Bargaining with Reference Dependent Preferences

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Abstract

We posit that parties assess bargaining outcomes not in absolute terms but in relative terms vis à vis reference points and we assume that reference points are affected by prior offers. In a simple bargaining model, we illustrate how such evolving preferences may be responsible for gradualism and delay in bargaining. We observe that the resulting inefficiency may not vanish even in the limit as the cost of waiting for one more period gets very small.

1 Introduction

Many factors influence the positions people take when negotiating. To proceed, both sides must adjust their positions throughout the negotiation, ultimately arriving at either agreement or impasse. Most observed bargaining patterns typically involve gradual adjustments of positions.

In this paper we take the view that agents assess final agreements not in absolute terms but in relative terms vis a vis some reference point. That is, any agreement above the reference point is valued as a gain (possibly discounted by when the agreement is reached) and any agreement below the reference point is valued as a loss (see Khaneman and Tversky (1979)). We also take the view that in bargaining contexts the reference point is

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affected by prior offers (see Bazerman and Neale (1992, p 24) for a support of this view). Specifically, we assume that a larger prior offer obtained in a previous bargaining phase results in a higher reference point for the current bargaining phase. To fix ideas we let the reference point used by a party coincide with the most generous offer received by this party in previous bargaining phases. Within a bargaining phase reference points do not adjust, but as one keeps bargaining there is always a risk that the current bargaining phases stops in which case one moves to a new bargaining phase, reference points adjust and parties incur a small cost c.[1]

Our main interest lies in understanding the bargaining dynamics with reference points evolving as just described. We adopt an equilibrium approach. That is, we assume that at any point in time parties understand the pattern of offers that will come next as a function of their own current move (and the history of offers). These patterns of moves are then assessed according to the preference that derives from the reference point that prevails in the current bargaining phase, and parties choose optimal moves given their criterion.

Our main result is that in an otherwise standard model, evolving reference points may induce gradualism in bargaining. Our paper thus shows the link between reference-dependent preferences and gradualism in bargaining, two well established features whose connection has not been noted (as far as we know).

Our result should be contrasted with the literature on bargaining with complete information and standard non-evolving preferences (Rubinstein (1982)). There, an agreement is reached immediately and thus no gradual pattern can arise. The reason for the immediate agreement result in Rubinstein's model is related to the so called Coase Theorem. If some inefficiency were to arise, then the proposer could offer an immediate agreement, thereby

[1] Our modeling is consistent with the view that reference points adjust only occasionally when a salient event (like a change of bargaining phase) occurs.

keeping the lost surplus for himself.[2]

The intuition as to why the immediate agreement result does not hold with evolving reference-dependent preferences is as follows. Suppose the negotiation is about the partition of a pie of size 1 between two parties A and B, and suppose by contradiction that there is immediate agreement at $(1/2, 1/2)$. If party A starts the negotiation, she should offer right away $(1/2, 1/2)$ to party B, and party B should accept.

But, party B has a better option. By not making any offer, party B would eventually trigger a switch to a new bargaining phase. This would have a cost. However, negotiation would start on new grounds, with party B's reference point now being equal to $1/2$ and party A's reference point remaining equal to 0 (remember that B has not made any offer yet). With such reference points, B would end up with a larger share of the pie,[3] and, from the viewpoint of the original bargaining phase, this larger share would plainly justify for B the cost of moving to another phase (if this cost is assumed small).

The failure of the Coase Theorem has already been noted in other frameworks with reference-dependent preferences. In particular, Kahneman, Knetch and Thaler (1990) report evidence on the fact that willingness to accept may exceed willingness to pay in simple transaction economies, thereby suggesting that the level of transactions may be inefficiently low (from an orthodox viewpoint). Such pathologies are referred to as the endowment effect and they are generally explained by the fact that one's preferences depend on one own's allocation (one gets attached to goods one possesses). The endowment effect clearly involves a form of reference-dependence (the reference being the endowment). Here, the reference-dependence is of a different nature: *prior offers* rather than *endowments* affect the reference

[2]This argument shows that there exist equilibria with immediate agreement. It does not prove though that there are no other equilibria (which Rubinstein (1982) shows).

[3]That is, assuming some symmetry in the subgame, a share equal to 3/4.

point.[4] Moreover, in our case, the departure from the Coase Theorem takes the form of delay in reaching agreements, which has no counterpart in the literature on the endowment effect.

This paper is not the first to develop a theoretical model with reference-dependent preferences. Compared to prospect theory (Khaneman and Tversky (1979)), our model makes no assumption as to whether agents are more sensitive to gains or losses, and it applies whatever the attitude towards gains versus losses. From this perspective, the focus of our work is more on the dynamics of reference points and how it affects the bargaining tactics.[5] In recent work, Köszegi and Rabin (2006) have proposed a model to endogenize the reference point by the expectations made in equilibrium (see also Shalev (2000)). In our approach, reference points are determined by the positions adopted by the parties in prior bargaining phases. Since these positions are endogenously determined, one might argue that the dynamics of reference points is endogenous in our model. Yet, the function specifying how the reference point depends on past positions is exogenous in our approach, which is different in spirit from the formulation of Köszegi and Rabin. It should be noted that if reference points were determined by the expected utility anticipated by agents in equilibrium then one equilibrium would result in immediate agreement as in Rubinstein (1982)'s original theory.[6] Thus, some dependence of the reference point on history rather than on expectations

[4]Nevertheless, one could argue that both effects have similar psychological foundations, with the view that prior offers are often taken for granted, and that one becomes attached to the plans one builds based upon these prior concessions.

[5]Previous works on evolving reference points considered non-strategic interactions, see Gilboa Smeidler (2001), and Strahilevitz-Loewenstein (1998) who study consumption models with changing reference points based on the ideas of evolving aspiration levels and the endowment effect, respectively.

[6]The reason is that as in Rubinstein's equilibrium, equilibrium utility would be independent of history and thus there would be no effect of prior offers on expectations and thus no effect on the ensuing subgame equilibrium. Stationarity would be preserved, resulting in immediate agreement.

seems to be required to explain gradualism in our bargaining model.

¿From an applied perspective, our analysis may be viewed as formalizing the commonly accepted idea that it may be risky to start off negotiations with generous offers. The risk is that, through the possible change of reference point, a generous offer will be taken for granted in the future, and eventually whet one's opponent appetite. The Camp David negotiations, as accounted for by President Carter (as reported in Kissinger (1979, p 392), offer a good illustration of this view (through the advice given to President Sadat):

> His (Sadat) own advisers had pointed out the danger in his signing an agreement with the United States alone. Later, if direct discussions were ever resumed with the Israelis, they could say: "The Egyptians have already agreed to all these points. Now we will use what they have signed as the original basis [our reference point] for all future negotiations".

¿From a theory perspective, our result provides an explanation for delay and gradualism in bargaining based on the adjustment of reference points. It differs from other explanations of delay that rely on some form of *incomplete information* (see Kennan and Wilson (1993)). It also differs from explanations of gradualism based on the presence of *outside options* that would depend on the history of offers (Compte and Jehiel (2004)).

Our finding however has close connections with the explanation based on history-dependent outside options. Even though there is no explicit outside option in our setup, the tactic of waiting for the arrival of a new bargaining phase (so that reference points adjust) plays a role very similar to that of an outside option. An important difference though is that in Compte and Jehiel (2004), the payoff derived from the outside option is exogenously specified as a function of the history of offers; whereas in our setup, the 'outside option' payoff is derived endogenously. In particular, the endogenous character of this payoff allows us to obtain that the inefficiency induced by the delay may be bounded away from 0 even as the bargaining friction (i.e. the cost of switching to a new bargaining phase) gets very small.

The paper is organized as follows. Section 2 presents the model. Section 3 presents the main insight. Section 4 deals with **the** equilibrium construction. Concluding remarks appear in Section 5.

2 The model

We consider a bargaining game described as follows. There are two parties $i = 1, 2$ who negotiate over the partition of a pie of size 1. The game consists of possibly several bargaining phases $n = 1, 2...$ taking place over dates $t = 1, 2...$ The start of the interaction corresponds to date $t = 1$, and bargaining phase 1. At the start of a new bargaining phase, say at date t, one of the parties is drawn at random with probability $1/2$ to make an offer to the other party. The responder may then accept or reject the offer. If he accepts, this is the end of the bargaining and the partition offer is implemented. Otherwise, the game moves to the next date $t + 1$.

As long as one stays within a bargaining phase, parties move in alternate order; so if party i made the offer at t, party j makes the offer at $t+1$. At the end of each date, there is an exogenous probability β of breakdown. In case of breakdown, a new bargaining phase starts. Moving from one bargaining phase to the next one is assumed to cost c to each party.

The key ingredients of this paper are that (i) *preferences are defined relative to reference points* and that (ii) *reference points move from one phase to another.*

Preferences: In any given phase, the utility that player j derives from the partition offer $y = (y_i, y_j)$ when his *current* reference point is r_j is defined by

$$v_j(y, r_j) = y_j - r_j.$$

Thus, only offers in excess of the reference point are valued positively. Offers below the reference point would be counted as a loss if accepted.[7] One possible interpretation of the reference point r_j is that it corresponds to the minimum aspiration level of the player, that is, a level of offer at which the utility he derives is zero.[8]

Let us now specify how players assess in any phase an agreement to take place in a later phase. We assume that the assessments are made relative to the reference point prevailing in the current phase.

That is, consider bargaining phase n and assume player j's reference point is equal to r_j in that phase. Player j assesses an agreement on $y = (y_i, y_j)$ that would take place in phase $n' \geq n$ according to $u_j^n(y, r, n')$ where:

$$u_j^n(y, r_j, n') = v_j(y, r_j) - (n' - n)c.$$

In particular, viewed from phase n, the maximum surplus that players can derive from agreement is equal to $K = 1 - r_i - r_j$.

Shifting reference points: We assume that reference points vary from one bargaining phase to another according to how generous the current offers are, given the current reference point. Formally, let $X_j^{(n)}$ denote the largest offer received by party j during phase n. The difference $X_j^{(n)} - r_j^{(n)}$ is a measure of the generosity of the offers received by player j in phase n. We assume that

$$r_j^{(n+1)} - r_j^{(n)} = \max\{X_j^{(n)} - r_j^{(n)}, 0\}. \tag{1}$$

[7]In prospect theory, one issue is how losses are counted compared to gains, that is, whether the slope of the utility function is the same for gains and losses. For our purpose, the slope is unimportant. It only matters that offers below the reference point are viewed as a loss (compared to the scenario in which no agreement is ever reached and one stays in the same bargaining phase for ever).

[8]See Yukl (1974) and White and Neale (1994) for experiments concerning the effect of offers on aspiration levels, and the effect of aspirations on negotiation outcomes.

That is, as long as offers to player j in stage n remained below the reference point $r_j^{(n)}$, player j's reference point may not rise. An equivalent formulation of equality (1), which we will use in the analysis, is that at any date, a player's reference point coincides with the most generous offer he received in the previous phases. That is, X_j^t denotes the most generous offer received up to date t in phase n. If breakdown occurred after date t offer was rejected, then

$$r_j^{(n+1)} - r_i^{(n)} = X_j^t - r_i^{(n)}.$$

Strategies and equilibrium: For each player i, a strategy in this game specifies an offer y_j^t at each date t, as a function of the history of the game (which consists of the previous offers, the dates at which breakdown occurred (if any), the identity of the player drawn to start making an offer after each breakdown).

We assume that at any point in time parties understand the pattern of offers that will come next as a function of their own current move (and the history of offers), and that parties choose the best move(s) given their current preferences and expectations about the following moves. From a technical viewpoint, we are considering the subgame perfect equilibria of the game in which party i in bargaining phase n is viewed as a different player from party i in bargaining phase n'. We refer to such equilibria as *subgame perfect equilibria.*

Our main purpose in the next Section will be to convey some intuition as to why shifting reference points lead to gradualism. We shall delay the explicit construction of a subgame perfect equilibrium involving inefficient delay to Section 4.

It should be noted that if the reference points were set at 0 in this model (as is assumed in virtually all formal models of bargaining) then in a symmetric (Markovian) equilibrium the proposer would offer $\frac{1}{2} - \frac{\beta c}{2-\beta}$ keeping the rest for herself so that the responder is indifferent between accepting and rejecting the deal.[9]

Comment: We have chosen not to introduce time discounting in the payoff specification. This is mostly to simplify the construction of the equilibrium in Section 4.[10]

3 Main insights

This section explains why shifting reference points lead to gradualism. It also provides some intuition on the size of the steps that players make in equilibrium.

Consider the first phase of bargaining and a date t. Suppose that bargaining breaks down right after date $t-$offer has been rejected thus inducing a switch to a second bargaining phase. The most generous offer received in this phase by player $i = 1, 2$ is X_i^t. In such a scenario, the phase 2 reference points would be given by

$$r_i = X_i^t \tag{2}$$

In this second phase, given the change in preferences, an offer (y_1, y_2) is valued at $y_i - r_i$ by player i. So it is as if players were now bargaining over a pie of size K with

$$K = 1 - r_1 - r_2 \tag{3}$$

where a partition (y_1, y_2) in the original game corresponds to $(y_1 - r_1, y_2 - r_2)$ in the new bargaining game.

In the symmetric setting that we consider, this surplus K will end up being shared equally between the two players in expectation. That is, each

[9] $\frac{1}{2} - \frac{\beta c}{2-\beta} = \beta(\frac{1}{2} - c) + (1-\beta)(\frac{1}{2} + \frac{\beta c}{2-\beta})$

[10] It should be noted that in the absence of time discounting the game with reference points set at 0 admits other equilibria beyond the stationary one described above.

party i will end up with a share y_i of the original pie such that

$$y_i - r_i = \frac{K}{2} \tag{4}$$

Of course, several periods and thus several bargaining phases may be required before the agreement is reached. For the sake of illustration, we assume that the expected number of breakdowns depends solely on the size of K and we denote this expected number by $\eta(K)$; we further assume that $\eta(\cdot)$ is non-decreasing in K (this will be the case for the equilibrium analyzed in Section 4).

It follows from (4) and the definition of $\eta(.)$ that from the viewpoint of phase 1, player i's expected payoff is equal to[11]

$$r_i + \frac{K}{2} - (1 + \eta(K))c$$

or, equivalently,

$$\frac{1}{2} + \frac{X_i^t - X_j^t}{2} - (1 + \eta(K))c \tag{5}$$

The interpretation of (5) is as follows. The difference $X_i^t - X_j^t$ represents how generous the offers made by player j were in phase 1 relative to those made by i. When this difference is positive, party i is in a favorable position in case the negotiation moves to a second phase. This is precisely why making a generous offer is a risky strategy. Suppose player i makes a generous offer, player j may next refrain from making any relevant offer. This would force a breakdown at some point. Player j's reference point would then increase, and this would hurt player i. As a result, the negotiation process has to be gradual.

It should be noted that while the analysis above applies to the first phase, it readily extends to any later phase. Indeed, consider phase n, with reference points $r_1^{(n)}$ and $r_2^{(n)}$. Let $K^n = 1 - r_1^{(n)} - r_2^{(n)}$. Assume that the

[11]The cost is c when moving to phase 2 and $\eta(K)c$ in expectation thereafter.

most generous offer received by player $i = 1, 2$ is X_i^t, and that the bargaining process breaks down after date t offer is rejected, thus inducing a switch to phase $n+1$. Equation (4) becomes $y_i - r_i^{(n+1)} = \frac{K^{n+1}}{2}$, with $r_i^{(n+1)} = X_i^t$. Defining x_i^t as the increment over the reference point, that is,

$$x_i^t \equiv X_i^t - r_i^n,$$

we get, from the viewpoint of phase n, that player i's expected payoff is equal to

$$\frac{K^n}{2} + \frac{x_i^t - x_j^t}{2} - (1 + \eta(K^{n+1}))c$$

As in phase 1, the difference $x_i^t - x_j^t$ represents how generous the offers (in excess of the current reference points) made by player j were relative to those made by i (in phase n).

How gradual does the process has to be?

The above observations are suggestive that the negotiation process must be gradual, but how gradual should it be? To address that issue, we derive an upper bound on how much parties are ready to improve their offer in each round.

Specifically, consider a date t in phase 1. We assume that in equilibrium, player i increases the generosity of his offer by an amount of Δ. We compare the payoff that player i would obtain by waiting till the process moves to the next phase, with the payoff he would obtain at most if he were to increase the generosity of his offer by an amount of Δ. We let $\Delta^t = X_i^t - X_j^t$ denote the difference between player i and player j's most generous offers received up to date t, and $K^t = 1 - X_1^t - X_2^t$. By refraining from making any better offer (until breakdown arises), player i would obtain at least:[12]

$$v_i \geq \frac{1 + \Delta^t}{2} - (1 + \eta(K^t))c \tag{6}$$

[12] Possibly, party j could make a more generous offer before breakdown arises, but this could only increase further the payoff party i obtains once breakdown arises.

If player i instead increases by Δ the generosity of his offer, player j may secure (again by waiting for a breakdown) a payoff equal to

$$v_j \geq \frac{1 - \Delta^t + \Delta}{2} - (1 + \eta(K^t - \Delta))c. \tag{7}$$

Since players cannot hope to get more than 1 in total, we must have

$$v_1 + v_2 \leq 1. \tag{8}$$

In equilibrium, only offers for which Δ satisfies the above inequalities can be made. Since $\eta(\cdot)$ is assumed to be non-decreasing, we infer by adding (6)-(7)-(8) and rearranging:

$$\Delta \leq 4(1 + \eta(K^t))c \tag{9}$$

Inequality (9) thus provides an upper bound on the extra generosity of a new offer made in equilibrium in phase 1. Clearly, the same logic applies to any later phase, so

$$\overline{\Delta} = 4(1 + \eta(1))c \tag{10}$$

provides a uniform upper bound on the steps that players make throughout the game. In particular, it implies that when c is small the expected number of phases cannot be small: if it were, then the steps would be small by (10), hence the expected number of rounds necessary to reach agreement would be large (that is, at least equal to $1/\overline{\Delta}$), and so would be the expected number of breakdowns (β times the number of rounds necessary to reach agreement).

More formally, this argument shows that $\overline{\Delta} \leq \Delta^*$ where[13] $\Delta^* = 4(1 + \frac{\beta}{\Delta^*})c$. This implies for small c:

$$\overline{\Delta} \lesssim 2(\beta c)^{1/2}. \tag{11}$$

[13] $\overline{\Delta} > d \Rightarrow \eta(1) < \frac{\beta}{d} \Rightarrow \overline{\Delta} < 4(1 + \frac{\beta}{d})c$ (by (10)), which is obviously absurd when $d > 4(1 + \frac{\beta}{d})c$.

It would seem from (11) that inefficiencies might be small as the cost c of moving to a new bargaining phase gets small (since $\frac{\beta}{(\beta c)^{1/2}} \cdot c = (\beta c)^{1/2} \underset{c \to 0}{\to} 0$). However, (11) only provides a very crude bound on the steps that players make in equilibrium. As in Compte and Jehiel (2004), two forces are at work to determine the bound on Δ: (i) the extent to which a generous offer benefits the other party in case of breakdown, and (ii) the efficiency loss associated with breakdown, as compared to following the equilibrium course of offers and counter-offers. Since the expected number of breakdowns cannot be small when c is small, following the equilibrium course of offers and counter-offers is also inefficient, and therefore inequality (8) can be refined.

Assuming that constant steps of size Δ are made throughout the game in equilibrium (this will turn out to be a feature of the equilibrium displayed in Section 4), we must have:

$$v_1 + v_2 \leq 1 - 2\beta c - 2c\eta(K^t - \Delta) \tag{12}$$

To see this, observe that after a further step of size Δ there is a probability β of breakdown resulting in an extra efficiency loss $2\beta c$; and whether or not there is breakdown, the expected number of new bargaining phases after this extra step is $\eta(K^t - \Delta)$ resulting in an expected efficiency loss of $2c\eta(K^t - \Delta)$ (we use here the feature that the same step Δ is made whether or not there is breakdown). By adding up (6), (7) and (12), we obtain a refinement of (9) as follows:

$$\begin{aligned} \Delta &\leq 2(1 + \eta(K^t))c + 2(1 + \eta(K^t - \Delta))c - 4\beta c - 4c\eta(K^t - \Delta) \\ &\leq 4c(1 - \beta) + 2c(\eta(K^t) - \eta(K^t - \Delta)). \end{aligned}$$

Since $\eta(K^t) = \beta + \eta(K^t - \Delta)$ (this is obtained using again our assumption that in equilibrium, a step of size Δ is made at K^t), we finally get:

$$\Delta \leq 2c(2 - \beta)$$

which implies that steps have a size comparable to the cost of breakdown.

In particular, this implies a number of rounds approximately equal to $\frac{1}{2c(2-\beta)}$, which corresponds to an expected number of breakdowns equal to $\frac{\beta}{2c(2-\beta)}$, thus an efficiency loss approximately equal to $\beta/(2-\beta)$. We note that the efficiency loss is positive for any fixed β, even at the limit when c gets arbitrarily small.

4 Equilibrium Construction

The main objective of this Section is to construct an equilibrium, and confirm for this equilibrium the insights developed in Section 3. The equilibrium we will construct has the feature that offers are gradually improved so as to make the responder indifferent between moving further or waiting for a breakdown (with no further move meanwhile). So inequalities (6) and (7) are tight. Besides, in this equilibrium, it will be the case that the size of offer increment remains unchanged throughout the game. Thus, the insight at the end of Section 3 applies, and the equilibrium is such that inefficiencies arise even at the limit as c is very small.

Specifically, we let $\alpha = \beta c/(1-\beta/2)$. We define the sequence

$$z^{(0)} = \alpha, z^{(1)} = 2c + 2\alpha \text{ and for all } k \geq 1,\ z^{(m+1)} = z^{(m)} + 2c(2-\beta).$$

Next, we let k be the largest integer such that $z^{(k)} < 1$, and let $z^{(k+1)} = 1$. So when c is small, k is approximately equal to $\frac{1}{2c(2-\beta)}$, which will be the number of rounds required to reach agreement.

We construct an equilibrium where (i) the distance between most generous offers gradually reduces from 1 to $z^{(k)}$ to $z^{(k-1)}$, and so on until $z^{(1)}$, and where (ii) in each phase, players take turn in increasing the generosity of their offers.

We define a candidate strategy profile σ as follows. Consider any date t. As before, we let X_1^t and X_2^t denote the most generous offers received by players 1 and 2 up to date t, and let $X^t \equiv 1 - X_1^t - X_2^t$. Assume it is player i's turn to move.

We start by describing strategies in the event $X^t > z^{(0)}$. We let $z^{(m)}$ denote the threshold satisfying

$$z^{(m)} < X^t \leq z^{(m+1)}.$$

According to σ, player i makes one of the following two moves (we will shortly define which one he makes, as a function of the history of the game): either he leaves his offer unchanged

$$(y_i^t, y_j^t) = (1 - X_j^t, X_j^t),$$

or he increases his offer to player j by an amount $X^t - z^{(m)}$, thereby ensuring that $X^{t+1} = z^{(m)}$ if his offer is rejected. That is,

$$(y_i^t, y_j^t) = (X_i^t + z^{(m)}, X_j^t + X^t - z^{(m)}).$$

Next, player j rejects all offers, except those that would lead to some $X^{t+1} \leq z^{(0)}$.

We now specify the rules that determines whether the player who moves, say player i, is supposed to increase his offer ($s^t = 1$) or not ($s^t = 0$).

(a) If this is the start of a new phase or if $z^{(m)} < X^t < z^{(m)} + d$, with $d = (1 - \beta)2c + \alpha$, then player i is supposed to increase his offer ($s^t = 1$).

(b) If (a) does not apply and $X^{t-1} > z^{(m+1)}$, player i is supposed to increase his offer ($s^t = 1$).

(c) If (a) does not apply and $X^{t-1} \leq z^{(m+1)}$, then either $s^{t-1} = 1$ and then $s^t = 0$, or $s^{t-1} = 0$ and then $s^t = 1$. That is, player i is supposed to increase his offer at t only if player j were supposed to increase her offer at $t - 1$, but not otherwise.

Rule (a) implies that whoever is drawn to start a phase is supposed to increase his offer. Rule (b) implies that when a player (say player j) is the first to make a generous move down to or below some threshold $z^{(m+1)}$, it is the other player (player i) who is expected to make a move to the next threshold $z^{(m)}$. Rule (c) implies that as long as this generous move does not come and X^t remains in the interval $(z^{(m)} + d, z^{(m+1)}]$, player j is supposed to wait for player i to make the generous move (down to $z^{(m)}$).

To conclude the description of the strategies, we specify what players do in the event $X^t \leq z^{(0)}$. The player who moves (i.e. player i) chooses an offer

$$(y_i^t, y_j^t) = (X_i^t + \frac{X^t}{2} + \frac{z^{(0)}}{2}, X_j^t + \frac{X^t}{2} - \frac{z^{(0)}}{2}), \tag{13}$$

This offer, as well as any offer that would be more generous, is accepted. Any offer that is less generous than y_i is rejected by player j.

We have:

Proposition: The strategy profile σ defined above is a subgame perfect Nash equilibrium. Each player's equilibrium payoff is no larger than $2c(1 - \beta)k$ where k is the largest integer such that $z^{(k)} < 1$.

The proof the Proposition appears in Appendix. As already noted, for a fixed value of β, there are efficiency losses, even at the limit when the cost of breakdown becomes arbitrarily small. This is to be contrasted with the analysis of Compte and Jehiel (2004) in which when the efficiency loss induced by the outside option is sufficiently small, there are no inefficiencies in equilibrium: parties must opt out in equilibrium.

This may seem contradictory, as in our set up, when the cost of breakdown vanishes, the cost associated with triggering the outside option (i.e. forcing a breakdown) vanishes too. However, unlike in Compte and Jehiel (2004), it cannot be that in equilibrium, parties find it optimal to opt out (and force a breakdown). Because if it were the case, then this would imply that no agreement would ever be found (because in the next phase, parties would still find it optimal to force breakdown and so on). This simple argument explains why forcing a breakdown serves only as a threat in the present model and can never be used in equilibrium no matter how small the cost of moving to a new bargaining phase is.

5 Discussion

We wish to discuss here possible extensions of our model. We have assumed that reference points coincide with the most generous offer received so far. A more general specification would allow the reference point to react to a lesser extent to prior offers. For example,

$$r_j^{(n+1)} - r_j^{(n)} = \mu \max\{X_j^{(n)} - r_j^{(n)}, 0\}, \tag{14}$$

where $\mu \in (0, 1)$ would measure the sensitivity of the reference point to prior offers.

We have also assumed that preferences are solely driven by the comparison of partition offers to reference points. As noted by Köszegi and Rabin (2006) a more general specification would allow both the absolute payoff and the relative payoff to enter the utility specification. For example, denoting by r_i player i's reference point, player i's utility associated with the partition offer $y = (y_i, y_j)$ may take the form:

$$v_i(y, r_i) = (1 - \alpha)y_i + \alpha(y_i - r_i) \tag{15}$$

where $\alpha \in (0, 1)$ captures the weight the agent puts on the reference-dependent part of his utility. Observe that $v_i(y, r_i) = y_i - \alpha r_i$ and thus this is formally similar to a situation in which preferences are as specified in Section 2 with a reference point defined as a fraction α of r_i. Assuming r_i evolves as in Section 2, this is thus equivalent to the situation in which reference points evolve as in (14) with $\mu = \alpha$.[14]

The analysis of equilibrium is more complicated in these extensions because the state of the game can no longer be solely described by the most generous offers made so far as it should distinguish between offers made in the current phase and offers made in earlier phases. However the qualitative

[14] If reference points do not react that strongly to prior offers then this will correspond to a smaller μ.

insights developed in Section 3 remain unchanged. To fix ideas, if the step by which offers are increased is constant throughout the game then the step size Δ should satisfy

$$\Delta \leq 2c(2-\beta)/\mu$$

when reference points evolve according to (14). Thus, bargaining will be gradual, there will be inefficiencies even in the limit as c gets small, and these inefficiencies will be more important as reference points are more sensitive to prior offers (μ is larger). The effect of more sensitivity is that it increases the payoff a party can guarantee by forcing a breakdown, thereby making this threat more stringent and the resulting delay larger.

References

[1] Bazerman, M. H. and M. A. Neale (1992): *Negotiating Rationally* Free Press.

[2] Compte O. and P. Jehiel (2004): “Gradualism in Bargaining and Contribution Games,” *Review of Economic Studies* **71**, 975-1000.

[3] Gilboa, I. and Schmeidler, D. (2001): ”Reaction to Price Changes and Aspiration Level Adjustments”, *Review of Economic Design*, **6**, 215-223.

[4] Kahneman, K., J. L. Knetch and R. H. Thaler (1990): ”Experimental Tests of the Endowment Effect and the Coase Theorem,” *Journal of Political Economy* **98**, 1325-1348.

[5] Kahneman, K. and Tversky, A.(1979): ”Prospect theory: an analysis of decision under risk” *Econometrica*, 47, 263–291.

[6] Kennan, J. and R. Wilson (1993): ”Bargaining with Private Information,” *Journal of Economic Literature*, **31**, 45-54.

[7] Kissinger, H., (1979), *White House Years*, Little, Brown & Company, Boston

[8] Köszegi, B. and M. Rabin (2006): ”Reference-Dependent Preferences,” forthcoming *Quarterly Journal of Economics*

[9] Rubinstein, A. (1982), “Perfect Equilibrium in a Bargaining Model,” *Econometrica* **50**, 97-109.

[10] Shalev, J. (2000), ”Loss aversion equilibrium,” *International Journal of Game Theory* **29**, 269-287.

[11] Strahilevitz, M., and Loewenstein, G. (1998) "The effects of ownership history on the valuation of objects" *Journal of Consumer Research*, 25, 276-289.

[12] White, S. and Neale, M. "The role of Negotiator Aspirations and Settlement expectancies in bargaining outcomes", *Organizational Behavior and Human Decision Processes*, 57, 303-317

[13] Yulk, G. (1974) "Effects of situations Variables and opponent concessions on a bargainer's perception, aspirations, and concessions", *Journal of Personality and Social Psychology* 29, 227-236

Appendix (Proof of the Proposition)

Fix X_1^t and X_2^t. Assume that $z^{(m)} < X^t \leq z^{(m+1)}$, player i is supposed to move, and make a more generous offer ($s^t = 1$). We let $v^{m+1}(X^t)$ denote the expected payoff in excess of $X_j^t - r_j^t$ that player j gets under σ, and by $\underline{v}^{m+1}$ the expected payoff in excess of $X_i^t - r_i^t$ that player i gets under σ.[15] We also let $\bar{v}^{m+1} = v^{m+1}(z^{(m+1)})$. These values are defined by induction on m.

When $m = 0$, player i makes an offer that is accepted, so we have:

$$\begin{aligned} \underline{v}^1 &= z^{(0)}, \\ v^1(X) &= X - z^{(0)}, \\ \bar{v}^1 &= z^{(1)} - z^{(0)} \end{aligned}$$

When $m > 0$, player i makes an offer that is rejected. With probability $(1 - \beta)$, there is no breakdown and player j is the next mover (and makes a more generous offer). With probability β, there is breakdown, in which case each player has equal chance of being the first mover. This implies:

$$\underline{v}^{m+1} = (1 - \beta)\bar{v}^m + \beta[\frac{\bar{v}^m + \underline{v}^m}{2} - c], \text{ and} \tag{16}$$

$$v^{m+1}(X) = X - z^{(m-1)} + (1 - \beta)\underline{v}^m + \beta[\frac{\bar{v}^m + \underline{v}^m}{2} - c] \tag{17}$$

Note that the thresholds have been chosen precisely to ensure that for all

[15] $\underline{v}^{(m+1)}$ is independent of X^t because, whatever X^t, player i is supposed to reach the threshold $z^{(m)}$.

$m \geq 1$,[16]

$$\bar{v}^m - \underline{v}^m = 2c, \tag{18}$$

or equivalently

$$\frac{\bar{v}^m + \underline{v}^m}{2} - c = \underline{v}^m.$$

This equality is key to check incentives. It ensures that at $X^t = z^{(m+1)}$ with $s^t = 1$ (i.e. player i is supposed to make a more generous offer), player i is indifferent between making that offer (and obtaining $\underline{v}^{m+1}$), and waiting for breakdown, in which case he gets $\frac{\bar{v}^{m+1}+\underline{v}^{m+1}}{2} - c$.

Before proceeding to check all possible deviations, it is worth noting that these equalities imply:

$$v^{m+1}(X) - \underline{v}^{m+1} = X - z^{(m)} + (1-\beta)2c. \tag{19}$$

and that $\underline{v}^{m+1} = \underline{v}^m + (1-\beta)2c$.

Let us now check for all possible one-shot deviations. We call $y^{i,m}$ the equilibrium offer that i is supposed to make.

If player i chooses a less generous offer y^t leading to $X^{t+1} \in (z^{(m)} + \Delta, X^t]$. In case of breakdown, he obtains

$$\frac{\underline{v}^{m+1} + v^{m+1}(X^{t+1})}{2} - c$$

which by construction is smaller than $\underline{v}^{m+1}$. In case no breakdown occurs, player j does not change her offer ($s^{t+1} = 0$), and player i is still supposed to offer $y^{i,m}$, and thus obtains $\underline{v}^{m+1}$. So the deviation is not profitable.

If player i chooses an offer y^t (less generous than $y^{i,m}$) leading to $X^{t+1} \in (z^{(m)}, z^{(m)} + \Delta)$, then $s^{t+1} = 1$ (rule (a)). If no breakdown occurs, player

[16] To see why (18) holds, observe that for $m = 1$, $\bar{v}^1 - \underline{v}^1 = z^{(1)} - 2z^{(0)} = 2c$. For $m > 1$, this is checked by induction on m, as equalities (16) and (17) imply:

$$\bar{v}^{m+1} - \underline{v}^{m+1} = z^{(m)} - z^{(m-1)} - (1-\beta)(\bar{v}^m - \underline{v}^m) = 2c(2-\beta) - 2c(1-\beta) = 2c.$$

j moves and player i gets $v^{m+1}(X)$. Otherwise player i pays c, and either player i is called upon moving again, and he gets $\underline{v}^{m+1}$ or player j moves and he gets $v^{m+1}(X^{t+1})$. So his expected gain is

$$(1-\beta/2)v^{m+1}(X^{t+1}) + \beta/2\underline{v}^{m+1} - \beta c$$

which we need to compare to $\underline{v}^{m+1}$. Since $X^{t+1} < z^{(m)} + \Delta$, by (19), this expression is strictly smaller than

$$\underline{v}^{m+1} + (1-\beta/2)[\Delta - (1-\beta)2c)] - \beta c$$

hence, given the definition of Δ, strictly smaller than $\underline{v}^{m+1}$. So the deviation is not profitable.

Finally, it is easy to check that offers that are more generous than $y^{i,m}$ cannot be profitable deviations either.

To conclude we need to check incentives when $X^t \leq z^{(0)}$. Recall that the equilibrium offer is given by (13). If a more generous offer is made, it is accepted, so it cannot be profitable. If a less generous offer is made, it is rejected. Either there is breakdown, and player i obtains (in excess to $X_i^{(t)} - r_i^t$) $X^t/2 - c$. Or there is no breakdown, and he gets $X^t/2 - z^{(0)}/2$. So in expectation, player i obtains

$$(1-\beta)(X^t/2 - z^{(0)}/2) + \beta(X^t/2 - c) < X^t/2$$

Now player j has incentives to reject a less generous offer, say $y_j^t < X_j^t + X^t/2 - z^{(0)}/2$, because if she does, she obtains (in excess to $X_j^{(t)} - r_j^t$)

$$(1-\beta)(X^t/2+z^{(0)}/2)+\beta(X^t/2-c) = X^t/2+(1-\beta)\frac{z^{(0)}}{2}-(1-\beta/2)z^{(0)} = X^t/2-\frac{z^{(0)}}{2}.$$