

Antitrust Enforcement Policy and Markets Interaction: Targeted or Concerted Interventions?*

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Abstract

We study the design of antitrust intervention policy in presence of horizontally imperfectly differentiated industries. Firms in a given industry may decide to collude, but inter-industry collusion is assumed not to be possible. We find that the enforcement policy depends critically on the nature of the differentiation and of the competition between industries.

With substitutes, the intervention policy should be targeted when firms are Cournot competitors. Indeed, in this case, enforcing a competitive behavior from one industry has a positive spill-over on the incentive to collude in the other industry: the stronger the substitutability, the more targeted the intervention. However, with Bertrand competition and sufficiently homogenous products, even two collusive industries make almost no profits. In this case, we show that the intervention is concerted across industries and decreases with the substitutability between products.

By contrast, with complements, these probabilities must be equal across markets since enforcing a competitive behavior in one industry reinforces the other industry's incentive to collude. This result carries over to the situation of vertically linked industries where outputs are technological complements. For sufficiently large degrees of complementarity, the antitrust authority is forced to intervene with probability one in both markets. These results do no longer depend on the nature of the competition.

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1 Introduction

The advent of competition is at the heart of important evolutions in many industries. Competition by new firms has been allowed, giving rise to a proliferation of actors, while, simultaneously, globalization of activities leads to a greater interaction between product markets.

This reorganization process should be accompanied with an adequate institutional design of the authorities in charge of the supervision of these industries. In particular, the supervision of one industry may impact in a non trivial way the supervision of other industries that interact with the former through product market competition. This is particularly true for antitrust authorities, which have a quasi-universal mandate, and must therefore pay attention to the potential spill-overs of their interventions.

In this paper, we shall determine the optimal design of the collusion deterrence policy in presence of multiple and related markets.

It is noticeable that this dimension has received little attention in the antitrust enforcement literature. For instance, Besanko and Spulber (1989) build on the pioneering work of Becker (1968) on crime deterrence to construct a model of optimal antitrust intervention. In that paper, a competition policy authority faces an industry composed of firms, with identical efficiency parameters, that produce a homogeneous good and that may decide to collude or to compete. Thus, the competition authority has to set a probability of investigating the industry, that depends on the observed price level, and a fine to be paid by the firms whenever they are convinced of a collusive behavior. A major intuition is that the audit takes place for high prices, not because these prices are necessarily collusive but to deter more efficient types of industries from charging these prices.

But, at the same time, competition authorities might well recognize that the enforcement policy implemented in one market may affect the incentives to collude of firms located in imperfectly differentiated markets. The question we ask in this paper is precisely: How does the interaction between different potentially collusive markets affect the enforcement policy implemented by a competition policy authority? That the intervention of competition authority affects the market structure is by now clear. Norman and Thisse (2000) offer a synthesis of theoretical works¹ showing that competition policy impacts in subtle ways the organization of industries under its supervision. Our purpose is different in that we want to study how the market structure (and in particular the degree of differentiation between products and the nature of competition between industries) affect the enforcement policy set by the antitrust authority.

Such a problematic has also some relevance for the design of the intervention undertaken by

¹See also the special issue of the *International Journal of Industrial Organization*, April 2001, on competition policy in dynamic markets.

a competition policy authority like, say, the DG IV in an integrated area such as the European Union. With the removal of trade barriers, the interaction between national markets of the member States, although not perfect yet, has been reinforced. It becomes then interesting to study whether one could find some principles for the antitrust intervention in such a context.

The framework we consider is the following. We consider two imperfectly differentiated markets, or industries. Products can be substitutes or complements. Each industry is composed of firms endowed with the capability of colluding (that is, firms in a given market can collude; however, we do not allow for inter-industry collusion since this would make our analysis similar to the one sector case). Firms in both markets have identical and privately known marginal cost of production.

The competition policy does not observe production or price levels and, therefore, has to decide of an antitrust intervention policy on the sole basis of the markets size, the cost of the audit, the fine and, importantly, on the degree of products differentiation. Indeed, anecdotal evidences incline us to think that competition policy authorities do not constantly monitor and gather information on all industries potentially under their jurisdictions. In particular, antitrust intervention is usually ‘one-shot’ and competition authorities might not always have the time and the resources needed to collect information about prices and production levels needed to implement such an enforcement policy².

Let us start with the case of substitutable products with inter-industry Cournot competition. In this case, the enforcement policy should follow a *targeted intervention principle*: the competition authority must devote all its resources to investigate one industry and let the other industry without any supervision. Indeed, enforcing a competitive behavior from one industry prevent the other collusive industry, that produces the same final good, from unduly charging a supra-competitive price. Therefore, it comes naturally that the stronger the substitutability is, the more the competition authority intervenes with a large probability in a given industry and a low probability in the other one.

However, results change with inter-industry Bertrand competition. In this situation, we find that the antitrust authority should intervene with similar probabilities in both markets, and that, the stronger the substitutability is, the smaller are the probabilities of intervention. The intuition becomes now the following: with Bertrand competition, even when both industries collude and act as a monopolist on their own respective market, when goods are strong substitutes they earn

²Two polar situations have been studied in the literature. Either the competition authority perfectly observes the pricing decisions of firms but remains uninformed on some dimension of the firms’ activities; in that case, that authority faces a mechanism design problem as in Besanko and Spulber (1989). Or the intervention of the competition authority is completely exogenous, as in Norman and Thisse (2000). As we argued previously, the former approach puts requirements on the side of the authority, which fit well the intervention of sectoral regulators but seem at odds with the practice of competition policy authorities. The latter approach is more pragmatic but relies on the (strong) assumption that the public intervention can be hardly modified.

almost no profits and produce almost the socially optimal quantities. Thus, with tough price competition, the need for the antitrust authority to intervene on the markets decreases with the substitutability since the industries' behavior becomes very close to the social optimum.

Thus, in presence of demand substitutes, the nature of the competition between markets appears as a crucial variable to determine the intervention of antitrust authority. As shown by d'Aspremont, Dos Santos Ferreira and Gérard-Varet (1991), Cournot competition implies some form of price coordination and therefore softer price competition. Therefore, from the viewpoint of competition authorities, fighting 'facilitating practices', i.e. contractual clauses or norms of conduct that enable to implement some price coordination, becomes a substitutable instrument to the direct antitrust intervention aiming at deterring collusive behavior. The substitutability at the market level generates a substitutability between the instruments at the disposal of the competition authority.

Let us turn on to the the case of demand complements. We show that the enforcement policy in that case should be guided by a *concerted intervention principle*: the optimal probabilities of intervention are identical across markets and depend positively on the degree of complementarity. The reason underlying this result is the following: with complements, enforcing a competitive behavior in one industry increases the demand for the good produced by the other industry, thereby providing that latter industry with greater incentives to collude. In this case, the intervention on one market has a negative spill-over on the other market; this explains why intervention must be concerted across markets. Interestingly, the results concerning complementary final products become insensitive to the nature of the competition.

Finally, we show that for sufficiently large degrees of complementarity, the antitrust authority is forced to intervene with probability one in both markets. This highlights the fact that competition policy (reduced to the supervision of the industries' conducts) might not be able to deal, alone, with industries characterized by a pattern of strong complementarities. Hence, in such situations, one should think of (re-)introducing some 'traditional' regulatory oversight on top of the supervision by competition authorities.

We build on this case to conclude with the design of the competition authority's intervention when markets are vertically integrated, i.e. when one market produces a good used as an intermediate input by the other market³. There, we show that the intervention must still be concerted across markets.

The outline of the model goes as follows. In the next section, Section 2, we present the model. Section 3 focuses on the choice by each industry to collude or to compete, depending on the antitrust intervention policy. Then, in Section 4, we study the benchmark of independent markets. We continue with the case of substitutable and complementary products in Section 5.

³This is the case for the access to a bottleneck segment for instance.

Section 6 studies the impact of the nature of the competition and Section 7 deals with the case of vertically related markets. Conclusions are gathered in Section 8. All the proofs are relegated in an Appendix.

2 The model

The demand side We consider consumers that derive a gross surplus or utility from the consumption of two goods, denoted by ‘1’ and ‘2’, equal to

$$GS(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2) + cq_1q_2.$$

Parameter $c \in (-1, 1)$ indicates that goods are either (demand) substitutes ($c \leq 0$) or complements ($c \geq 0$). The associated inverse demand function for good i is therefore given by⁴

$$p_i(q_1, q_2) = \frac{\partial GS}{\partial q_i}(q_1, q_2) = a - q_i + cq_j, \quad j \neq i.$$

Net consumers’ surplus is then given by $NS(q_1, q_2) = GS(q_1, q_2) - p_1q_1 - p_2q_2$.

The case of imperfect substitutes is clear: think, for instance, of producers of the same product but with a spatial differentiation. Competition between producers of complementary products is also prevalent: this is the case, among others, for the hardware and software industries.

The industry side The production of good i is ensured by a set of firms, called ‘industry i ’. Block and Feinstein (1986) focus both theoretically and empirically on the informational spillover effect of antitrust enforcement when firms are engaged in different horizontally linked markets. To abstract from this effect, we assume that firms in each industry i produce the corresponding good only.

We consider in a first time that firms are Cournot competitors and relegate the analysis of Bertrand competition later in the paper. The difference in the intensiveness of competition between industries will impact the antitrust intervention, especially with substitutes.

To abstract from considerations based on a difference of productive efficiency, we consider that industry i and industry j have the same marginal cost θ . Parameter θ is private information of the firms and is uniformly distributed on $[\underline{\theta}, \bar{\theta}]$ with $\Delta\theta \equiv \bar{\theta} - \underline{\theta}$.

Since we do not want to introduce a difference between the Cournot and Bertrand cases based on the value of the competitive outcome, we also assume that if industry i behaves competitively (denoted with a superscript ‘ nc ’ for non-cooperative behavior), then firms equate

⁴Throughout the paper, we assume that a is sufficiently large to ensure positivity of quantities. The slopes of the demand functions are taken to be equal to one for simplicity.

price with marginal cost and the industry's profit is driven down to zero. This enables to isolate the impact of the nature of the inter-industry competition on the antitrust intervention policy. Obviously, this is always the case with price competition since marginal costs are equal and constant; with Cournot competition, this only holds in the limit case of an infinite number of firms.

Finally, we assume that firms in each industry are able to perfectly collude; however, collusion between industries is not possible⁵. Differently stated, intra-industry collusion is perfect and firms in a given market can decide to collude and to coordinate their decisions to maximize their joint profit⁶ given by $\pi_i = [p_i(q_1, q_2) - \theta]q_i$ but take as given the behavior of the other industry. This is the situation of 'cooperation' in industry i (denoted with a superscript 'c').

The antitrust agency Let us now describe the intervention of the competition policy authority. Following the literature on antitrust enforcement in a static setting, we assume that the instruments at the disposal of the antitrust agency are defined by:

- A probability $\beta_i \in (0, 1)$ of intervening in market i and discovering evidences of collusive behavior (if any) by this industry⁷.
- A constant⁸ fine $F_i \in [0, \bar{F}]$ that must be paid by firms in market i whenever they are convinced of collusive behavior. This fine cannot exceed an upper bound \bar{F} because, for instance, firms may have limited liability.

Finally, we make the following assumption.

Assumption 1 *The antitrust authority does not observe production levels in the markets.*

This is a strong assumption that we justify as follows. First, our aim is to obtain some intuitions about the impact of markets interaction on the antitrust intervention policies. Through the assumption of unobservable quantities, we simplify drastically the mechanism design problem

⁵Indeed, otherwise, our model boils down to a one-industry framework. Moreover, as a first approximation, it seems that intra-market collusion is more easily achievable and enforceable than inter-markets collusion: Think, for instance, of geographically distant markets.

⁶This a convenient but also extreme assumption, which has always been embraced by the antitrust enforcement literature. However, the literature on cartel formation under incomplete information does not provide definitive answers on the possibility to reach an efficient cartel: Roberts (1985) and Kihlstrom and Vives (1989, 1992) have exhibited some environments in which it was indeed possible; Cramton and Palfrey (1990) obtain more ambiguous results. In a related but different perspective, Laffont and Martimort (2000) show that asymmetric information within the coalition can be a source of inefficiencies.

⁷We shall only consider interior solutions.

⁸Since we assume later on that production levels are unobservable by the antitrust agency, it is natural to consider that the fine she commits to ex ante is constant.

of the antitrust agency⁹. Second, and more importantly, it is not clear (at least to us) that competition policy authorities, with quasi-universal mandates, have the expertise to perfectly and constantly gather all the relevant information concerning production levels on all markets potentially under her jurisdiction. Third, Kühn (2000) forcefully argues that detecting collusion from quantity (or price) observations is by no mean an easy task and, therefore, such observation might not be the main element that triggers the intervention.

As usual, antitrust intervention is socially costly: parameter $K > 0$ denotes the social cost of all the resources devoted to antitrust enforcement. We assume that $K > \bar{F}$ to ensure that the competition policy authority always wants to minimize the probability of intervention.

Finally we assume that the objective of the competition policy authority consists simply in the sum of net consumers' surplus, minus the cost of enforcement, plus the fine collected in case of collusion detection, or

$$SW = NS(q_1, q_2) + \sum_{i=1,2} \beta_i [\mathfrak{S}_i F_i - K],$$

where \mathfrak{S}_i equals 1 if industry i is convinced of collusive behavior and 0 otherwise.¹⁰

3 Industries' behavior

No antitrust intervention Let us assume for the moment that the antitrust agency does not intervene. Because the profit in industry i depends on the behavior of industry j , there are four possible configurations, depending on whether each industry behave cooperatively or not:

1. If both industries cooperate, then the quantity produced by, and the corresponding profit of, industry i are $q_i^{c,c}(\theta) = \frac{a-\theta}{2-c}$ and $\pi_i^{c,c}(\theta) = [\frac{a-\theta}{2-c}]^2$.
2. If, say, industry 1 cooperates whereas industry 2 does not cooperate, then quantities are $q_1^{c,nc} = \frac{1+c}{2-c^2}(a-\theta)$ and $q_2^{c,nc} = \frac{2+c}{2-c^2}(a-\theta)$, and the corresponding profits are $\pi_1^{c,nc}(\theta) = [\frac{1+c}{2-c^2}(a-\theta)]^2$ and $\pi_2^{c,nc}(\theta) = 0$. A similar case occurs when industry 1 does not cooperate while industry 2 does.

⁹Many works in mechanism design have shown that a central planner facing agents with correlated information parameters can use this correlation to bridge his informational gap: see Crémer and McLean (1984) or McAfee and Reny (1988) among others. However, as shown by Laffont and Martimort (2000) the power of such so-called yardstick mechanisms may disappear when agents can collude. Also, the implementation of such yardstick competition schemes requires to use potentially large penalties and rewards; wether antitrust authorities, which can not transfer money to the industries, can nevertheless exploit the correlation is the topic of an ongoing research.

¹⁰Results remain unaltered qualitatively if the competition authority values the industries' profits as long as the weight attached to these profits is not too large.

3. Finally, if none of the industries cooperate, then the quantity produced by, and the profit of industry i , are $q_i^{nc,nc}(\theta) = \frac{a-\theta}{1-c}$ and $\pi_i^{nc,nc}(\theta) = 0$.

Therefore, industry i is better off if industry j colludes when goods are substitutes, but prefers that industry j behaves non-cooperatively when goods are complements: $\pi_i^{c,c}(\theta) \geq \pi_i^{c,nc}(\theta) \Leftrightarrow c^2 + 2c \leq 0$. Indeed, a collusive industry j always tends to contract its production, which in turn decreases (respectively increases) the residual demand faced by industry i with substitutes (respectively complements).

Antitrust intervention Let us now assume that the competition policy authority implements an enforcement policy $\{\beta_i, F_i\}$ in market i . Given our assumptions, firms in this market will decide to cooperate if the gain to collude minus the risk of detection is larger than the gain if they behave competitively. As the gains earned by industry i depend on the behavior of industry j we define the following thresholds¹¹:

- $\tilde{\theta}_i$ is the value of the cost parameter such that industry i with this type is indifferent between cooperating or not when industry j behaves cooperatively: $\pi_i^{c,c}(\tilde{\theta}_i) - \beta_i F_i = 0$.
- $\hat{\theta}_i$ is the value of the cost parameter such that industry i with this type is indifferent between cooperating or not when industry j behaves non-cooperatively: $\pi_1^{c,nc}(\hat{\theta}_1) - \beta_1 F_1 = 0$ and $\pi_2^{nc,c}(\hat{\theta}_2) - \beta_2 F_2 = 0$.

Finally, we also assume that the maximal fine is large enough so that even the most efficient industry does not find it profitable to collude if the antitrust agency intervenes with probability 1¹².

4 Antitrust intervention: The benchmark of independent markets

When markets are independent (i.e., $c = 0$), the problem of the antitrust authority becomes separable since the incentive to collude of one industry depends only on the deterrence policy implemented in this market, and not on the production level in the other market. Let us rewrite, for this section only, the net consumers' surplus as $NS(q_1, q_2) = NS_1(q_1) + NS_2(q_2)$, where $NS_i(q_i)$ is the net surplus attached to the production of good i when markets are independent.

¹¹We consider parameters values such that these thresholds belong to $[\underline{\theta}, \bar{\theta}]$.

¹²This will be the case if $\max\{\pi_1^{c,c}(\underline{\theta}), \pi_1^{c,nc}(\underline{\theta})\} \leq \bar{F} \Leftrightarrow a - \underline{\theta} \leq \frac{1}{2}\sqrt{\bar{F}}$.

In this situation, we also have $\tilde{\theta}_i = \hat{\theta}_i \equiv \theta'_i$ and an industry i with low efficiency ($\theta \geq \theta'_i$) decides to compete, while an industry i with high efficiency ($\theta \leq \theta'_i$) prefers to collude. This is represented in Figure 1.

Insert Figure 1 Here

Thus, the problem of the competition policy authority in market i can be rewritten as follows (where superscript ‘*ind*’ stands for ‘independent goods’):

$$\mathcal{P}_i^{ind} : \max_{\{\beta_i, F_i\}} \left\{ \int_{\underline{\theta}}^{\theta'_i} [NS_i(q_i^c(\theta)) + \beta_i F_i] \frac{d\theta}{\Delta\theta} + \int_{\theta'_i}^{\bar{\theta}} [NS_i(q_i^{nc}(\theta))] \frac{d\theta}{\Delta\theta} - \beta_i K \right\}.$$

At this stage, we notice that, in equilibrium, the fine F_i has to be set at the highest possible level. Indeed, firms are risk-neutral and therefore only care about the expected fine; thus, since enforcement is costly, if the fine were not at the highest possible value, then it could be possible to reduce the cost of intervention by decreasing the probability of intervention and increasing the fine, keeping the expected fine constant.

The first-order condition associated to β_i can be rewritten as follows:

$$-\frac{\partial \theta'_i}{\partial \beta_i} [NS_i(q_i^{nc}(\theta'_i)) - NS_i(q_i^c(\theta'_i)) - \beta_i \bar{F}] = \Delta\theta K - (\theta'_i - \underline{\theta}) \bar{F}.$$

The trade-off that solves the optimal probability of intervention is intuitive: an increase in the probability of intervention enables to deter more types of firms from colluding, but decreases the amount of fines collected from firms convinced of anti-competitive behavior and increases the cost of antitrust enforcement.

Solving \mathcal{P}_i^{ind} yields the following proposition.

Proposition 1 *Assume that markets are independent (i.e., $c = 0$). Then, the optimal probability of intervention in market i is*

$$\beta_i^{ind} = \frac{4}{9\bar{F}} \left(a - \underline{\theta} - \Delta\theta \frac{K}{\bar{F}} \right)^2.$$

Proof See Appendix A.1. ■

The comparative static results on β_i^{ind} are straightforward. The larger the market is (i.e., parameter a), the more the competition authority finds it valuable to intervene and deter collusion; the larger the maximum fine is, the smaller is the probability of intervention since it enables to

reduce the expected fine; the more socially costly the intervention is, the smaller is the probability of intervention. Finally, the larger the uncertainty with respect to the industries' type is, the smaller is β_i^{ind} .¹³

In the following, the probabilities of intervention that we will study will always follow the same variations as function of a , \bar{F} , K and $\Delta\theta$.

5 Antitrust intervention: The impact of markets interaction

Since they lead to qualitatively different results, we focus first on the case of substitutes and then turn on to the case of complements. We assume without loss of generality that $\beta_1 \geq \beta_2$ and that fines are maximal, or $F_1 = F_2 = \bar{F}$.

5.1 Substitutable goods and the targeted intervention principle

Before turning on to the determination of the optimal probability of intervention, we must focus on the choice of collusion/competition by one industry given the enforcement policy announced by the competition policy authority and the behavior of the other industry.

When products are substitutes (and given that $\beta_1 \geq \beta_2$), direct computations show the following inequalities: $\hat{\theta}_i \leq \tilde{\theta}_i$, $\hat{\theta}_1 \leq \hat{\theta}_2$ and $\tilde{\theta}_1 \leq \tilde{\theta}_2$. More interestingly, we have

$$\hat{\theta}_2 \geq \tilde{\theta}_1 \Leftrightarrow (2 - c^2 + c) \sqrt{\beta_1} \geq (2 - c^2) \sqrt{\beta_2}. \quad (1)$$

Whether Condition (1) holds or not depends endogenously of the optimal probabilities of intervention. However, intuitively, it seems that when the substitutability degree is weak (i.e., close to 0) then Condition (1) is likely to hold. Conversely, when goods are strong substitutes then this condition is likely to be violated. As we explain below, this condition will impact the design of the optimal probability of intervention, leading us to distinguish the following two cases: weak substitutes and strong substitutes.

Weak substitutes Let us consider first that Condition (1) is satisfied. Then, given the definitions of the thresholds $\tilde{\theta}_i$ and $\hat{\theta}_i$, the choice of collusion/competition is as follows: (i) For $\theta \in [\underline{\theta}, \hat{\theta}_1]$, it is a dominant strategy for both industries to collude. In the same vein, for $\theta \in [\tilde{\theta}_2, \bar{\theta}]$, both industries have the same dominant strategy, namely to behave competitively. (ii) For $\theta \in [\hat{\theta}_1, \tilde{\theta}_1]$, industry 2 always prefers to collude; industry 1 is willing to collude if industry 2 also colludes, but prefers to compete when industry 2 competes. Therefore, both industries end up cooperating, since industry 2's collusive behavior provides industry 1 with the incentive

¹³This comes from our assumption $K > \bar{F}$.

to behave cooperatively. A similar result holds when $\theta \in [\tilde{\theta}_1, \hat{\theta}_2]$: the competitive behavior of industry 1 prevents industry 2 from colluding. Thus, in this region, there is a positive spillover of the antitrust intervention in industry 1 on industry 2. (iii) Finally, for $\theta \in [\hat{\theta}_1, \tilde{\theta}_2]$, industry 1 always competes, whereas industry 2 always colludes.

With substitutes, when Condition (1) holds, the industries' behavior is represented in Figure 2.

Insert Figure 2 Here

The expected net consumers' surplus is then

$$\mathbf{E}_\theta \{NS^{ws}\} = \int_{\underline{\theta}}^{\hat{\theta}_1} NS(q_1^{c,c}(\theta), q_2^{c,c}(\theta)) \frac{d\theta}{\Delta\theta} + \int_{\hat{\theta}_1}^{\hat{\theta}_2} NS(q_1^{nc,c}(\theta), q_2^{nc,c}(\theta)) \frac{d\theta}{\Delta\theta} + \int_{\hat{\theta}_2}^{\bar{\theta}} NS(q_1^{nc,nc}(\theta), q_2^{nc,nc}(\theta)) \frac{d\theta}{\Delta\theta}.$$

Thus, the competition authority's problem can be stated as follows (where superscript 'ws' stands for 'weak substitutes'):

$$\mathcal{P}^{ws} : \begin{cases} \max_{\{\beta_1, \beta_2\}} \mathbf{E}_\theta \{SW^{ws}\} = \left\{ \mathbf{E}_\theta \{NS^{ws}\} + \left(\frac{\hat{\theta}_1 - \underline{\theta}}{\Delta\theta}\right) \beta_1 \bar{F} + \left(\frac{\hat{\theta}_2 - \underline{\theta}}{\Delta\theta}\right) \beta_2 \bar{F} - (\beta_1 + \beta_2) K \right\}, \\ \text{subject to Condition (1).} \end{cases}$$

Unconstrained optimization with respect to β_1 and β_2 yields the following probabilities of intervention:

$$\begin{cases} \beta_1^{ws} &= \frac{16(2-c^2)^4}{(24-4c-30c^2+9c^3+10c^4-4c^5)^2 \bar{F}} \left(a - \underline{\theta} - \Delta\theta \frac{K}{\bar{F}}\right)^2, \\ \beta_2^{ws} &= \frac{16(1-c)^2(1+c)^4}{(6-11c^2+4c^4)^2 \bar{F}} \left(a - \underline{\theta} - \Delta\theta \frac{K}{\bar{F}}\right)^2. \end{cases}$$

However, we must check that, in equilibrium, both the second-order conditions and Condition (1) are satisfied. This will effectively constrain the competition authority's problem as explained in the the following proposition.

Proposition 2 *Assume that goods are substitutes. Then, there exists a threshold $c' \in (0, 1)$ such that, when products are weakly substitutes (i.e., $c' \leq c \leq 0$), the optimal probabilities of intervention are $\{\beta_1^{ws}, \beta_2^{ws}\}$.*

Proof See Appendix A.2. ■

To interpret this proposition we need a few additional notations. $\Delta NS_{c,c}^{nc,c} \equiv NS(q_1^{nc,c}(\theta), q_2^{nc,c}(\theta)) - NS(q_1^{c,c}(\theta), q_2^{c,c}(\theta))$ is the gain, from the viewpoint of net consumers' surplus, to move from the

region where both industries collude to the region in which only industry 2 colludes. Define similarly $\Delta NS_{nc,c}^{nc,nc} \equiv NS(q_1^{nc,nc}(\theta), q_2^{nc,nc}(\theta)) - NS(q_1^{nc,c}(\theta), q_2^{nc,c}(\theta))$.

First, notice that it becomes difficult to obtain a detailed knowledge of the shape of the optimal probabilities of intervention since the differences in net surplus $\Delta NS_{c,c}^{nc,c}$ is not monotonic in the parameter of differentiation.

However, one can show that β_2^{ws} is increasing with c . Indeed, the stronger the substitutability is, the smaller becomes the difference in net surplus between the (nc, c) -region and the (nc, nc) one; therefore, since the competitive pressure exerted by a competitive industry 1 propagates in industry 2, the intervention in market 2 is reduced. In other words, enforcing a competitive behavior in one market has a positive spill-over on the other market when goods are substitutable. This positive spill-over increases with the substitutability between goods produced by the different industries.

From Figure 2, the probability of intervention on market 2 will be guided (among others) by the gain for consumers to move from the region in which only industry 2 colludes (i.e., (nc, c)) to the region in which both industries compete (i.e., (nc, nc)). Simple computations show that the associated difference in net consumers' surplus is

$$\Delta NS_{nc,c}^{nc,nc} = \frac{(1+c)[3-2c^2]}{2(1-c)(2-c^2)^2}(a-\theta)^2, \quad (2)$$

which is increasing in c , and goes to 0 when markets become more and more substitutes: at the extreme, with homogenous products, if one industry behaves competitively, there is no need to prevent the other from colluding. On the other hand, even with perfect substitutes, there always remains a positive gain to move from the region (c, c) , in which the two industries collude and act as Cournot competitors, to the region in which one of the industries does not cooperate and puts a competitive pressure on the other one.

Nonetheless, even if β_2^{ws} decreases when goods become more and more substitutable, Condition (1) will not be satisfied for strongly substitutable outputs. Indeed, for strong substitutes, a non-cooperative industry 1 puts a strong competitive pressure on a collusive industry 2: $\hat{\theta}_2$ sharply decreases with the substitutability degree. The condition $c \geq c'$ ensures that this condition is satisfied in equilibrium. It is also sufficient to ensure that second-order conditions are met¹⁴.

¹⁴Indeed, the second-order condition with respect to β_2 is not satisfied when goods are strong substitutes. This appears not to be a simple technical problem. As we explained earlier on, the gains for net consumers' surplus to move from the region (nc, c) to the region (nc, nc) tends to 0 when goods are more and more substitutable. Thus, the optimization with respect to β_2 , which impacts $\hat{\theta}_2$, leads to corner solution.

Strong substitutes Let us now consider the situation in which Condition (1) is not satisfied:

$$\tilde{\theta}_1 \geq \hat{\theta}_2 \Leftrightarrow (2 - c^2 + c) \sqrt{\beta_1} \leq (2 - c^2) \sqrt{\beta_2}. \quad (3)$$

How does the choice of collusion/competition change? It is straightforward to see that we end up with the following three zones of efficiency parameters: (i) For $\theta \in [\underline{\theta}, \hat{\theta}_2]$, each industry finds it profitable to collude. (ii) For $\theta \in [\hat{\theta}_2, \tilde{\theta}_1]$, industry i is willing to collude if industry j also colludes; however, the former prefers to behave competitively if the latter competes. As a result, two Nash equilibria exist: (c, c) and (nc, nc) . (iii) For $\theta \in [\hat{\theta}_2, \bar{\theta}]$, each industry decides to compete.

The new feature of this situation is that antitrust intervention creates inter-industry coordination problems. Indeed, for intermediate values of the efficiency parameter, two possible equilibria emerge: one in which firms make no profit, and one in which firms earn the collusive profit. This is summarized in Figure 3.

Insert Figure 3 Here

This coordination problem is interesting. Indeed, it unveils a potentially important dimension of the competition policy authority's problem, namely that of the tacit coordination between industries on a particular equilibrium. Unfortunately, our model does not have enough structure to address this issue. Moreover, it is also unappealing since our focus is on the determination of the optimal probabilities of intervention. We thus have to choose one among the two possible equilibria. We focus on the equilibrium where firms of each market choose to collude for the $[\hat{\theta}_2, \tilde{\theta}_1]$ -region¹⁵.

Given this choice of equilibrium, the net consumers' surplus in this case can be restated as follows (where superscript 'ss' stands for 'strong substitutes')

$$\mathbf{E}_\theta \{NS^{ss}\} = \int_{\underline{\theta}}^{\tilde{\theta}_1} NS(q_1^{c,c}(\theta), q_2^{c,c}(\theta)) \frac{d\theta}{\Delta\theta} + \int_{\tilde{\theta}_1}^{\bar{\theta}} NS(q_1^{nc,nc}(\theta), q_2^{nc,nc}(\theta)) \frac{d\theta}{\Delta\theta}.$$

Thus, the program of the competition policy authority can be restated as follows:

$$\mathcal{P}^{ss} : \begin{cases} \max_{\{\beta_1, \beta_2\}} \mathbf{E}_\theta \{SW^{ss}\} = \left\{ \mathbf{E}_\theta \{NS^{ss}\} + \left(\frac{\tilde{\theta}_1 - \underline{\theta}}{\Delta\theta} \right) (\beta_1 + \beta_2) \bar{F} - (\beta_1 + \beta_2) K \right\}, \\ \text{subject to Condition (3).} \end{cases}$$

But, since the probability of intervention on market 2 does no longer affect the size of the region where collusion occurs, the competition policy authority prefers to set β_2 at the lowest

¹⁵The other case is somewhat pathological and is available upon request from the authors.

possible value consistent with Condition (3), which becomes binding in equilibrium.

In the Appendix, we solve \mathcal{P}^{ss} and obtain the following optimal probabilities of intervention:

$$\begin{cases} \beta_1^{ss} &= \frac{4(1-c)^2(8+4c-7c^2-2c^3+2c^4)}{(2-c)^2(12-4c-21c^2+7c^3+9c^4+4c^5)^2 F} \left(a - \underline{\theta} - \Delta\theta \frac{K}{F} \right)^2, \\ \beta_2^{ss} &= \frac{4(1-c)^2(1+c)^2(8+4c-7c^2-2c^3+2c^4)}{(2-c)^2(12-4c-21c^2+7c^3+9c^4+4c^5)^2 F} \left(a - \underline{\theta} - \Delta\theta \frac{K}{F} \right)^2. \end{cases}$$

This enables to state the following proposition.

Proposition 3 *Assume that goods are strong substitutes (i.e., $-1 < c \leq c'$). Then, the optimal probabilities of intervention are given by $\{\beta_1^{ss}, \beta_2^{ss}\}$.*

Proof See Appendix A.3. ■

Comparisons When goods are substitutes, we explained previously that it is necessary to account for the degree of the substitutability. That these probabilities of intervention are continuous in c' should not come at a surprise; however, as shown in the next corollary, they are not differentiable in c' .

Corollary 1 *The optimal probabilities of intervention exhibit a kink in c' : with respect to the case of weak substitutes (i.e., $c' \leq c \leq 0$), when goods become strong substitutes (i.e., $-1 < c \leq c'$) then the probability of intervention in market 1 strongly increases and the probability of intervention in market 2 strongly decreases.*

Proof See Appendix A.4. ■

When the degree of substitutability is below this threshold, then the intervention in industry 2 remains at the smallest possible level consistent with Condition (3) and starts to decrease sharply. On the other hand, the competition policy authority devotes the maximal possible resources to investigate industry 1, anticipating that a competitive industry 1 will drastically reduce the negative impact of a collusive industry 2 when goods are strongly substitutable.

Proposition 2 and 3 are not entirely surprising. Indeed, in the limit case of perfect substitutability, there is only one market; thus, enforcing a perfectly competitive behavior from a subset of firms is sufficient to prevent the remaining firms from colluding.

In a similar vein, let us consider the following re-interpretation of our model: there are two firms, one in each market, and the intervention of the competition authority can force the a firm to behave in a socially efficient way (that is, to set a quantity such that price equals marginal cost). Our results indicates that, when goods are perfect substitutes, the competition authority should devote all her resources in the intervention on one firm only. This obviously carries over to the case of an arbitrary number of firms producing an homogeneous good.

5.2 Complementary goods and the concerted intervention principle

We now consider the case of complements. Let us first start with the following result.

Proposition 4 *Assume that goods are complements. Then, in equilibrium, the competition policy authority intervenes with the same probability in both markets.*

Proof See Appendix A.5. ■

Notice first that, with complements, the probabilities of intervention are qualitatively different than with substitutes. The reason for this ‘symmetry’ goes as follows.

In this situation, the thresholds satisfy $\tilde{\theta}_i \leq \hat{\theta}_i$ since industry i now prefers that industry j competes in order to increase the price of good i . Therefore, this is now the respective positions of $\tilde{\theta}_2$ and $\hat{\theta}_1$ that matter. Moreover, we show in the Appendix that, if we assume $\beta_1 \geq \beta_2$ and $\tilde{\theta}_2 \geq \hat{\theta}_1$, then the regions for the choice of collusion/competition are the same as in the case of weak substitutes and are depicted in Figure 2. But it turns out that the unconstrained maximization leads to $\beta_2 > \beta_1$: indeed, when outputs are complements, the gain for consumers to reduce the (nc, c) -region and to expand the (nc, nc) -region is larger than the gain to reduce the (c, c) -zone and to expand the (nc, c) one. When industry 1 does not collude, the market power of industry 2 that colludes is highly reinforced through the complementarity; this calls for a larger probability of intervention on market 2... leading the antitrust authority to intervene with the same probability on both industries.

Indeed, forget for the moment the cost of intervention (which is the force that drives the probabilities of intervention down) and the gain related to the fines collected from collusive industries. Assume also that $\tilde{\theta}_2 \geq \hat{\theta}_1$; then, it is immediate to check that the three regions of the efficiency parameter that appear are the same as for the case of weak substitutes. Consequently, since the objectif is separable in β_1 and β_2 , the probability of intervention in market 1 (respectively 2) is driven up by the gain for consumers from an increase in the region (nc, c) (respectively (nc, nc)) and a decrease of the region (c, c) (respectively (nc, c)). From Equation (2), it is immediate to see that

$$\Delta NS_{nc,c}^{nc,nc} \geq \Delta NS_{nc,c}^{c,c} \quad \forall c \geq 0,$$

since, with complements, the competitive behavior of one industry increases the willingness to pay of consumers for the other good. Consequently, when industry 1 is deterred from colluding, the stronger the complementarity is, the larger is the demand for the good produced by a collusive industry 2, and the larger is the gain for consumers to prevent that latter from exercising its market power.

Thus, the gain for consumers' surplus associated to β_2 is always larger than the one associated to β_1 ... and it is never optimal to intervene in market 2 with a smaller probability than in market 1. But since we started with the assumption that $\beta_1 \geq \beta_2$, in equilibrium with complements, both probabilities must be equal.

This explains that, with complements, the competition policy authority is forced to intervene in both markets with identical probabilities. Then, provided that the second-order condition is satisfied, the common probability is given by¹⁶

$$\beta^c = \frac{64(1-c)^2(1+c)^4(2-c^2)^4}{(48+20c-126c^2-41c^3+119c^4+27c^5-50c^6-6c^7+8c^8)^2\bar{F}} \left(a - \underline{\theta} - \Delta\theta \frac{K}{\bar{F}} \right)^2.$$

However, this not yet the end of the story as shown in the next proposition.

Proposition 5 *Assume that goods are complements. Then, there exists a threshold $c''' \in (0, 1)$ such that the common optimal probability of intervention for each market is equal to*

$$\begin{cases} \beta^c & \text{if } c \leq c''', \\ 1 & \text{if } c > c'''. \end{cases}$$

Proof See Appendix A.6. ■

As shown in the previous proposition, with strong complements, a corner solution emerges in which the competition policy authority intervenes with probability 1 in both markets. Coming back to the differences in net consumers' surplus between regions, we have

$$\lim_{c \rightarrow 1} \Delta NS_{nc,c}^{nc,nc} = +\infty,$$

whereas

$$\lim_{c \rightarrow 1} \Delta NS_{nc,nc}^{nc,c} < +\infty.$$

Therefore, the larger the level of complementarity is, the larger is the gain to increase the probability of intervention. And, at some point, it always offset the (finite) cost of intervention: indeed, we show in the Appendix that for strong complements the competition authority's

¹⁶In the determination of the optimal probabilities of intervention with complements, we need to consider the case where $\hat{\theta}_1 \geq \hat{\theta}_2$. Under this condition, multiple equilibria emerge when $\theta \in [\hat{\theta}_2, \hat{\theta}_1]$: one in which industry 1 competes whereas industry 2 colludes, and one in which industry 1 colludes and industry 2 competes. Since they are equivalent from the viewpoint of net consumers' surplus, we decided to select the (nc, c) equilibrium. Notice that this choice is the most natural since for the interval just before and the interval just after the $[\hat{\theta}_2, \hat{\theta}_1]$ region, the unique equilibrium is (nc, c) . A similar multiplicity arises in the other cases (see the Appendix for the details) and we always choose the 'most natural' equilibrium, that is, we choose the equilibrium which is consistent with the ones that arise in the neighbouring regions.

objective is a convex function of the optimal probability of intervention. Since the marginal gain always exceeds the marginal cost of intervention, in equilibrium the competition authority decides to intervene with probability 1 in both markets.

6 The impact of the nature of the competition

We now assume that industries choose prices instead of quantities. The differences between the polar situations of Cournot and Bertrand competition are not caused by a change in the nature of the inter-industry strategic interaction, but are rather due to a change in the ‘toughness’ of the inter-markets competition.

Let us start with the case of substitutes. We obtain the following proposition¹⁷.

Proposition 6 *Assume that industries are Bertrand competitors. Then, for all degrees of substitutability, the competition authority intervenes with the following differentiated probabilities*

$$\begin{cases} \beta_1^{s,Bertrand} &= \frac{256}{(24-4c+14c^2-27c^3-17c^4+7c^5+3c^6)^{2F}} \left(a - \underline{\theta} - \Delta\theta \frac{K}{F} \right)^2, \\ \beta_2^{s,Bertrand} &= \frac{4}{(1-c+c^2-c^3)^{2F}} \left(a - \underline{\theta} - \Delta\theta \frac{K}{F} \right)^2. \end{cases}$$

With substitutes, these probabilities of intervention are monotonically increasing in the parameter of product differentiation.

Proof *Left to the reader.* ■

This proposition deserves some comments. First, with Bertrand competition, there is only one case to consider now, which corresponds to the situation of weak substitutes studied under Cournot competition. This is a first difference due to the nature of the competition. Second, and more importantly, the stronger the substitutability between outputs is, the smaller are the probabilities of intervention on both markets. This result sharply contrasts with the case of Cournot competition. Let us explain the underlying intuition.

As we explained in the previous sections, in the case of weak substitutes and Cournot competition, the probability of intervention in market 1 strongly depends on the difference in net surplus $\Delta NS_{c,c}^{nc,c}$, whereas the probability for market 2 relies mainly on $\Delta NS_{nc,c}^{mc,nc}$. Consequently, the differences between those two differences roughly gives the ‘importance’ of intervening in

¹⁷First, the proof for the case of substitutes follows the same line as for the case of Cournot competition and weak substitutes. It turns out that with Bertrand competition and substitutes, Condition (1) is always satisfied. Second, the proof for the case of complements is identical to the one with Cournot competition and complements. The threshold level of the complementarity parameter is obtained by simply remarking that $\lim_{c \rightarrow 1} \beta_c^{s,Bertrand} = +\infty$.

market 1 with respect to the intervention in market 2. We have

$$\Delta NS_{\text{Cournot}} \equiv \Delta NS_{c,c}^{nc,c} - \Delta NS_{nc,c}^{nc,nc} = \frac{-c(8 - 5c - 5c^2 + 3c^3)}{(2 - c)^2(1 - c)(2 - c^2)^2} (a - \theta)^2.$$

Therefore, for substitutes, $\Delta NS_{\text{Cournot}}$ is monotonically decreasing with c . This confirms our previous result that the stronger the substitutability is, the more ‘targeted’ becomes the antitrust intervention since, with strong substitutes, deterring collusion in one industry puts a strong competitive pressure on the other one.

With Bertrand competition, things turn out to be different. Simple computation show that

$$\Delta NS_{\text{Bertrand}} \equiv \Delta NS_{c,c}^{nc,c} - \Delta NS_{nc,c}^{nc,nc} = \frac{-c(1 + c)(8 + 3c)}{4(1 - c)(2 + c)^2} (a - \theta)^2,$$

which, for substitutes, is a concave function of the differentiation parameter c (with an interior maximum).

Notice first that the difference in net surplus associated to the intervention in market 1 and in market 2 is always smaller with Bertrand competition than under Cournot competition. Therefore, the probabilities of intervention will tend to be more ‘balanced’ across markets.

Second, for strong substitutes, the gain for net consumers’ surplus of moving from the (c, c) region to the (nc, c) one (associated with the antitrust intervention in market 1) is very close to the gain for net consumers’ surplus of moving from the (nc, nc) region to the (nc, c) one (associated to the intervention in market 2). Indeed, consider that industries are producing almost perfectly homogenous goods. In this case, price competition by two collusive industries leads to zero profit in equilibrium, and, in the limit, there is even no need to intervene! The nature of the competition between industries critically impacts the shape the optimal antitrust intervention policy.

We now consider the case of complementary goods, and obtain the following proposition.

Proposition 7 *Assume that industries are Bertrand competitors. Then, with complements, there exists a threshold $c_B''' \in (0, 1)$ such that the competition authority intervenes with the following common probability on both markets*

$$\beta^{c, \text{Bertrand}} = \begin{cases} \frac{1024}{(1-c)^2(48+20c+58c^2+7c^3-10c^4-3c^5)^2 F} \left(a - \underline{\theta} - \Delta\theta \frac{K}{F} \right)^2 & \text{if } c \leq c_B''', \\ 1 & \text{if } c > c_B'''. \end{cases}$$

Proof *Left to the reader.* ■

Hence, our results in the case of Cournot competition carry over the case of Bertrand competi-

tion. Since the argument is very similar, we do not repeat it for the sake of conciseness.

7 Antitrust intervention with vertically linked industries

We make now a slight digression and consider the case of vertical linkages between industries. In this context, goods produced by the industries will be perfect complements.

We consider the simplest framework of vertical interaction¹⁸ in which an upstream industry (indexed by ‘ u ’) produces an intermediate good at marginal cost θ and sells it to a downstream industry (indexed by ‘ d ’) at unit price p_u ; that industry transforms this input at no cost¹⁹ into a final consumption good, which is sold on a market where prevails a linear inverse demand $p(q) = a - q$.

In this setting, we can define the quantity produced by the downstream sector, as well as the profits earned by the downstream and the upstream industries; they depend on the behavior of each industry, which in turn depends on the probabilities of intervention across industries and can be characterized with thresholds as previously. To alleviate the exposition, this, and the definition of the different thresholds, are relegated in Appendix A.7.

There, we show that for vertically linked industries, and as for the case of complementary final goods, the optimal probabilities of intervention for the different industries must be equal. The underlying intuition is very similar: enforcing a competitive behavior from, say, the upstream industry, reinforces the incentives to collude of the downstream industry, and reciprocally.

However, the asymmetry between the roles played by the downstream and the upstream segments implies that letting the upstream part collude while the downstream one compete is no longer equivalent to letting the downstream collude while enforcing a competitive behavior from the upstream part. Indeed, while in both case the final quantity produced by the downstream industry is the same²⁰, and then both situations are equivalent from the viewpoint of consumers’ surplus, the definition of the thresholds are different depending on the configuration.

In fact, the optimal antitrust intervention policy can follow two regimes. The similarities between these regimes are that for high efficiency levels both industries collude while for low efficiency parameters both industries behave competitively. However, for intermediate efficiency levels, either the upstream industry colludes while the downstream one behaves competitively or the reverse. The next proposition summarizes this and provides a comparison of the two regimes.

¹⁸See Tirole (1988) for instance.

¹⁹The analysis carries over to the case where the downstream industry transforms the input at a strictly positive constant marginal cost.

²⁰This relies on the specification of our model.

Proposition 8 *With vertically linked industries, the optimal antitrust intervention policy is characterized as follows:*

- *The probability of intervention on the upstream and the downstream industries are equal: $\beta_u = \beta_d \equiv \beta^p$.*
- *The probability of intervention in each industry can take two values:*
 1. *Either $\beta^p = \beta_1^p$ and for intermediate efficiency levels the upstream industry colludes while the downstream industry behaves competitively.*
 2. *Or $\beta^p = \beta_2^p < \beta_1^p$ and for intermediate efficiency levels the upstream industry behaves competitively while the downstream industry colludes.*
- *The first antitrust intervention policy yields a larger expected welfare than the second one.*

Proof See Appendix A.7. ■

To understand the reason underlying the welfare analysis of the different regimes, let us first note that $NS(q_d) = \frac{1}{2}q_d^2$; since (with obvious notations) $q_d^{c,c}(\theta) = \frac{a-\theta}{4} \leq q_d^{c,nc}(\theta) = q_d^{nc,c}(\theta) = \frac{a-\theta}{2} \leq q_d^{nc,nc}(\theta) = \frac{a-\theta}{2}$, we have $\Delta NS_{c,nc}^{nc,nc} \geq \Delta NS_{c,c}^{nc,c}$. The first regime, which puts a larger competitive pressure on the downstream industry, leads to a larger welfare than the second one since it leads to a larger region in which at most one industry colludes. Consequently, the optimal antitrust intervention policy is such that, for intermediate efficiency levels, the upstream industry is induced to collude while the downstream industry competes.

8 Conclusions

The theory on optimal antitrust enforcement in a static setting has provided economists with a deeper understanding of the way antitrust authorities should design their intervention to deter collusive behavior. Although our model imposes stronger restrictions on the policy instruments at the disposal of the competition policy authorities, it also yields several new insights that are worth summarizing now.

First, the presence of interactions across markets affects the intervention by the antitrust authority: the knowledge of the nature and the degree of the interaction between industries is crucial.

Second, the nature of the competition at the industries' level is far from being neutral on the intervention too. Then, there is a need to determine the nature of the competition between these firms. Although we acknowledge this is not an easy task in practice, there are some indicators,

like the so-called ‘facilitating practices’: for instance, contractual clauses or norms of conduct enable to implement some price coordination²¹.

Third, our model clearly shows that antitrust intervention alone is not enough in presence of strong complementarities. In particular, our model clearly shows that as soon as an antitrust authority is concerned with collusion in a given industry, the intervention of that competition authority should not be confined to that market only, but should be extended to markets producing complementary products.

In such a case, a constant monitoring of the industries is needed, an intervention which amounts to re-introducing a form of regulatory intervention. The same conclusion carries over to the case of vertically linked industries (in which goods are complements in a technological sense). In this case, the ideal way to supervise the industries seems to combine both a regulatory intervention, through price-cap mechanisms for instance, with a monitoring of collusive practices, through an antitrust intervention policy. However, whether the combination between such qualitatively different policies leads to a social optimum still remains an open question, even from a theoretical point of view.

Finally, it is worth re-emphasizing that our analysis rests on an over-simplified model of antitrust intervention; in particular, our assumptions on the informational structure of the competition authority departs from the usual ones and future research should study the robustness of our results to more ‘sophisticated’ (i.e., using the correlation between technological parameters of the industries) antitrust enforcement policies.

A Appendix

A.1 Proof of Proposition 1

It is straightforward to solve the first-order condition associated to \mathcal{P}_i^{ind} and to obtain the corresponding optimal probability of intervention. The second-order condition is $-\frac{3\bar{F}^2}{4\sqrt{\beta_i\bar{F}}} \leq 0$, which is trivially satisfied.

A.2 Proof of Proposition 2

The first-order conditions associated to \mathcal{P}^{ws} are trivial to compute and to solve to obtain $\{\beta_1^{ws}, \beta_2^{ws}\}$.

Let us consider the second-order conditions of \mathcal{P}^{ws} . First, notice that $\frac{\partial^2}{\partial\beta_1\partial\beta_2}\mathbf{E}_\theta\{SW^{ws}\} = 0$.

²¹See also Holt and Scheffman (1987).

Therefore, the second-order conditions amount to

$$\frac{\partial^2}{\partial \beta_1^2} \mathbf{E}_\theta \{SW^{ws}\} \leq 0 \Leftrightarrow -\frac{(24 - 4c - 30c^2 + 9c^3 + 10c^4 - 4c^5)\bar{F}^2}{8(2 - c^2)^2 \sqrt{\beta_1 \bar{F}}} \leq 0,$$

which is trivially satisfied, and

$$\frac{\partial^2}{\partial \beta_2^2} \mathbf{E}_\theta \{SW^{ws}\} \leq 0 \Leftrightarrow -\frac{(6 - 11c^2 + 4c^4)\bar{F}^2}{8(1 - c)(1 + c)^2 \sqrt{\beta_2 \bar{F}}} \leq 0$$

which is satisfied if and only if $|c| \leq \frac{\sqrt{3}}{2}$.

Now, we must also check that Condition (1) is satisfied in equilibrium. Straightforward computations show that

$$\tilde{\theta}_1 \leq \hat{\theta}_2 \Leftrightarrow \frac{(2 - c^2)^4}{(12 + 4c - 13c^2 - 2c^3 + 4c^2)^2} \geq \frac{(1 - c^2)^2}{(3 - 4c^2)^2} \quad (4)$$

Since we must assume $c \geq -\frac{\sqrt{3}}{2}$ in order that the second-order conditions are satisfied, Condition (4) is equivalent to

$$\frac{-c(4 + 3c - 6c^2 - 2c^3 + 2c^4)}{(3 - 4c^2)(12 + 4c - 13c^2 - 2c^3 + 4c^2)} \geq 0,$$

which is equivalent to (still given that $0 \geq c \geq -\frac{\sqrt{3}}{2}$) $P_1(c) \equiv 4 + 3c - 6c^2 - 2c^3 + 2c^4 \geq 0$. Let $c' \approx -0.722453$ be the unique root of $P_1(\cdot)$ that belongs to $(-1, 0]$.

Since $c' \geq -\frac{\sqrt{3}}{2}$, as soon as $c \in [c', 0]$ Condition (1) as well as the second-order conditions associated to \mathcal{P}^{ws} are satisfied.

A.3 Proof of Proposition 3

Consider \mathcal{P}^{ss} . Assume first that Condition (3) is not binding. Then, the optimization with respect to β_2 yields

$$\frac{\partial}{\partial \beta_2} \mathbf{E}_\theta \{SW^{ss}\} = (\tilde{\theta}_1 - \underline{\theta})\bar{F} - \Delta\theta K,$$

which is negative under the assumption $\bar{F} \leq K$. Therefore, β_2 has to be set at the lowest possible value. Since $\beta_2 = 0$ violates Condition (3), Condition (3) is binding, or

$$\beta_2 = \left(\frac{2 - c^2 + c}{2 - c^2} \right)^2 \beta_1.$$

Now, replace this value in the objective and optimize with respect to β_1 . Tedious but

straightforward computations show that the candidate probabilities of intervention are:

$$\begin{cases} \beta_1^{ss} &= \frac{4(1-c)^2(8+4c-7c^2-2c^3+2c^4)}{(2-c)^2(12-4c-21c^2+7c^3+9c^4+4c^5)^2\bar{F}} \left(a - \underline{\theta} - \Delta\theta\frac{K}{\bar{F}} \right)^2, \\ \beta_2^{ss} &= \frac{4(1-c)^2(1+c)^2(8+4c-7c^2-2c^3+2c^4)}{(2-c)^2(12-4c-21c^2+7c^3+9c^4+4c^5)^2\bar{F}} \left(a - \underline{\theta} - \Delta\theta\frac{K}{\bar{F}} \right)^2. \end{cases}$$

It remains to check that the second-order condition is satisfied in equilibrium is also satisfied. This amounts to

$$\frac{\partial^2}{\partial\beta_1^2}\mathbf{E}_\theta\{SW^{ss}\} \leq 0 \Leftrightarrow -\frac{(2-c)(12-4c-21c^2+7c^3+9c^4-4c^5)}{4(1-c)(2-c^2)^2} \frac{\bar{F}^2}{\sqrt{\beta_1\bar{F}}} \leq 0,$$

which obviously holds.

A.4 Proof of Corollary 1

First, it is trivial to show that $\beta_i^{ws}|_{c=c'} = \beta_i^{ss}|_{c=c'}$, that is, that the probabilities of intervention are continuous in c' .

Let us look now at the slopes of those probabilities in c' . We have, first,

$$\frac{\partial\beta_1^{ss}}{\partial c}|_{c=c'} \leq \frac{\partial\beta_1^{ws}}{\partial c}|_{c=c'} \leq 0,$$

and, second,

$$\frac{\partial\beta_1^{ws}}{\partial c}|_{c=c'} \geq 0 \geq \frac{\partial\beta_1^{ss}}{\partial c}|_{c=c'}.$$

A.5 Proof of Proposition 4

With demand complements, we have $\tilde{\theta}_i \leq \hat{\theta}_i$. Therefore, and contrary to the case of substitutes, this is now the position of $\hat{\theta}_1$ with respect to $\tilde{\theta}_2$ that may impact the different zones. Let us therefore introduce the following condition:

$$\tilde{\theta}_2 \geq \hat{\theta}_1 \Leftrightarrow (2-c^2+c)\sqrt{\beta_2} \leq (2-c^2)\sqrt{\beta_1}. \quad (5)$$

Assume first that Condition (5) holds. Then, three zones appear: (i) For $\theta \in [\underline{\theta}, \tilde{\theta}_1]$, each industry decides to collude. (ii) For $\theta \in [\tilde{\theta}_1, \hat{\theta}_2]$, industry 1 behaves competitively and industry 2 colludes. (iii) For $\theta \in [\hat{\theta}_2, \bar{\theta}]$, each industry decides to compete.

This situation is similar to the case of weak substitutes. Provided that the second-order condition with respect to β_2 is satisfied (i.e., $|c| \leq \frac{\sqrt{3}}{2}$), when Condition (5) is not taken into account, the optimal probabilities of intervention are given by $\{\beta_1^{ws}, \beta_2^{ws}\}$.

However, direct computations show that for $c \in [0, \frac{\sqrt{3}}{2}]$

$$\beta_1^{ws} - \beta_2^{ws} > 0 \Leftrightarrow -20c - 10c^2 + 41c^3 + 13c^4 - 27c^5 - 4c^6 + 6c^7 > 0,$$

which never holds.

Thus, we must consider the possibility that $\beta_2 = \beta_1$. But, when goods are complements, this contradicts Condition (5).

Consequently, let us assume that

$$\hat{\theta}_1 \geq \tilde{\theta}_2 \Leftrightarrow (2 - c^2 + c) \sqrt{\beta_2} \geq (2 - c^2) \sqrt{\beta_1}. \quad (6)$$

With respect to the previous case, the zone $[\tilde{\theta}_1, \hat{\theta}_2]$ has to be modified as follows: (i) For $\theta \in [\tilde{\theta}_1, \tilde{\theta}_2]$, industry 1 competes and industry 2 colludes. (ii) For $\theta \in [\tilde{\theta}_2, \hat{\theta}_1]$, industry i prefers to collude when industry j competes, and prefers to compete when industry j colludes. Thus, two equilibria appears: (nc, c) or (c, nc) . (iii) For $\theta \in [\hat{\theta}_1, \hat{\theta}_2]$, industry 1 competes and industry 2 colludes.

The choice (nc, c) for the equilibrium in the region $[\tilde{\theta}_2, \hat{\theta}_1]$ simplifies the analysis since the regions of the different equilibria $((c, c), (nc, c)$ and $(nc, nc))$ do no longer depend on the position of $\hat{\theta}_1$ with respect to $\tilde{\theta}_2$. Hence, with complementary goods, the problem of the competition policy authority has again the same objective as in \mathcal{P}^{ws} .

But, we know now that, with complements, this problem leads to $\beta_1^{ws} \leq \beta_2^{ws}$, which contradicts Condition (6). Therefore, in equilibrium it must be the case that $\beta_1^{ws} = \beta_2^{ws}$.

A.6 Proof of Proposition 5

From the proof of Proposition 4, we know that the problem of the competition policy authority is similar to \mathcal{P}^{ws} with the additional requirement that $\beta_1 = \beta_2 \equiv \beta^c$.

Taking into account this consideration and optimizing the objective with respect to the unique probability, we obtain:

$$\beta^c = \frac{64(1-c)^2(1+c)^4(2-c^2)^4}{(48+20c-126c^2-41c^3+119c^4+27c^5-50c^6-6c^7+8c^8)\bar{F}} \left(a - \underline{\theta} - \Delta\theta \frac{K}{\bar{F}} \right)^2.$$

Now, we must check that the second-order condition is satisfied, or (after simple computations)

$$\frac{\partial^2}{\partial(\beta^c)^2} \mathbf{E}_\theta \{SW^{ws}\} \leq 0 \Leftrightarrow$$

$$P_2(c) = 48 + 20c - 126c^2 - 41c^3 + 119c^4 + 27c^5 - 50c^6 - 6c^7 + 8c^8 \geq 0.$$

The polynomial expression $P_2(\cdot)$ admits only one root c''' that belongs to $(0, 1)$ and, thus, is positive for $c \leq c'''$ and negative otherwise.

Let us now consider the case of strong complements, or $c \geq c'''$. In this case, the objective of the competition policy authority is a convex function of the common probability of intervention β^c . Moreover, we also have

$$\frac{\partial}{\partial \beta^c} \mathbf{E}_\theta \{SW^{ws}\} |_{\beta^c=0} \propto a - \underline{\theta} - \Delta\theta \frac{K}{F} > 0.$$

Thus, the objective is a strictly increasing function of the common probability of intervention β^c : the optimal solution is therefore the corner solution $\beta^c = 1$.

A.7 Proof of Proposition 8

In this setting, the profits are given by (the first superscript refers to the behavior of the upstream industry and the second to the downstream one):

1. If both industries cooperate, then $\pi_u^{c,c}(\theta) = \frac{1}{2}[\frac{a-\theta}{2}]^2$ and $\pi_d^{c,c}(\theta) = [\frac{a-\theta}{4}]^2$. The quantity produced by the downstream industry in that case is $q_d^{c,c}(\theta) = \frac{a-\theta}{4}$.
2. If the upstream cooperates whereas the downstream one does not, then $\pi_u^{c,nc}(\theta) = [\frac{a-\theta}{2}]^2$ and $\pi_d^{c,nc}(\theta) = 0$. The production of the downstream industry is $q_d^{c,nc}(\theta) = \frac{a-\theta}{2}$.
3. If the upstream does not cooperate whereas the downstream one does, then $\pi_u^{nc,c}(\theta) = 0$ and $\pi_d^{nc,c}(\theta) = [\frac{a-\theta}{2}]^2$. The production of the downstream industry is $q_d^{nc,c}(\theta) = \frac{a-\theta}{2}$.
4. Finally, if none of the industries cooperate, then $\pi_u^{nc,nc}(\theta) = 0$ and $\pi_d^{nc,nc}(\theta) = 0$. The production of the downstream industry is $q_d^{nc,nc}(\theta) = a - \theta$.

As previously, for a given antitrust intervention policy $(\beta_u, \beta_d, \bar{F})$, we must define the following thresholds to study the industries' behavior: $\tilde{\theta}_u = a - 2\sqrt{2\beta_u\bar{F}} \leq \hat{\theta}_u = a - 2\sqrt{\beta_u\bar{F}}$ and $\tilde{\theta}_d = a - 4\sqrt{\beta_d\bar{F}} \leq \hat{\theta}_d = a - 2\sqrt{\beta_d\bar{F}}$. Notice that for all β_i , $\tilde{\theta}_i \leq \hat{\theta}_i$, $i = u, d$.

Now, consider first that $\beta_d \geq \beta_u$ (case 1). This implies that $\hat{\theta}_u \geq \hat{\theta}_d$, $\tilde{\theta}_u \geq \tilde{\theta}_d$ and $\hat{\theta}_u \geq \tilde{\theta}_d$. Two subcases have to be considered: if $\beta_d \geq 2\beta_u$ then $\tilde{\theta}_u \geq \hat{\theta}_d$ (case 1.1); otherwise $\tilde{\theta}_u \leq \hat{\theta}_d$ (case 1.2):

- *Case 1.1:* It is immediate to see that for $\theta \in [\underline{\theta}, \tilde{\theta}_d]$ both industries collude; for $\theta \in [\tilde{\theta}_d, \hat{\theta}_u]$ the upstream industry colludes and the downstream one does not; for $\theta \in [\hat{\theta}_u, \bar{\theta}]$ both industries compete.
- *Case 1.2:* It is immediate to see that for $\theta \in [\underline{\theta}, \tilde{\theta}_d]$ both industries collude; for $\theta \in [\tilde{\theta}_d, \tilde{\theta}_u] \cup [\hat{\theta}_d, \hat{\theta}_u]$ the upstream industry colludes and the downstream one does not; for

$\theta \in [\hat{\theta}_u, \bar{\theta}]$ both industries compete; for $\theta \in [\tilde{\theta}_u, \hat{\theta}_d]$ either the upstream industry colludes and the downstream competes or the upstream competes and the downstream colludes: we choose the former equilibrium. In that case, case 1.2 reduces to case 1.1.

Thus, unconstrained maximization of the associated welfare leads to probabilities of intervention that violate the initial condition $\beta_d \geq \beta_u$; this constraint must therefore be binding. Optimizing then leads to

$$\beta_d = \beta_u = \beta_1^p = \frac{16}{81\bar{F}} \left(a - \underline{\theta} - \frac{K}{\bar{F}} \Delta\theta \right)^2.$$

Notice that even when $\beta_d = \beta_u$, $[\hat{\theta}_u, \tilde{\theta}_d]$ is non-empty and, in this region, the upstream industry colludes and the downstream industry competes.

Now consider that $\beta_d \leq \beta_u$ (case 2). This implies that $\hat{\theta}_u \leq \hat{\theta}_d$ and $\tilde{\theta}_u \leq \hat{\theta}_d$. We have to distinguish three subcases: if $\beta_d \leq \frac{1}{4}\beta_u$ then $\hat{\theta}_u \leq \tilde{\theta}_d$ and $\tilde{\theta}_u \leq \tilde{\theta}_d$ (case 2.1); if $\frac{1}{2}\beta_u \geq \beta_d \geq \frac{1}{4}\beta_u$ then $\hat{\theta}_u \geq \tilde{\theta}_d$ and $\tilde{\theta}_u \leq \tilde{\theta}_d$ (case 2.2); if $\beta_u \geq \beta_d \geq \frac{1}{2}\beta_u$ then $\hat{\theta}_u \geq \tilde{\theta}_d$ and $\tilde{\theta}_u \geq \tilde{\theta}_d$ (case 2.3).

Let us describe the first two subcases:

- *Case 2.1:* It is immediate to see that for $\theta \in [\underline{\theta}, \tilde{\theta}_u]$ both industries collude; for $\theta \in [\tilde{\theta}_u, \hat{\theta}_d]$ the upstream industry competes and the downstream one colludes; for $\theta \in [\hat{\theta}_d, \bar{\theta}]$ both industries compete.
- *Case 2.2:* It is immediate to see that for $\theta \in [\underline{\theta}, \tilde{\theta}_u]$ both industries collude; for $\theta \in [\tilde{\theta}_u, \tilde{\theta}_d] \cup [\hat{\theta}_u, \hat{\theta}_d]$ the upstream industry competes and the downstream one colludes; for $\theta \in [\hat{\theta}_d, \bar{\theta}]$ both industries compete; finally, for $\theta \in [\tilde{\theta}_d, \hat{\theta}_u]$ either the upstream industry does not collude and the downstream one does or the upstream industry colludes and the downstream one does not: we choose the former equilibrium.

Thus, subcase 2.2 reduces to subcase 2.1. Optimizing the corresponding social welfare yields probabilities of intervention that violates the constraint $\beta_d \leq \beta_u$, which must therefore be binding in equilibrium.

Imposing that $\beta_d = \beta_u$ and optimizing the corresponding associated welfare, we obtain

$$\beta_d = \beta_u = \beta_2^p = \frac{32(11 - 6\sqrt{2})}{441\bar{F}} \left(a - \underline{\theta} - \frac{K}{\bar{F}} \Delta\theta \right)^2.$$

Notice that even when $\beta_d = \beta_u$, $[\hat{\theta}_u, \tilde{\theta}_d]$ is non-empty and, in this region, the upstream industry competes and the downstream industry colludes.

Now consider subcase 2.3. In that case, it is immediate to check that the relevant regions are as follows: for $\theta \in [\underline{\theta}, \tilde{\theta}_d]$, both industries collude; for $\theta \in [\tilde{\theta}_d, \hat{\theta}_u]$, the upstream industry colludes and the downstream one competes; for $\theta \in [\tilde{\theta}_u, \hat{\theta}_u]$ two equilibria emerge in which one

industry colludes and the other competes; for $\theta \in [\hat{\theta}_u, \hat{\theta}_d]$ the upstream industry competes and the downstream one colludes; for $\theta \in [\hat{\theta}_d, \bar{\theta}]$ both industries compete.

Whatever the choice of equilibrium for the $[\hat{\theta}_u, \hat{\theta}_u]$ -region, the relevant regions for consumers' net surplus are $[\underline{\theta}, \hat{\theta}_d]$, $[\hat{\theta}_d, \hat{\theta}_d]$ and $[\hat{\theta}_d, \bar{\theta}]$. These regions only depend on β_d and not on β_u .

Now consider that for the $[\tilde{\theta}_u, \hat{\theta}_u]$ -region we choose the equilibrium in which the upstream industry colludes and the downstream one competes. The (unconstrained) maximization of the corresponding welfare with respect to β_u leads to the following first-order condition

$$\frac{\partial \hat{\theta}_u}{\partial \beta_u} (\beta_u - \beta_d) \bar{F} + (\hat{\theta}_u - \underline{\theta}) \bar{F} - \Delta \theta K < 0,$$

since β_u must be greater than β_d and $K \geq \bar{F}$. Therefore, we must have $\beta_u = \beta_d$. But then, subcase 2.3 reduces to case 1. A similar reasoning can be made if the other equilibrium for the $[\tilde{\theta}_u, \hat{\theta}_u]$ -region is selected.

Finally, let $\mathbf{E}_\theta\{SW_i^p\}$ be expected welfare when the probability of intervention on both industries is equal to β_i^p , $i = 1, 2$. Tedious, but straightforward, computations yield

$$\mathbf{E}_\theta\{SW_1^p\} - \mathbf{E}_\theta\{SW_2^p\} = \Gamma \left(a - \underline{\theta} - \Delta \theta \frac{K}{\bar{F}} \right)^2,$$

where $\Gamma = \frac{32}{83349\bar{F}}(-3815 + 2268\sqrt{2} - (252 + 432\sqrt{2})\sqrt{22 - 12\sqrt{2}} + 1620\sqrt{11 - 6\sqrt{2}}) > 0$.

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