptation of the second-price auction to

the optimal auction of Myerson. In the asymmetric regular case, the functions $c_i(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$ are increasing in v_i , and the optimal auction requires allocating the object to bidder $i^* \in \arg \max_{i \in I} c_i(v_i)$ whenever $c_{i^*}(v_{i^*}) > v_s$ (and otherwise the seller should keep the object). This is achieved in a direct mechanism implementable in dominant strategy in which bidder i^* is required to pay $\max_{j \neq i} [c_i^{-1}(c_j(v_j)), c_i^{-1}(v_s)]$. It is easily checked that this payment is always less than v_{i^*} by the monotonicity of $c_i(\cdot)$. A similar construction can be achieved in the general non- necessarily regular in which intervals of valuations are treated alike.⁴⁸ **Q. E. D.**

The rest of the argument goes as follows. Consider a monotonic bijection ψ from $[c, d]$ into itself, and let M^ψ be the mechanism obtained from the mechanism M^D identified in step 1 as follows: in M^ψ , every bidder i submits a bid b_i and mechanism M^D is applied to the profile of announcements $(\psi(b_i))_{i=1}^n$. Clearly, M^ψ falls in the class of admissible mechanisms and reporting $\psi^{-1}(v_i)$ for bidder i with valuation v_i is a weakly dominant strategy. Besides, M^ψ achieves Myerson's optimal auction revenues and no bidder is willing to exercise his ex-post quitting rights in M^ψ .

Consider now the following auction design. Format M^ψ is used with probability $1 - \varepsilon$ and the first-price auction with v_s reserve price referred to as *FPA* is used with probability ε . Besides, bidders get only to know the aggregate distribution of bids of all bidders across both formats. That is, we consider the bidder-anonymous and format-anonymous analogy partition in which for all i , $\bigcup_{(j,k)} \{(j, k)\}$ forms the unique analogy class of P_i . We will show that for a suitable choice of ε and ψ this auction design generates strictly more expected payoff to the seller than Myerson's optimal auction. First, we observe that the payoff generated in this auction design can be written as $(1 - \varepsilon)R^\psi + \varepsilon R^*$ where R^ψ is the expected payoff generated in this auction design when M^ψ prevails and R^* is the corresponding expected payoff when *FPA* prevails. It is clear that R^ψ is equal to Myerson's optimal auction payoff R^M , since the behaviors in M^ψ are unaffected by the rest of the auction design given that bidders have (weakly) dominant strategies in M^ψ . Thus, it suffices to show that $R^* > R^M$ for suitable choices of ε and ψ .

⁴⁸One can easily perturb the format so as to make incentives strict in all cases (even in the non-regular case).

To this end, let ψ be defined such that for all $b > v_s$.⁴⁹

$$\prod_{i=1}^n F_i(\psi(b)) = \left(\frac{b - \hat{c}}{d - \hat{c}} \right)^{\frac{n}{n-1}m}$$

for some $\hat{c} > v_s$ and m .

In the limit case in which $\varepsilon = 0$, the (perceived) optimal bid in *FPA* for a bidder with valuation $v > v_s$ would be $\arg \max_{b > v_s} (v - b) \left(\frac{b - \hat{c}}{d - \hat{c}} \right)^m$ as $\left(\frac{b - \hat{c}}{d - \hat{c}} \right)^m$ would represent the perceived probability that all other bidders' bids are below b . This expression is maximized at b^{opt} such that

$$b^{opt} - \hat{c} = \frac{m}{m+1}(v - \hat{c})$$

Let b^* be such that $b^* - \hat{c} = \frac{m}{m+2}(v - \hat{c})$ and consider $\varepsilon > 0$. A bidder with valuation $v > v_s$ will perceive to get at most

$$(1 - \varepsilon)(v - b^*) \left(\frac{b^* - \hat{c}}{d - \hat{c}} \right)^m + \varepsilon(v - \hat{c}) \quad (3)$$

by bidding $b < b^*$.

By bidding b^{opt} , a bidder with valuation $v > v_s$ will perceive to get at least:

$$(1 - \varepsilon)(v - b^{opt}) \left(\frac{b^{opt} - \hat{c}}{d - \hat{c}} \right)^m \quad (4)$$

Hence, whenever (4) is larger than (3) we can be sure that a bidder with valuation v bids no less than $\hat{c} + \frac{m}{m+2}(v - \hat{c})$. The difference between (4) and (3) writes

$$\Delta(v) = (1 - \varepsilon) \frac{(v - \hat{c})^{m+1}}{(d - \hat{c})^m} \left[\frac{1}{m+1} \left(\frac{m}{m+1} \right)^m - \frac{2}{m+2} \left(\frac{m}{m+2} \right)^m \right] - \varepsilon(v - \hat{c})$$

Given that $\frac{1}{m+1} \left(\frac{m}{m+1} \right)^m - \frac{2}{m+2} \left(\frac{m}{m+2} \right)^m > 0$, this allows us to obtain that:

Step 2: $\forall \underline{v} > \hat{c}, \forall m, \exists \bar{\varepsilon} > 0$ such that $\forall \varepsilon < \bar{\varepsilon}, \forall v > \underline{v}, \Delta(v) > 0$.

From step 2 and the above considerations, we infer that for all $v > \underline{v}$, $b^{FPA}(v) - \hat{c} > \frac{m}{m+2}(v - \hat{c})$ in the above auction design as defined by ψ and $\varepsilon < \bar{\varepsilon}$. The corresponding

⁴⁹It is clear that such ψ exists and satisfies $\psi(c) = c$ and $\psi(d) = d$.

value of R^* converges to the full information revenue R^F as m converges to infinity and \underline{v} converges to \hat{c} . It follows that one can find m large enough, \hat{c} close enough to v_s , \underline{v} close enough to \hat{c} and $\varepsilon > 0$ so that $R^* > R^M$. This completes the proof of the proposition. **Q. E. D.**

Proof of Proposition 4:

Consider an auction design assumed to deliver an expected payoff that is ε -close to R^F . We will show that this is not possible for ε small enough.

To simplify the notation, we consider the case of two symmetric bidders $i = 1, 2$ and we allow only for auction designs with format-anonymous analogy partitions. The argument easily generalizes to the n asymmetric bidder case with arbitrary analogy partitions (by restricting attention to those formats that are pooled together into one analogy class of say bidder i).

Observe that in all formats, whenever $v_j < v_s$, we must have $\beta_j^k(v_j) < v_s$ given the payment rules of the auction (and the fact that there is a tiny cost to cancelling the transaction). We further let $\gamma = \Pr(v_j < v_s, j \neq i) > 0$, and we let $\Delta^k(v)$ denote the expected revenue loss incurred by the designer in format M_k when bidder i has valuation v as compared with the full information case.

Let m be large enough and let $e < \frac{1}{2} \Pr(\frac{d+v_s}{3} < v_j < \frac{d+v_s}{2})$. Define $\underline{d} > \frac{d+v_s}{3}$ such that $e = \Pr(\underline{d} < v_i < \frac{d+v_s}{2})$. Finally, let $f = \min(\Pr(v_i > \frac{3d+v_s}{4}), e)$, and $\eta = \frac{m\varepsilon\gamma}{f}$. We define

$$\Gamma = \left\{ k \text{ s.t. } \exists v > \frac{3d}{4} \text{ and } \exists v' \in (\underline{d}, \frac{d+v_s}{2}), \Delta^k(v) < \eta \text{ and } \Delta^k(v') < \eta \right\}$$

Given that the auction design delivers an expected payoff that is ε close to R^F , it is readily verified that $\sum_{k \in \Gamma} \lambda_k \geq 1 - \frac{1}{m}$.

Perceived equilibrium payoff:

We note that for $v > \underline{d}$, if $\Delta^k(v) < \eta$, then a bidder with valuation v should in format M_k win whenever $b_j < v_s$ (which happens with probability γ) and pay at least $v - \frac{\eta}{\gamma}$. By monotonicity of the payment rule, this implies that in M_k bidder i with valuation v

perceives in equilibrium to get at most

$$\frac{\eta}{\gamma} = \frac{m\varepsilon}{f}$$

(the payment when i wins, bids b_i and $b_j > v_s$ must be at least as large as when i wins, bids b_i and $b_j < v_s$).

We also note that $\Delta^k(v) < \eta$ implies that in M_k bidder i with valuation v should win against some $v_j \in (\underline{d}, \frac{d+v_s}{2})$ with probability at least $1 - \frac{\eta}{fv} = 1 - \frac{m\varepsilon\gamma}{f^2v}$. We will choose ε small enough so that this probability is no smaller than $\frac{1}{2}$.

Perceived equilibrium from downward deviation:

One can rank the various $k \in \Gamma$ by decreasing order of $\beta^k(\frac{d+v_s}{3})$, and let \bar{r} denote the maximum l such that the sum of λ_k over the first $l - 1$ formats in Λ is strictly below $\frac{1}{2} \sum_{k \in \Gamma} \lambda_k$. We denote by Λ^{sup} the formats in Λ which correspond to the first \bar{r} formats in this induced order.

Consider $k \in \Lambda^{\text{sup}}$ and let $v' \in (\underline{d}, \frac{d+v_s}{2})$, $\Delta^k(v') < \eta$. Consider any $v > \frac{3d+v_s}{4}$ and let v submits a bid $b_i = \beta_i^k(v')$. Bidder i with valuation v in format M_k must perceive to be winning with probability at least $\frac{1}{4}(1 - m) \Pr(v_j < \frac{d+v_s}{3})$,⁵⁰ and he must be paying at most $\frac{d}{2}$ whenever he wins.⁵¹ Given that $v > \frac{3d+v_s}{4}$ (and thus $\frac{3d+v_s}{4} - \frac{d+v_s}{2} = \frac{d-v_s}{4}$), overall such a deviation makes bidder i feel he can get at least $\frac{1}{2}(1 - m)\frac{d}{4} \Pr(v_j < \frac{d+v_s}{3})$ in M_k .

Given that ε can be chosen so that $\frac{m\varepsilon}{f} < \frac{1}{2}(1 - m)\frac{d}{4} \Pr(v_j < \frac{d+v_s}{3})$ we get a contradiction to the definition of an analogy-based expectation equilibrium (since a bidder should obviously feel that his perceived payoff obtained by following his equilibrium strategy is no smaller than his perceived payoff obtained by following any other strategy). **Q. E. D.**

Proof of Proposition 5:

We consider the following formats M_1 and M_2 both used with probability $\frac{1}{2}$. In both formats, the good is never allocated whatever the bids, $\varphi_i^k(b) = 0$ for all i, b and $k = 1, 2$.

⁵⁰This is because in the format anonymous feedback partition, all the bids $\beta_j^{k'}(v_j)$ with $v_j < \frac{d+v_s}{3}$ and $k' \in \Lambda \setminus \Lambda^{\text{sup}} \cup \{\bar{r}\}$ must be below $\beta_i^k(v')$ and by construction $\Lambda \setminus \Lambda^{\text{sup}} \cup \{\bar{r}\}$ has a probability at least $\frac{1}{2}(1 - m)$.

Moreover in M_k , i with valuation v should win against some $v' \in (\underline{d}, \frac{d+v_s}{2})$ with a probability at least $\frac{1}{2}$ (see above), and thus by the monotonicity of $\varphi_i^k(b)$ with respect to b_j he should also win against all bids which are below $\beta_k^i(\underline{d})$ with a probability at least $\frac{1}{2}$.

⁵¹This is because he is mimicking type v' who never pays more than v' when he wins.

In format M_1 , bidder 1 wins ε if $b_1 = 1$ and 0 otherwise. In format M_2 , bidder 1 wins ε if $b_1 = 2$ and 0 otherwise. In format M_2 , bidder 2 pays $\frac{A}{2} > 0$ if $b_1 = 2$ and $b_2 = 1$ and receives A if $b_1 = 1$ and $b_2 = 1$, and receives nothing otherwise, i.e.

$$\tau_2^2(b) = \begin{cases} \frac{A}{2} & \text{if } (b_1, b_2) = (2, 1) \\ -A & \text{if } (b_1, b_2) = (1, 1) \\ 0 & \text{if } (b_1, b_2) \neq (2, 1), (1, 1) \end{cases}$$

and the analogy partition is the anonymous-format analogy partition.

Clearly, in this auction design, bidder 1 will bid $b_1 = 1$ in M_1 and $b_1 = 2$ in M_2 . Given that $\lambda_1 = \lambda_2 = \frac{1}{2}$, and the format-anonymous analogy partition is being used, bidder 2 will believe that in M_2 , bidder 1 bids $b_1 = 1$ or 2 each with probability $\frac{1}{2}$. Based on this belief, bidder 2 will find it optimal to bid $b_2 = 1$ in M_2 (because $\frac{1}{2}(A - \frac{A}{2}) > 0$).

In such an analogy-based expectation equilibrium, the designer gets a revenue equal to $-\varepsilon$ in M_1 and $\frac{A}{2} - \varepsilon$ in M_2 so an overall expected revenue of $\frac{A}{4} - \varepsilon$. Since A can be chosen arbitrarily large, we get the desired result. **Q. E. D.**

Proof of Proposition 6:

Consider the format-anonymous analogy partition in the auction design in which a share $1 - \varepsilon$ of SPA is mixed with ε of the format $M(\eta, \mu)$ in which the bidder with highest bid, say bidder 1 if $b_1 > b_2$, wins the auction and pays a price

$$\tau(b_1, b_2) = (1 - \eta(1 + \mu b_2))b_2 + \eta(1 + \mu b_2)b_1$$

(Note that as $\eta = 0$, $M(\eta, \mu)$ is the second-price auction.)

We subtract c from all bids and valuations. As ε tends to 0, in format $M(\eta, \mu)$, a bidder with valuation v chooses his bid $b(v)$ so that:

$$v - b - \eta \int_0^b (1 + \mu x) dx = 0$$

or

$$b(v) = \frac{-(1 + \eta) + ((1 + \eta)^2 + 2\eta\mu v)^{\frac{1}{2}}}{\eta\mu}$$

given that bidders expect bids to be uniformly distributed on $(0, 1)$. The revenue so generated in format $M(\eta, \mu)$ writes:

$$\begin{aligned} R(\eta, \mu) = & \int_0^1 \int_0^v \left\{ \eta(1 + \mu x) \frac{((1 + \eta)^2 + 2\eta\mu v)^{\frac{1}{2}} - (1 + \eta)}{\eta\mu} + \right. \\ & \left. (1 - \eta - \eta\mu x) \frac{((1 + \eta)^2 + 2\eta\mu x)^{\frac{1}{2}} - (1 + \eta)}{\eta\mu} \right\} dx dv \end{aligned}$$

Simple algebra yield that $R(\eta, \mu = 0)$ is identical to the revenue in the second-price auction. Moreover,

$$\frac{\partial R}{\partial \mu}(\eta, \mu = 0) = \left(\frac{\varepsilon}{1 + \varepsilon} \right)^3.$$

Thus, there must exist $\mu > 0$ and $\eta > 0$ sufficiently small so that $R(\eta, \mu) > 0$. The manipulative design as described above with ε sufficiently small induces a strictly higher revenue than the second price auction. Given that the revenue in the second price auction is the optimal revenue by Myerson's analysis, we conclude as desired (observe that for η and μ small enough, $\eta(1 + \mu x) \in (0, 1)$ so that $M(\eta, \mu)$ belongs to the required space of mechanisms). **Q. E. D.**

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