Access Pricing for Interconnected Vertically Separated Industries*  

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Abstract  

We study the interaction between railroad infrastructure managers in charge of pricing the access to their networks. The infrastructures are used by downstream firms to provide two types of service: domestic and international. The latter requires the use of both networks. Each infrastructure manager must ensure the financial viability of his own network.  

We study the Nash equilibrium of the game played by noncooperative infrastructure managers and characterize the strategic interaction between their access pricing decisions. Then, we allow infrastructure managers (or their political principal) to choose to finance the infrastructure’s common costs either through a subsidy or solely through user charges. We show that an infrastructure manager sometimes has an incentive to adopt the budget-balance system in order to free-ride on the access prices imposed by the other infrastructure manager.  

Keyword: Ramsey Pricing, Interconnected Infrastructures, Financing System.  

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1 Introduction

The railway sector in Europe is experiencing a significant reorganization process following the application of EC Directive 91/440. The vertical separation of network management from transport service provision prescribed by the Directive requires a clear definition of the terms of access to railroad infrastructures for downstream transport operators.

The application of the Directive in European Member States has followed different paths, reflecting the heterogeneous nature of the railway networks in the various countries as well as significant differences in pre-existing market structures. In particular, the pricing schemes for infrastructure access vary across countries with respect to the level of infrastructure costs' coverage by users. For instance, the French charging system has enabled RFF to cover about 25% of its total cost, while the percentage is 40% for SCHIG in Austria. On the other hand, the access pricing system implemented by NETZ in Germany has been set with the aim of recovering all costs, excluding those related to new or enhanced infrastructure. Therefore, the role played by infrastructure access pricing can also be markedly dissimilar depending on the objective of the infrastructure manager (or the one of his political principal), or, more generally, on the choice of the mode of regulation.

In view of these differences, and given the Directive's objectives of promoting intra-European traffic, particular attention should be devoted to infrastructure access pricing for international traffic between bordering countries. A proposal for a Council Directive on railway capacity assignment upholds that dissimilar objectives of the infrastructure managers and consequently varying charging systems, lacking acknowledgement of the different markets in which freight services operate and thus inability to grant its international competitiveness require coordination on the side of infrastructure managers in order to avoid heavy impacts on service efficiency and market share.

The aim of this paper is to study the interaction between infrastructure managers in charge of pricing the access to their infrastructures, which are exploited by downstream firms to provide transport services. Our stylized model considers two bordering countries. There are two types of transport services with independent demands, namely domestic and international services. The latter require the use of the railroad infrastructures of both countries. Infrastructure managers maximize national welfare while financing their network deficit.

We argue that a basic distinction should be made according to the type of cost recovery principle adopted. In fact, the fraction of network costs which is not covered by access charges could be funded through taxes levied on the economy as a whole: This kind of approach is named 'taxpayers-pay' financing system. Alternatively, access charges imposed on railway users could be meant to recover the total infrastructure cost: This approach is called 'users-pay' financing system. The main difference is that under the taxpayers-pay financing system the cost of the infras-
ture deficit is evaluated at the shadow cost of public funds, whereas under a users-pay financing system it is evaluated at the shadow cost of the budget-balance constraint. Throughout the paper, we will assume that the shadow cost of public funds is smaller than the shadow cost of the budget constraint under a users-pay financing system because of the presence of large fixed costs.

If the infrastructure managers were perfectly cooperating, the optimal access prices would obey standard Ramsey-Boiteux formulas. Moreover, the taxpayers-pay system is socially preferred to the users-pay one since it provides one additional instrument (the subsidy) to the perfectly cooperating infrastructure managers.

However, in an open economy with interconnected infrastructures, the two financing systems entail access charges which, though similar in their Ramsey-Boiteux structure, can differ both in level and strategic nature.

Our results are driven by two basic effects. First, the infrastructure manager of each country will only internalize the fraction of social benefits deriving from international services that accrues to his own consumers. Hence, the international service creates a negative externality on access tariffs: This is the constituency effect. Second, the ‘perceived’ infrastructure cost of the international service for an infrastructure manager differs from the total infrastructure cost of this service because each infrastructure manager is concerned with the financial viability of his network only: This is the double marginalization effect. These two effects lead to excessive access prices for the international service.

Let us first mention that with noncooperative infrastructure managers, from the point of view of (total) social welfare, the taxpayers-pay system is even more preferred to the users-pay one. Indeed, under a users-pay system in both countries, since noncooperative access charges are too high, the sum of the shadow costs of the budget constraint in each country is larger than the shadow cost of the budget constraint under perfect cooperation. Therefore, the provision of both the domestic and international services is affected. Consequently, it would be optimal that both infrastructure managers decided simultaneously to finance the infrastructure deficit through a subsidy.

On this basis, we study the strategic choice of the mode of regulation and analyze the individual incentives of the infrastructure managers to adopt one of the two financing systems before setting their access prices. The decision to adopt a particular financing system triggers two effects. First, the change in access prices in a country has a direct effect on the welfare in this country. Second, the change in the international access price in one country entails a modification of the access prices set in the other country; this indirect effect depends on the strategic interaction between infrastructure managers.

A first and obvious result is that, when the infrastructure deficit is sufficiently large, each infrastructure manager prefers the taxpayers-pay system since the access charges needed to balance the budget without subsidy are too high, leading to too large a loss of consumers’ surplus. In this case, the direct effect is more important.
than the indirect strategic effect.

However, an infrastructure manager can sometimes have an incentive to adopt the socially sub-optimal users-pay system. Indeed, consider that, say, country $i$ decides to adopt the users-pay financing system instead of the taxpayers-pay one. For fixed foreign access charges, the taxpayers-pay system is always preferred. Therefore, in this case the direct effect is negative and adopting the users-pay system leads to higher access charges in country $i$.

Under the strategic substitutability property, when country $i$ increases its access price for the international service, the infrastructure manager in country $j$ is led to decrease his international access charge. This in turn has the following effects: It tends to alleviate the increase in the price of the international service (which benefits to country $i$'s consumers) and to decrease the infrastructure deficit in country $i$ as well as the shadow cost of the budget-balance constraint in this country. The indirect strategic effect is positive when access charges are strategic substitutes. If the infrastructure deficit is initially small, then the positive indirect strategic effect might offset the negative direct effect and the infrastructure manager in country $i$ might prefer to adopt the users-pay financing system because the loss in consumers' surplus might be more than offset by the reduction in the subsidy required to ensure the viability of the infrastructure under a taxpayers-pay system. By contrast, with strategic complements, the indirect strategic effect related to the adoption of a users-pay system is negative. Both effects are then negative and provide each infrastructure manager with the incentive to finance the infrastructure deficit with a subsidy. For two polar cases, we show that depending on (i) the strategic interaction between international access charges, (ii) the level of infrastructure deficit and (iii) the difference between the domestic and the international demands, the incentives to adopt a particular financing system are radically different.

Our paper borrows from two distinct economic literatures. First, we use the extensive works on regulation under a budget constraint, pioneered by Boiteux (1956) and Ramsey (1927) in a different context. This literature has recently been extended to the telecommunication industry, the focus being on the regulation of a vertically integrated industry in which a dominant firm controlling a bottleneck is required to provide interconnection to entrants competing in a complementary segment. For an extensive account of this literature, see Laffont and Tirole (1999). Chang (1996) studies the problem of pricing the access in a vertically separated industry but does not consider the issue of interconnection between infrastructures. Laffont, Rey and Tirole (1998a,b) study the negotiation of access agreements between two networks that need interconnection. Our work differs since we are considering vertically separated industries and our focus concerns more the choice of the mode of regulation. Our model also borrows from the insights obtained by the strategic trade literature, initiated by the seminal paper by Brander and Spencer (1985), in which governments
seek to provide their domestic firms with a strategic advantage\(^1\).

The outline of the paper is as follows. In Section 2 we present the model. Then, in Section 3, we study the Ramsey-Boiteux benchmark in which all access pricing decisions are coordinated across countries. This leads, in Section 4, to the determination of the access charges when infrastructure managers behave in a noncooperative way. The nature of the strategic interaction between infrastructure managers is also assessed. In Section 5, we introduce a two-stage game in which, first, infrastructure managers choose the financing system (taxpayers- or users-pay) and, second, determine their access charges; then, we study the equilibria of this game. Finally, Section 6 gathers some concluding remarks. All proofs are relegated to an Appendix.

2 The model

We consider two countries denoted by \(i = 1, 2\). In each country an infrastructure manager sets access charges, while downstream firms use the network to provide transport services to final consumers.

The final demand We assume that there are two types of demand, labeled as domestic and international.

Domestic demand corresponds to purely national transport services. Let \(q_i(p_i)\) represent the demand function for domestic services in country \(i\) (with \(\frac{dq_i}{dp_i} < 0\)) and \(S_i(q_i)\) the related net consumers’ surplus, with \(\frac{dS_i}{dp_i} = -q_i\). All benefits associated to this service accrue to the consumers of country \(i\) only.

Similarly, let \(q_* (p_*)\) be the international demand for transport services (with \(\frac{dq_*}{dp_*} < 0\)) and \(S_*(q_*)\) the related net total consumers’ surplus, that is, the net consumers’ surplus of both countries when a total quantity \(q_* (p_*)\) of international services is consumed at price \(p_*\). We then have \(\frac{dS_*}{dp_*} = -q_*\) and we assume that country \(i\) only internalizes a part \(\theta_i\) of this surplus. This hypothesis can be justified by assuming that \(q_*\) is the total level of round-trip demand for transport (for example, from Paris to Brussels and back to Paris), and \(\theta_i\) is the fraction of consumers of country \(i\) that originates this demand. Then \(\theta_1 + \theta_2 = 1\) and the surplus from international services accruing to country \(i\) amounts to \(\theta_i; S_*(q_*)^2\).

\(^1\)Usually, this literature assumes that firms compete in a third market, implying that governments only care about the domestic firm’s profit and subsidy.

\(^2\)Other interpretations could easily be thought of. For example, let \(\theta_{ij}\) be the fraction of consumers having a demand for transport from \(i\) to \(j\) that belongs to country \(i\) and \(q_{ij}^i\) the related demand. For \(i, j = 1, 2\) and \(i \neq j\), we have that \(\theta_{ij} + \theta_{ji} = 1\), while \(q_{ij} = q_{ij}^i + q_{ij}^j\) is the total demand for international transport from country \(i\) to country \(j\). Thus, we are able to define the (net) surplus \(S_{ij}(q_{ij})\) related to the demand for international transport. Under the assumption of an isotropic travel pattern \((q_{ij} = q_{ji})\) and with equal prices, we have \(S_*(q_*) = S_1(q_{11})\) and the surplus of consumers in country \(i\) related to international transport can be written as \(\theta_{ij} S_{ij}(q_{ij}) + \theta_{ji} S_{ji}(q_{ji}) = \theta_i; S_*(q_*)\), where \(\theta_i = \theta_{ij} + \theta_{ji}\) and \(q_{ij} = q_{ji} = q_*\).
The infrastructure managers Each infrastructure manager wants to maximize the welfare of his country, which is composed of three terms: (i) the net consumers’ surplus, (ii) the infrastructure deficit and (iii) the domestic downstream firms’ profits. Let us now describe the last two terms.

To simplify the exposition, we assume that international services travel in each country half of the total number of kilometers\(^3\). In the absence of subsidies, the profit of the infrastructure in country \(i\) is given by

\[
\pi_i^{\text{infra}} = (a_i - c_u)q_i + (a_{si} - c_u)q_s - k_i, \tag{1}
\]

where \(a_i\) and \(a_{si}\) are, respectively, the access charges for a unit of domestic and international transport demand, while \(c_u\) is the (constant) marginal cost of the infrastructure in both countries (we assume that it does not depend on the type of service) and \(k_i\) is the fixed cost of the network. Notice that for each infrastructure manager the perceived marginal cost for each unit of international demand is \(c_u\), whereas the total marginal cost is actually \(2c_u\).

We now discuss an important institutional feature, namely the possibility to use a subsidy to finance the infrastructure deficit. In what follows, we shall consider two possible financing systems:

- Under the ‘users-pay’ system, the infrastructure manager cannot directly subsidize the infrastructure, and access pricing alone must ensure that infrastructure access charges cover total (fixed and variable) costs. This case is labeled with a superscript ‘\(u\)’.

- In contrast, under the ‘taxpayers-pay’ system the infrastructure manager is allowed to finance the infrastructure through taxes levied on the rest of the economy. This case is labeled with a superscript ‘\(t\)’. Taxation is imperfect and has distortionary effects on the rest of the economy. In our partial equilibrium approach, we denote by \(\lambda_{pf}\) the shadow cost of public funds which captures this effect, and we assume that \(\lambda_{pf}\) is the same in both countries.

Denoting by \(\pi_i^d\) the profits of country \(i\)’s downstream firms, the program of the infrastructure manager in country \(i\) will be

\[
(P_{IM}^u) \left\{ \begin{array}{l}
\max_{\{a_i,a_{si}\}} \left\{ S_i(q_i) + \theta_i S_s(q_s) + \pi_i^{\text{infra}} + \pi_i^d \right\} \\
\text{subject to } \pi_i^{\text{infra}} \geq 0
\end{array} \right.
\]

under a users-pay system, and

\[
(P_{IM}^t) \left\{ \begin{array}{l}
\max_{\{t_i,a_i,a_{si}\}} \left\{ S_i(q_i) + \theta_i S_s(q_s) - (1 + \lambda_{pf})t_i + \left(t_i + \pi_i^{\text{infra}}\right) + \pi_i^d \right\} \\
\text{subject to } t_i + \pi_i^{\text{infra}} \geq 0
\end{array} \right.
\]

\(^3\)This entails no loss of generality with respect to the general case where the international traffic travels, say, in country 1 for a fraction \(\alpha\) of the total kilometers and in country 2 for a fraction \(1 - \alpha\).
under a taxpayers-pay system, where \( t_i \) is the subsidy to the infrastructure in country \( i \).

**The downstream firms’ behavior** Throughout the paper, we will assume that downstream firms in each sector, namely the two domestic and the international ones, behave competitively\(^4\). Thus, letting \( c_d \) be the constant marginal cost for these firms (we implicitly assume it is the same for both services)\(^5\), the price for the domestic service in country \( i \) will be

\[
p_i = a_i + c_d.
\]  

Since we consider round-trip travel, the resulting price in the market for international transport services will be given by

\[
p_s = a_{s1} + a_{s2} + c_d.
\]  

Under the assumption of downstream competitive behavior, downstream firms make no profit and the infrastructure budget constraint coincides with the industry budget constraint (that includes the infrastructure deficit as well as downstream firms’ profits).

### 3 Social optimum and the Ramsey-Boiteux principles for access pricing

As a preliminary to the forthcoming analysis, we consider the benchmark case where the infrastructure managers perfectly cooperate.

We first assume that the unique infrastructure manager adopts a taxpayers-pay financing system. The optimal access charges must therefore solve the following

\(^4\)The assumption of perfectly competitive behavior in the downstream segments significantly simplifies computations.

With downstream market power and linear access prices, there are conflicting forces in presence: on the one hand, an infrastructure manager wants to subsidize at the margin the downstream operators through low access charges to counter their incentive to under-produce and because he cares about the fraction of the downstream operators’ profit that accrues to his country; on the other hand, low access prices generate low access revenues. As concerns the international service, the tendency to subsidize the firm might be reduced because (i) each infrastructure manager cares about a fraction of the international surplus and (ii) only a fraction of the downstream operators’ profits might accrue to the consumers of his country.

If infrastructure managers can use two-part tariffs, then the fixed part can be used to capture the downstream operators’ profits. In this case, another coordination problem arises since, for the international service, both infrastructure managers might be tempted to try to capture the profit made by the downstream operators on this service.

\(^5\)We implicitly assume that the cost of the downstream firms does not depend on travel length. Our setting could be immediately extended to incorporate such considerations.
program

\[
\begin{align*}
\max_{\{t, a_1, a_2, a_\ast\}} & \quad \left\{ S_1(q_1) + S_2(q_2) + S_\ast(q_\ast) - (1 + \lambda_{p,f})t + \left( t + \pi_{1,fr} + \pi_{2,fr} \right) \right\} \\
\text{subject to} & \quad t + \pi_{1,fr} + \pi_{2,fr} \geq 0,
\end{align*}
\]

where \(a_\ast\) is the unique access charge imposed on the international service. The necessary first-order conditions to be satisfied in interior solutions yield the following optimal access pricing formulas:

\[
\frac{p_{R,i} - c}{p_{R,i}} = \frac{\lambda_{p,f}}{1 + \lambda_{p,f} \eta_i}, \quad (4)
\]

for domestic services in country \(i = 1, 2\) and

\[
\frac{p_{R,i} - c_\ast}{p_{R,i}} = \frac{\lambda_{p,f}}{1 + \lambda_{p,f} \eta_\ast}, \quad (5)
\]

where superscript ‘\(R\)’ stands for ‘Ramsey’; \(c = c_d + c_u\) and \(c_\ast = c_d + 2c_u\) are the social marginal costs of the domestic and international services respectively, and \(\eta = -\frac{\delta q}{\delta p}\) denotes the elasticity associated to demand \(q(p)\). These formulas exemplify the Ramsey-Boiteux recommendations for access pricing.

We would have obtained analogous formulas had we assumed that the (unique) infrastructure manager adopts a users-pay financing system. In this case, denoting by \(\tilde{\lambda}\) the shadow cost of the budget constraint, the optimal access charges are given by (4) and (5) in which \(\lambda_{p,f}\) is replaced by \(\tilde{\lambda}\). Notice that the shadow cost of the infrastructure is now endogenous.

4 The game between infrastructure managers

Let us now consider the situation in which the infrastructure managers act independently. We begin this section with the determination of access charges when both countries adopt the taxpayers-pay or the users-pay financing system, and compare the welfare under the two systems. Then, we determine the nature of the strategic interaction between the infrastructure managers.

4.1 Equilibrium access charges

We will always assume that parameters are such that we obtain interior solutions characterized by first-order conditions.

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\(^6\)With interdependent demands similar formulas are obtained except that elasticities are replaced by the so-called super-elasticities. See e.g. Laffont and Tirole (1999).

\(^7\)With cooperation between infrastructure managers, as soon as the total fixed cost is strictly positive, the budget-balance constraint is binding in equilibrium and \(\tilde{\lambda}\) is strictly positive.
Taxpayers-pay financing system The program of the infrastructure manager in country $i$ is then given by $(P^i_{1M_i})$. Standard computations yield the following proposition.

**Proposition 1** The equilibrium access charges under a taxpayers-pay financing system in both countries are characterized by

\[
\frac{p_i^{it} - c}{p_i^{it}} = \frac{\lambda_{pj} - 1}{\lambda_{pj} \eta_i},
\]

for the domestic service in country $i = 1, 2$ and

\[
\frac{a_i^{it} - c_u}{p_u^{it}} = \frac{1 + \lambda_{pj} - \theta_i - 1}{1 + \lambda_{pj} \eta_u},
\]

for the international service in country $i = 1, 2$.

A sufficient condition for local stability and uniqueness of the equilibrium is

\[
(S - \mathcal{U}) 3 \left( \frac{dq_s}{dp_s} \right)^2 - 2q_s \frac{d^2 q_s}{dp_s^2} > 0.
\]

The national access tariff is set at its Ramsey-Boiteux level and is therefore optimal. This is intuitive as the infrastructure manager in country $i$ entirely internalizes the surplus generated by this service. However, this also rests on the fact that access pricing in country $j$ does not affect the domestic service in country $i$ because (i) domestic demands are independent from the international one and (ii) the shadow cost of public funds, which gives the social cost of infrastructure financing, is exogenous.

From the perspective of the international demand, matters are different. First, since $\theta_i \in [0, 1]$, each infrastructure manager does not fully internalize the effect of his decision on total international consumers’ surplus. The international access charge in country $i$ will thus be excessive: This is the constituency effect.

The second effect that guides the pricing of access for the international service in country $i$ is due to the fact that each infrastructure manager does not account for the increase in deficit incurred by the other when he decides to increase his international access charge. With respect to the Ramsey-Boiteux benchmark, the equilibrium price will thus be excessive: This is the double marginalization effect.

In order to illustrate our main results, we will later employ specific demand functions. When they are linear or iso-elastic (with an elasticity parameter strictly greater than 1), Condition $(S - \mathcal{U})$ for local stability and uniqueness is always satisfied.

**Users-pay financing system** Usually, the taxpayers-pay system and the users-pay system are strongly analogous. However, there is a slight difference that needs to be mentioned.
Under a taxpayers-pay system, the fictitious cost of the budget-balance constraint is the shadow cost of public funds and is given exogenously. On the other hand, under a users-pay system, the cost of the budget constraint is endogenous and depends on the equilibrium configuration. However, we will always have $\lambda \leq \lambda_1 + \lambda_2$, where $\lambda_i$ is the Lagrange multiplier associated to the budget constraint in country $i$. Indeed, the budget-balance condition as well as the objective of the unique infrastructure manager under cooperation are respectively the sum of the budget constraints and the objectives of the noncooperative infrastructure managers.

Immediate computations show that under a users-pay financing system in both countries the final price is such that

$$\frac{p_{uu}^m - c_u}{p_{uu}^m} = \left( \frac{1 + \lambda_1 - \theta_1}{1 + \lambda_1} + \frac{1 + \lambda_2 - \theta_2}{1 + \lambda_2} \right) \frac{1}{\eta_u},$$

which is therefore larger than the final price under a taxpayers-pay system in both countries. Hence, under a users-pay financing system there is an additional distortion on international services. Moreover, this affects access pricing decisions for domestic services, even though domestic and international demands are independent$^8$.

### 4.2 Comparison of financing systems

From the point view of total social welfare, the taxpayers-pay system is preferred to the users-pay system since, beyond the fact that it provides the infrastructure manager with an additional instrument, the non-internalized externalities in the access pricing of international services do not affect domestic charges. Differently stated, given the access charges imposed in country $j$, the taxpayers-pay system dominates from the point of view of country $i$, and it does not create a negative externality on country $j$.

**Proposition 2** From the point of view of total welfare, the simultaneous adoption of the taxpayers-pay financing system is Pareto-superior to the simultaneous adoption of the users-pay financing system$^9$.

Let us now introduce another assumption that will be useful for the rest of the analysis.

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$^8$ We cannot compare $\lambda_i$ with $\lambda$. For instance, when $k_i = 0$, we have $\lambda_i = 0 < \lambda$. However, since $\lambda \leq \lambda_1 + \lambda_2$, the sum of the net consumers’ surplus associated to the domestic services is larger under cooperation than under noncooperation.

$^9$ Let us mention that, as noted in Laffont and Tirole (1999) in more general environments, the taxpayers-pay financing system might be ‘dangerous’ for the following reasons. First, it does not prevent to undertake an undesirable activity (which is financed through taxes on the whole economy). Second, it dilutes the incentives of consumers of the final services to act as watchdogs, that is, to monitor the infrastructure manager’s behavior.
**Assumption 1** In each country $\lambda_{pf} \leq \tilde{\lambda}_i$, where $\tilde{\lambda}_i$ is the Lagrange multiplier associated to the budget constraint of the infrastructure in country $i$.\(^{10}\)

Since in the users-pay system the Lagrange multiplier is endogenous, it is quite difficult to come up with a definitive comparison of the two financing systems. This explains the need for this assumption, which seems natural for the railway sector. First, this industry is characterized by the presence of large fixed costs. Second, if this assumption were not satisfied, then we should expect a railway infrastructure manager to finance (part of) the State expenses since raising funds from a distortion in the provision of railway services would have a lower social cost than distortionary taxation on the rest of the economy; this is far from being the case.

### 4.3 Strategic interaction between infrastructure managers

In this section we analyze the nature of the strategic interaction between the infrastructure managers. In particular, we want to determine how a change in the access prices set in one country (following, say, the adoption of a different mode of regulation) affects the access prices imposed in the other country. In our setting, since domestic demands are independent from the international one, this interaction derives from the access charges set on the international service.

**Taxpayers-pay financing system** In equilibrium, since transfers are socially costly, the budget constraint will be binding. Replacing the value of the transfer yields the following social welfare in country $i$

$$SW_i(a_{si}, a_{sj}) = S_i(q_i) + \theta_i S_u(q_u) - (1 + \lambda_{pf})[k_i - (a_i - c_u)q_i - (a_{si} - c_u)q_u].$$

The previous equation can be decomposed as the sum of a ‘profit’ term (the net consumers’ surplus, which is affected by the access pricing choice of the other infrastructure manager) and a ‘cost’ term (the infrastructure deficit, which is also affected by the actions undertaken by the other infrastructure manager). As in a standard IO setting, the sign of $\frac{\partial SW_i}{\partial a_{sj}}$ depends on the effect of a marginal variation of $a_{sj}$ on the marginal welfare in country $i$. Differentiating the welfare function of the infrastructure manager in country $i$ with respect to $a_{si}$ and $a_{sj}$ we get

$$\frac{\partial^2 SW_i}{\partial a_{si}\partial a_{sj}} = (1 + \lambda_{pf} - \theta_i) \frac{dq_u}{dp_u} + (1 + \lambda_{pf})(a_{si} - c_u) \frac{d^2 q_u}{dp_u^2}. \quad (8)$$

\(^{10}\)This condition implicitly requires that with noncooperative infrastructure managers the budget constraint under a users-pay system in each country is binding in equilibrium. This will be the case whatever the valuation for the international service in country $i$ if the maximal access revenue generated by the international service in country $i$ does not enable to recover the infrastructure fixed cost. In this case, even for a country which does not value the international service the budget constraint will be binding, implying that $\tilde{\lambda}_i > 0$. We make from now on this assumption.
Using the optimality condition on the international access charge in country \( i \) (see (7)), we can rearrange (8) to obtain:

\[
\frac{\partial^2 SW_i}{\partial a_{ai} \partial a_{aj}} \propto q_a \frac{d^2 q_a}{dp_a^2} - \left( \frac{dq_a}{dp_a} \right)^2
\]  

(9)

This enables us to state the following proposition.

**Proposition 3** Under a taxpayers-pay financing system, access charges are strategic substitutes (respectively strategic complements) if \( q_a \frac{d^2 q_a}{dp_a^2} - \left( \frac{dq_a}{dp_a} \right)^2 \leq 0 \) (respectively \( > 0 \)).

The condition on the strategic interaction relates to the log-concavity or log-convexity of the international demand. For instance, if the international demand is log-concave, then access charges are strategic substitutes. Although we do not rule out the case of strategic complements, we will favor the strategic substitutability assumption because (i) log-concavity is more economically appealing and (ii) this condition ensures that the stability-uniqueness condition is always satisfied in equilibrium.

With a concave or linear international demand function \( q_a \frac{d^2 q_a}{dp_a^2} \leq 0 \) access charges for the international service will be strategic substitutes whereas for an iso-elastic parameterization \( q_a = p_a^{-\eta} \), \( q_a > 1 \) they become strategic complements.

**Users-pay financing system** In this case, there is an additional difficulty, namely that the Lagrange multiplier associated to the budget-balance condition is endogenous.

However the following simple observation simplifies the analysis: If the access charge for the international service in country \( j \) increases, then the profit of the infrastructure in country \( i \) decreases, and the budget constraint becomes harder to satisfy. Hence, the Lagrange multiplier associated to the budget-balance condition in country \( i \) (which reflects the shadow cost associated to this constraint) is an increasing function of the access charge imposed on the international service in the other country.

Under a users-pay system in country \( i \), the cross-derivative of the social welfare function in country \( i \) can be rewritten as follows

\[
\frac{\partial^2 SW_i}{\partial a_{ai} \partial a_{aj}} = \frac{1 + \lambda_i - \theta_i}{-\frac{dq_a}{dp_a}} \left[ q_a \frac{d^2 q_a}{dp_a^2} - \left( \frac{dq_a}{dp_a} \right)^2 \right] + \frac{\partial \lambda_i}{\partial a_{aj}} \left[ q_a + (a_{ai} - c_u) \frac{dq_a}{dp_a} \right].
\]  

(10)

The second bracketed term corresponds to the marginal profit of the infrastructure in country \( i \) with respect to the international access charge \( a_{aj} \). The previous condition simply states that under a users-pay system, the strategic interaction must account
for the change in the fictitious cost of the budget constraint in country $i$. Moreover, using the optimality condition (7) we immediately see that in equilibrium

$$q_s + (a_{si} - c_u) \frac{dq_s}{dp_s} = \frac{\theta_i q_s}{1 + \lambda_i} \geq 0.$$ 

Hence, this implies that with respect to the taxpayers-pay system, access charges tend to be more strategic complements due to the effect on the fictitious cost of the budget constraint. Notice finally that under a users-pay system (weak) concavity of the international demand is no longer sufficient to obtain the strategic substitutability property.

This also has a consequence on the properties of the equilibrium with strategic substitutes. Indeed, in the Appendix we show that under a users-pay financing system in both countries, Condition ($S - U$) is also sufficient to guarantee that the equilibrium is unique and locally stable only when access charges are strategic complements. With strategic substitutability, conditions such that the equilibrium is unique or locally stable are much harder to find and are left to future research.

## 5 Strategic financing of the infrastructure deficit

In this section we study the individual incentives for infrastructure managers to choose one of the two financing systems we consider. We analyze the following two-stage game:

1. The infrastructure managers independently choose a financing system.
2. The infrastructure managers noncooperatively set access charges in their countries.

The outcome of the game will strongly depend on the strategic interaction between the international access charges.

### 5.1 The direct and the indirect strategic effects

Let us assume that country $i$ has to decide whether to adopt a taxpayers-pay or a users-pay financing system. There are two (sometimes conflicting) effects that guide this decision.

First, consider that foreign access charges are fixed. In this case, the infrastructure manager always prefers a taxpayers-pay system because it provides an additional instrument. This is the direct effect which provides each infrastructure manager with an incentive to adopt the taxpayers-pay instead of the users-pay system.

Notice also that under Assumption 1 the choice of a users-pay system in country $i$ leads to higher access charges in this country (with respect to the taxpayers-pay
system). Therefore, for fixed access prices in country $j$, the adoption of the users-pay system in country $i$ leads to an increase in the infrastructure revenue (since in the relevant domain $\frac{\partial}{\partial a_i} \pi^{infra}_i \geq 0$ and $\frac{\partial}{\partial a_j} \pi^{infra}_i \geq 0$) but to a decrease in the welfare of this country (because consumers' surplus is negatively affected).

The indirect effect depends on the nature of the strategic interaction between international access charges. If access charges are strategic substitutes, then the increase in $a_i$ triggers a decrease in $a_j$. This variation in the international access price in country $j$ has the following effects in country $i$: it increases the infrastructure revenue (since $\frac{\partial}{\partial a_i} \pi^{infra}_i \leq 0$), decreases the international price and therefore increases the surplus of consumers in country $i$, and decreases the shadow cost of the budget constraint in country $i$. When access charges are strategic substitutes, the indirect strategic effect is positive and makes the adoption of the users-pay system attractive for the infrastructure managers by creating an incentive to save on the subsidy bestowed on the infrastructure and to free-ride on the access prices set in the other country. On the other hand, when access charges are strategic complements the effects are reversed and the indirect effect becomes negative.

### 5.2 Equilibria of the two-stage game

We now want to determine the equilibria of the two-stage game presented above. In the following, the term ‘infrastructure deficit’ denotes the amount of subsidy needed to ensure the financial viability of the network under a taxpayers-pay system. Let us start with a first and somewhat obvious result.

**Proposition 4** For a sufficiently large infrastructure deficit, an infrastructure manager prefers to adopt the taxpayers-pay financing system.

The intuition behind this result is that the shadow cost of the budget constraint becomes large when the infrastructure deficit is large. In this case, the users-pay system entails too large distortions on the access charges and in the unique equilibrium of the two-stage game both infrastructure managers choose a taxpayers-pay financing system, which coincides with the Pareto-optimal outcome.

Since the Lagrange multiplier of the budget constraint is endogenous, it is difficult to determine the general conditions under which a given equilibrium can arise. However, we can obtain a good understanding of the infrastructure managers' incentives by focusing on two polar cases. First, when demands are linear ($q_i(p_i) = \alpha_i - \gamma_i p_i$, $i = 1, 2$ and $q_a(p_a) = \alpha_a - \gamma_a p_a$), implying that international access charges are strategic substitutes. Second, when demands are iso-elastic

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11 For instance, assume that parameters' values are such that it is impossible to balance the infrastructure deficit without a subsidy. The users-pay system yields a welfare equal to 0 and the taxpayers-pay system is always preferred.
\[ q_i(p_i) = p_i^{-\alpha_i}, i = 1, 2, q_*(p_*) = p_*^{-\alpha_*} \], so that international access charges are strategic complements. We shall assume that \( c_d = 0 \)\(^{12}\).

We also assume that the shadow cost of public funds is equal to \( 0 \)\(^{13}\). This implies that (under a taxpayers-pay system) all the access revenue is generated only by the international demand (because domestic access charges are equal to marginal infrastructure costs).

When the infrastructure subsidy is not too large, the equilibria of our two-stage game are markedly different depending on the strategic interaction between infrastructure managers.

**Proposition 5** Assume that demands are linear so that international access charges are strategic substitutes\(^{14}\). For a small infrastructure deficit in country \( i \):

- If country \( j \) adopts the taxpayers-pay financing system, then country \( i \) prefers to adopt the users-pay system.

- Assume that the infrastructure deficit in country \( j \) is also small. If country \( j \) adopts the users-pay system, then country \( i \) prefers to adopt the users-pay system if and only if demands are such that:

\[
\left( \frac{\alpha_j - \gamma_j \gamma_j}{\alpha_j} \right) > \left( \frac{\alpha_* - 2 \gamma_j \gamma_j}{4 \gamma_*} \right).
\]

The first part of the proposition exemplifies the incentive of the infrastructure manager in country \( i \) to free-ride on his rival: By adopting the users-pay system he increases the infrastructure revenue and triggers a decrease in the international access price in the other country. This reaction is favorable to country \( i \) as it further increases the infrastructure revenue and tends to alleviate the increase in the international price and the distortion on the domestic service in this country due to the adoption of the users-pay system. In this case, country \( i \) wins from the reduction in its infrastructure deficit whereas country \( j \) suffers from the increase in its subsidy. The indirect effect more than offsets the direct effect.

Let us now consider the impact of the valuations for the international service. In the Appendix, we show that the larger the valuation of country \( i \), the larger the incentives of the infrastructure manager to adopt the users-pay financing system. In fact, in our example, the optimality condition (7) for country \( j \) can be rewritten as

\[
a_{s,j} - c_u \equiv \lambda_{y_j} = 0 \left( 1 - \theta_j \right) \frac{\alpha_* - \gamma_* p_*}{\gamma_*}
\]

\(^{12}\)Given the symmetry of the model, this normalization is without loss of generality.

\(^{13}\)This assumption is only made for convenience. It could be relaxed but we would obtain much more complex expressions.

\(^{14}\)As explained in Section 4, the linearity of the international demand in general is not sufficient to have the strategic substitutability property under the users-pay system. In the numerical illustrations proposed in the Appendix one can immediately check that access charges are always strategic substitutes.
Therefore, at the margin, the smaller the valuation of country $j$, the larger the decrease in the international access price in country $j$ following the adoption of the user-pay system in country $i$ (since the larger was $a_{ij}$ initially). Since the two valuations add up to one, this entails that when country $i$ has a large valuation for the international service, it has a strong incentive to adopt the users-pay system, because the reduction in $a_{ij}$ will be large.

The second part of the proposition shows that when country $j$ adopts the users-pay system there is an additional effect: the increase in the international charge in country $i$ distorts the access pricing decisions in country $j$ through an effect on the shadow cost of the budget constraint in this country. This makes the infrastructure manager in country $j$ less willing to reduce his international charge, especially when he internalizes a large fraction of the international surplus, or when the domestic demand in his country is small with respect to the international one. These two effects combine at the equilibrium\textsuperscript{15}.

The proof of this proposition (and the following one) relies on the following observation. Assume that country $j$ has chosen the taxpayers-pay system. Then, there exists a value of the fixed cost of infrastructure in country $i$ such that the infrastructure deficit in this country is null. For this value of the fixed cost, denoted by $k_i$, the shadow cost of the budget-balance constraint in country $i$ is equal to the shadow cost of public funds and both financing systems yield the same welfare in country $i$. In a neighborhood of $k_i$, we can compute the derivative of the difference of country $i$'s welfare under a taxpayers-pay and a users-pay system. This gives the first part of the previous proposition.

However, when country $j$ adopts the users-pay system, the value of the fixed cost such that both financing systems are equivalent for country $i$ depends on the endogenous shadow costs of the budget constraint in country $j$ (which also depends on country $i$'s first period choice). By assuming that the infrastructure deficit in country $j$ is small, we can get rid of this dependency on endogenous variables (since the shadow costs in country $j$ are almost equal to the shadow cost of public funds). In this case, the infrastructure deficit in country $i$ will be null if the fixed cost of infrastructure in this country is equal to $k_i$. Repeating the same procedure, we obtain the second part of the previous proposition which accounts for the effects on the endogenous shadow costs in country $j$.

If no assumption were made on the infrastructure deficit in country $j$, then, for the second part of the proposition, we would have obtained a condition which depends on the endogenous shadow costs in country $j$ (and the value of the infrastructure's fixed cost such that both systems yield the same welfare in country $i$ would be dependent on those shadow costs and smaller than $k_i$).

Summarizing, in this example it is therefore possible that each infrastructure manager always tries to free-ride on his rival, ending up in an equilibrium in which

\textsuperscript{15}It appears to be difficult to clearly separate these effects.
both infrastructure managers choose the users-pay system. In the Appendix, we
give three numerical illustrations for each of the possible equilibrium configurations.
In particular, multiple (pure strategy) equilibria sometimes emerge\textsuperscript{16}: One country
chooses a users-pay system whereas the other country sticks to the taxpayers-pay
system because it would be too costly to adopt the users-pay system, and conversely.

Figure 1 illustrates the previous proposition in the case \( \frac{(a_i - c_i)\alpha_i}{c_i} > \frac{(a_i - 2c_i)\alpha_i^2\beta_i}{4c_i} \); if this condition is violated, then the first zone in which adopting the users-pay
system is a dominant strategy for country \( i \) disappears.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{Figure 1 here}
\end{figure}

With iso-elastic demands and the strategic complementarity property, the infra-
structure managers’ incentives are radically different.

**Proposition 6** Assume that demands are iso-elastic so that international access
charges are strategic complements. For small infrastructure deficits each infra-
structure manager prefers to adopt the taxpayers-pay system.

In this case, the strategic reaction of country \( j \) after country \( i \) adopts the users-pay
system affects negatively country \( i \)’s welfare. With respect to the linear demands
case, the logic of the argument is reversed since both the direct and the indirect
effects are negative. Under the strategic complementarity assumption, the unique
equilibrium of the two-stage game involves each infrastructure manager choosing a
taxpayers-pay financing system.

### 5.3 The creation of international corridors

The European Union strongly encourages the creation of corridors as a means to de-
velop international (especially freight) traffic (see EC White Paper “A Strategy for
Revitalising the Community’s Railways”). Let us note that our previous modeling
captures some of the elements inherent to corridor creation. Indeed, each infra-
structure manager will have to determine the access charge to be paid for the part of the
corridor that concerns his infrastructure. Then, the sum of these access charges will
determine the unique access charge for the use of the corridors. Therefore, if infras-
tructure managers do not succeed in reaching a high level of cooperation, we should
expect the international traffic to be excessively charged, due to the constituency and
the double marginalization effects and also possibly to socially sub-optimal choices
of the mode of regulation.

Indeed, the creation of the corridor between Germany, Netherlands, Switzerland,
Austria and Italy has not succeeded in developing freight traffic. According to Le
Monde (18/07/2000) the two main reasons for this failure are that (i) access charges

\textsuperscript{16}In this case, a mixed strategy equilibrium also exists.
paid in Germany are prohibitively high and (ii) Germany has refused to reserve tracks for this corridor.

The creation of the Belifret corridor (Belgium, Luxembourg, France, Italy and recently Spain) is somewhat more successful. However, another coordination problem arises due to the ‘pass-through’ nature of the services using this corridor: Some countries that are part of the corridor do not value the surplus associated to international services running on the corridor and therefore tend to lobby in favor of high access charges. This raises another issue, namely that of supra-national cooperation between national infrastructure managers.

6 Conclusions

In this paper, we have studied the interaction between railroad infrastructure managers when an international service requires the use of both infrastructures. We have isolated two effects: The constituency effect, related to the fact that each infrastructure manager only internalizes a fraction of the surplus generated by the international service, and the double marginalization effect, due to the difference between the infrastructure cost perceived by each infrastructure manager and the total infrastructure cost of the international service.

The interaction created by the international service can provide an infrastructure manager with an incentive to adopt a sub-optimal financing system in order to free-ride on the other country and decrease the amount of subsidy bestowed on the infrastructure.

We have remained silent on a number of questions.

For instance, our model implicitly assumes that networks are interconnected: Firms can always go from one country to the other. However, it has been argued that the development of the international traffic also suffers from a poor quality of interconnection. This is the so-called interoperability problem, which appears to be critical for the development of intra-European networks.

Another question concerns the investment decisions undertaken by the infrastructure managers. More specifically, the decisions to create or to close lines should be incorporated in our framework, and should not be neutral with respect to the strategic interaction between infrastructure managers or the possibility to choose a particular financing system.

Finally, our model has highlighted many coordination failures between national infrastructure managers. Future work should study the design of supra-national institutions or rules that enable to implement a certain level of cooperation. We leave these extensions to future research.

\footnote{For instance, the fixed cost of maintaining the line is much lower for an only-freight line because of lower safety standards.}
7 References


8 Appendix

Throughout this Appendix we omit arguments of functions for simplicity.

8.1 Social optimum and the Ramsey-Boiteux principles for access pricing

In equilibrium the budget-balance condition will be binding. This determines the value of the transfer. Optimizing with respect to the access charges and rearranging terms yields (4) and (5).

Using these first-order conditions the second-order conditions of the maximization problem will be satisfied if
\[ (1 + 2 \lambda_{p,f}) \left( \frac{d q_i}{dp_i} \right)^2 \geq \lambda_{p,f} \frac{d^2 q_i}{dp_i^2}, \]
i = 1, 2 and
\[ (1 + 2 \lambda_{p,f}) \left( \frac{d q_s}{dp_f} \right)^2 \geq \lambda_{p,f} \frac{d^2 q_s}{dp_f^2}. \]

In the case of an iso-elastic international demand \( q_s = p_s^{-\eta} \), the second-order condition amounts to \( \eta_s \geq \frac{1}{1 + \lambda_{p,f}}. \)

8.2 Equilibrium access charges

Taxpayers-pay in both countries In equilibrium the budget-balance condition will be binding in both countries. This defines the value of the transfer in each country. Then, optimizing with respect to the access charges and rearranging terms yields (6) and (7).

The second-order conditions will be satisfied if
\[ (SOC_i) \quad (1 + 2 \lambda_{p,f}) \left( \frac{d q_i}{dp_i} \right)^2 \geq \lambda_{p,f} \frac{d^2 q_i}{dp_i^2}, \]
\[ (SOC_{ai}) \quad (2 + 2 \lambda_{p,f} - \theta_i) \left( \frac{d q_s}{dp_s} \right)^2 \geq (1 + \lambda_{p,f} - \theta_i) \frac{d^2 q_s}{dp_s^2}. \]

We will always assume that these two conditions are satisfied whatever \( \theta_i \in [0, 1] \). This implies that an equilibrium exists.

The equilibrium will be locally stable if
\[ \frac{\partial^2 SW_i}{\partial a_i \partial a_j} \frac{\partial^2 SW_j}{\partial a_i \partial a_j} < \frac{\partial^2 SW_i}{\partial a_i^2} \frac{\partial^2 SW_j}{\partial a_j^2}. \]

Using the first-order condition with respect to \( a_{ai} \) we immediately obtain
\[ \frac{\partial^2 SW_i}{\partial a_i \partial a_j} = \frac{1}{\frac{d q_s}{dp_s}} \left( 1 + \lambda_{p,f} - \theta_i \right) \left( \frac{d q_s}{dp_s} \right)^2 - \frac{d^2 q_s}{dp_s^2}, \]
\[ \frac{\partial^2 SW_i}{\partial a_i^2} = \frac{1}{\frac{d q_s}{dp_s}} \left( 2 + 2 \lambda_{p,f} - \theta_i \right) \left( \frac{d q_s}{dp_s} \right)^2 - (1 + \lambda_{p,f} - \theta_i) \frac{d^2 q_s}{dp_s^2}. \]
for $i \neq j$. Direct computations show that the equilibrium will be locally stable whenever $\theta_i \in [0, 1]$ if

$$
(2 + 3\lambda_{pf}) \left( \frac{dq_a}{dp_a} \right)^2 - (1 + 2\lambda_{pf}) q_a \frac{d^2 q_a}{dp_a^2} > 0.
$$

(11)

A sufficient condition for the equilibrium to be unique is

$$
\left| \frac{\partial^2 SW_i}{\partial a^2_i} \right| > \left| \frac{\partial^2 SW_i}{\partial a_{ai} \partial a_{aj}} \right|.
$$

(12)

Using the first-order condition with respect to $a_{ai}$, this condition can be rewritten as follows

$$
\frac{2 + 2\lambda_{pf} - \theta_i}{1 + \lambda_{pf} - \theta_i} \left( \frac{dq_a}{dp_a} \right)^2 - q_a \frac{d^2 q_a}{dp_a^2} > \left( \frac{dq_a}{dp_a} \right)^2 - q_a \frac{d^2 q_a}{dp_a^2}.
$$

This condition is trivially satisfied when international access charges are strategic substitutes. Under the strategic complementarity property, using the second-order condition with respect to $a_{ai}$ ($SOC_{ai}$), the condition for uniqueness can be rewritten as follows

$$
(3 + 3\lambda_{pf} - 2\theta_i) \left( \frac{dq_a}{dp_a} \right)^2 > 2(1 + \lambda_{pf} - \theta_i) q_a \frac{d^2 q_a}{dp_a^2},
$$

which will be satisfied whenever $\theta_i \in [0, 1]$ if

$$
3 \left( \frac{dq_a}{dp_a} \right)^2 - 2 q_a \frac{d^2 q_a}{dp_a^2} > 0.
$$

Notice that the sufficient condition for uniqueness implies the local stability property.

In the case of an iso-elastic international demand $q_a = p_a^{-\eta_i}$, ($SOC_i$) amounts to $\eta_i \geq \frac{\lambda_{pf}}{1 + \lambda_{pf}}$, ($SOC_{ai}$) amounts to $\eta_a \geq \frac{1 + \lambda_{pf} - \theta_i}{1 + \lambda_{pf}}$, local stability amounts to $\eta_a > \frac{1 + 2\lambda_{pf}}{1 + \lambda_{pf}}$, uniqueness amounts to $\eta_a > 2$, existence (i.e. positivity of $a_{ai}$) amounts to $\eta_a \geq \frac{1 + 2\lambda_{pf}}{1 + \lambda_{pf}}$, the infrastructure profit in country $i$ will be a concave function of $a_{ai}$ if $\eta_a \geq 1$.

**Users-pay in both countries** In this case, we have to account for the fact that the Lagrange multiplier associated to the budget constraint in country $i$ depends on the access charge for the international service set in country $j$. Using the optimality condition for the international access charge, simple manipulations show that

$$
\frac{\partial^2 SW_i}{\partial a^2_i} = \frac{d^2 q_a}{dp_a^2} \left[ (2 + 2\lambda_i - \theta_i) \left( \frac{dq_a}{dp_a} \right)^2 - (1 + \lambda_i - \theta_i) q_a \frac{d^2 q_a}{dp_a^2} \right],
$$

(13)

$$
\frac{\partial^2 SW_i}{\partial a_{ai} \partial a_{aj}} = \left[ \left( \frac{dq_a}{dp_a} \right)^2 - q_a \frac{d^2 q_a}{dp_a^2} \right] + \frac{\partial \lambda_i}{\partial a_{aj}} \frac{\theta_i q_a}{1 + \lambda_i}.
$$

(14)
We have a partial result on the Lagrange multiplier.

**Lemma 1** The Lagrange multiplier $\tilde{\lambda}_i$ associated to the budget-constraint in country $i$ under a users-pay financing system is such that $\frac{\partial \tilde{\lambda}_i}{\partial a_{ij}} \geq 0$.

This is intuitive. *Ceteris paribus* an increase in $a_{ij}$ decreases the infrastructure revenue and therefore hardens the budget-balance constraint.

We look for a condition such that the sufficient condition (12) that ensure the uniqueness, and consequently the local stability, of the equilibrium is satisfied. Under the strategic complementarity property, we always have

$$\frac{\partial^2 SW_i}{\partial a_{si} \partial a_{sj}} \geq 0.$$  

Using the second-order condition ($SOC_{ai}$), this implies that (12) can be rewritten as follows

$$\frac{1}{d_{p_a}} \left[ (3 + 3\tilde{\lambda}_i - 2\theta_i) \left( \frac{dq_a}{dp_a} \right)^2 - 2(1 + \tilde{\lambda}_i - \theta_i)q_a \frac{d^2 q_a}{dp_a^2} \right] + \frac{\theta_i q_a}{1 + \tilde{\lambda}_i} \frac{\partial \tilde{\lambda}_i}{\partial a_{ij}} < 0.$$  

From Lemma 1, a sufficient condition to ensure that the last inequality is satisfied is

$$(3 + 3\tilde{\lambda}_i - 2\theta_i) \left( \frac{dq_a}{dp_a} \right)^2 - 2(1 + \tilde{\lambda}_i - \theta_i)q_a \frac{d^2 q_a}{dp_a^2} > 0.$$  

(15)

Condition (15) will be satisfied for all $\theta_i \in [0,1]$ if

$$3 \left( \frac{dq_a}{dp_a} \right)^2 - 2q_a \frac{d^2 q_a}{dp_a^2} > 0,$$

which is the same condition we found when both countries adopt the taxpayers-pay financing system.

Under the strategic substitutability property, we cannot conclude because we were not able to sign $\frac{\partial^2 SW_i}{\partial a_{si} \partial a_{sj}}$.

### 8.3 Strategic financing of the infrastructure deficit

We use the same methodology for the cases of linear and iso-elastic demands. We study the incentives for the infrastructure manager in country 1 to adopt the users-pay financing system instead of the taxpayers-pay one, given the financing system adopted in country 2. For each case (taxpayers-pay or users-pay in country 2), we start by identifying the conditions on the fixed cost such that the Lagrange multiplier associated to the budget-balance constraint in country 1 is equal to the shadow cost.
of public funds. Then we compute the difference in welfare in country 1 under a taxpayers-pay and a users-pay system. Finally, we study the derivative of this difference with respect to the fixed cost in country 1 when the Lagrange multipliers are close to the shadow cost of public funds. We will give a numerical illustration of every type of equilibrium we will exhibit. The discussion is couched in terms of fixed costs but only the infrastructure deficit is relevant.

Let us introduce the following notation: $\tilde{\lambda}_{id}$ is the Lagrange multiplier associated to the budget-balance constraint in country $i$ when country 1 adopts the users-pay system and country 2 adopts the taxpayers-pay one. A similar notation is used for country $j$. From the perspective of the first stage of the game, there are four possible states: $(u, u)$, $(u, t)$, $(t, u)$ and $(t, t)$. We assume that $\lambda_{pf} = 0$ and $c_d = 0$. Note that $\theta_1 + \theta_2 = 1$, so that the country 2's valuation for the international service is $1 - \theta_1$.

8.3.1 Strategic substitutability and linear demands

Let us assume that the domestic demand functions are given by $q_i(p_i) = \alpha_i - \gamma_i p_i$, $i = 1, 2$. The international demand is given by $q_s(p_s) = \alpha_s - \gamma_s p_s$.

Taxpayers-pay in country 2 If country 1 also adopts the taxpayers-pay system, then the optimal access charges are given by (6) and (7). Given that $\lambda_{pf} = 0$ the access charge for the domestic service in country 1 is equal to the marginal cost of the infrastructure $c_u$ and the profit of the infrastructure in country 1 can be rewritten as

$$\frac{(\alpha - 2c_u \gamma_u)^2}{4\gamma_u} (1 - \theta_1) - k_1.$$  

Hence, if the fixed cost of the infrastructure is equal to $\bar{k}_1 \equiv \frac{(\alpha - 2c_u \gamma_u)^2}{4\gamma_u} (1 - \theta_1)$, then the optimal access charges are such that the infrastructure breaks even without any subsidy. Accordingly, we have $\tilde{\lambda}_{id} = \lambda_{pf} = 0$ and the infrastructure manager in country 1 is indifferent between the two financing systems.

Let us denote by

$$\Delta SW^1_t (\tilde{\lambda}_{id}, k_1) \equiv SW^1_t (k_1) - SW^1_{ut} (\tilde{\lambda}_{id}, k_1)$$

the difference in welfare in country 1 under a taxpayers-pay system and a users-pay system when country 2 adopts the taxpayers-pay system. We have, after simple computations,

$$\frac{d\Delta SW^1_t}{dk_1} = \frac{\partial \Delta SW^1_t}{\partial k_1} + \frac{\partial \Delta SW^1_t}{\partial \tilde{\lambda}_{id}} \frac{\partial \tilde{\lambda}_{id}}{dk_1}$$

$$= -1 + (1 + \tilde{\lambda}_{id}) \left[ \frac{\omega_1}{(1 + 2\tilde{\lambda}_{id})^3} + \frac{\omega_2 \theta_1^2}{[2 + (2 + \theta_1)\tilde{\lambda}_{id}]^3} \right] \frac{\partial \tilde{\lambda}_{id}}{dk_1},$$

(17)
where \( \omega_i = \frac{a_i - c_i a_i}{\gamma_i} \) and \( \omega_s = \frac{a_s - 2c_s a_s}{\gamma_s} \).

When country 1 adopts the users-pay financing system, replacing the optimal access charges as function of the Lagrange multiplier, the budget-balance condition can be rewritten in equilibrium as follows

\[
\omega_1 \frac{\lambda_{1i}^1 (1 + \lambda_{1i}^1)}{(1 + 2\lambda_{1i}^1)^2} + \omega_s \frac{(1 + \lambda_{1i}^1)(1 + \lambda_{1i}^1 - \theta_1)}{2 + (2 + \theta_1)\lambda_{1i}^1} = k_1. \tag{18}
\]

Totally differentiating (18) and taking the limit when \( k_1 \) goes to \( k \) (implying that \( \lambda_{1i}^1 \) goes to \( \lambda_{pf} = 0 \)), we obtain

\[
\frac{\partial \lambda_{1i}^1}{\partial k_1} \bigg|_{k_1 = k} = \frac{1}{\omega_1 + \omega_s \theta_1^2 - 4}. \tag{19}
\]

Finally, replacing (19) in (17) we obtain

\[
\frac{d\Delta SW_1^1}{dk_1} \bigg|_{k_1 = k} \propto -\frac{\omega_s \theta_1^2}{8} < 0.
\]

This enables us to state the following lemma.

**Lemma 2** Assume that demands are linear, \( c_i = 0 \), \( \lambda_{pf} = 0 \) and that country 2 adopts the taxpayers-pay system. In a neighborhood of \( k_1 \) country 1 prefers to adopt the users-pay system than the taxpayers-pay one.

Finally, notice that to obtain this result we do not need to make any assumptions on the infrastructure deficit in country 2.

**Users-pay in country 2** There is now an additional difficulty. When both countries adopt the users-pay financing system, a variation in the fixed cost of the infrastructure in country 1 affects the Lagrange multipliers in both countries.

As previously, denote by

\[
\Delta SW_1^1 (\lambda_{1u}, \lambda_{1u}^2, \lambda_{1u}^3, k_1) \equiv SW_1^1 (\lambda_{1u}, k_1) - SW_1^1 (\lambda_{1u}, k_1) - SW_1^1 (\lambda_{1u}, \lambda_{1u}^2, k_1)
\]

the difference in welfare in country 1 under a taxpayers-pay and a users-pay financing system when country 2 adopts the users-pay system. We are interested in the sign of

\[
\frac{d\Delta SW_1^1}{dk_1} = \frac{\partial \Delta SW_1^1}{\partial k_1} + \left[ \frac{\partial \Delta SW_1^1}{\partial \lambda_{1u}^1} \frac{\partial \lambda_{1u}^1}{\partial k_1} + \frac{\partial \Delta SW_1^1}{\partial \lambda_{1u}^2} \frac{\partial \lambda_{1u}^2}{\partial k_1} + \frac{\partial \Delta SW_1^1}{\partial \lambda_{1u}^3} \frac{\partial \lambda_{1u}^3}{\partial k_1} \right]. \tag{20}
\]
If country 1 adopts the taxpayers-pay system then there will be no need to finance the infrastructure if

\[ k_1 = \bar{k}_1 (\bar{\lambda}_{tu}^2) = \omega_s \frac{(1 - \theta_1)(1 + \bar{\lambda}_{tu}^2)}{[2 + (3 - \theta_1)\bar{\lambda}_{tu}^2]^2}. \]

This condition depends now on \( \bar{\lambda}_{tu}^2 \), the endogenous multiplier associated to the budget-balance condition in country 2. The budget-balance condition in country 2 can be rewritten as follows

\[ \omega_2 \frac{\bar{\lambda}_{tu}^2(1 + \bar{\lambda}_{tu}^2)}{(1 + 2\bar{\lambda}_{tu}^2)^2} + \omega_s \frac{(1 + \bar{\lambda}_{tu}^2)(\bar{\lambda}_{tu}^2 + \theta_1)(1 + \bar{\lambda}_{tu}^2)}{[2 + (3 - \theta_1)\bar{\lambda}_{tu}^2 + \bar{\lambda}_{tu}^2(2 + 3\bar{\lambda}_{tu}^2 + \theta_1)]^2} = k_2. \]  
(21)

It is immediate to notice that when \( k_2 = \bar{k}_2 \equiv \frac{\omega_s \theta_1}{4} \) then \( \bar{\lambda}_{tu}^2 = 0 \) and \( \bar{k}_1(0) = \bar{k}_1 \).

From now on, we assume that \( k_2 \) is in a neighborhood of \( \bar{k}_2 \), which is equivalent to a small infrastructure deficit in country 2. Notice that this assumption is not necessary but simplifies dramatically the computations.

Moreover, this also implies that when \( k_2 = \bar{k}_2 \) if \( k_1 = \bar{k}_1 \) then country 1 does not need to subsidize the infrastructure and, as previously, we have \( \bar{\lambda}_{tu}^1 = \lambda_{p,f} = 0 \).

Finally, when both countries adopt the users-pay system the budget-balance condition in country 2 can be rewritten as follows

\[ \omega_2 \frac{\bar{\lambda}_{tu}^2(1 + \bar{\lambda}_{tu}^2)}{(1 + 2\bar{\lambda}_{tu}^2)^2} + \omega_s \frac{(1 + \bar{\lambda}_{tu}^2)(\bar{\lambda}_{tu}^2 + \theta_1)(1 + \bar{\lambda}_{tu}^2)}{[2 + (3 - \theta_1)\bar{\lambda}_{tu}^2 + \bar{\lambda}_{tu}^2(2 + 3\bar{\lambda}_{tu}^2 + \theta_1)]^2} = k_2. \]  
(21)

When \( k_1 \) goes to \( \bar{k}_1 \) and \( k_2 \) goes to \( \bar{k}_2 \), we already know that \( \bar{\lambda}_{tu}^2 \) goes to 0. This entails that \( \bar{\lambda}_{tu}^1 \) defined by (21) also goes to 0.

All these considerations will simplify the forthcoming computations. Indeed, totally differentiating (21) we get

\[ \frac{\partial \bar{\lambda}_{tu}^2}{\partial \bar{\lambda}_{tu}^1} |_{k_1 = \bar{k}_1} = \frac{\omega_s \theta_1^2}{4 \omega_2 + (1 - \theta_1)^2 \omega_s}. \]  
(22)

When country 1 also adopts the users-pay system, the budget-balance condition can be rewritten as

\[ \omega_1 \frac{\bar{\lambda}_{tu}^1(1 + \bar{\lambda}_{tu}^1)}{(1 + 2\bar{\lambda}_{tu}^1)^2} + \omega_s \frac{(1 + \bar{\lambda}_{tu}^1)(1 + \bar{\lambda}_{tu}^1 - \theta_1)(1 + \bar{\lambda}_{tu}^2)}{[2 + 3(3 - \theta_1)\bar{\lambda}_{tu}^2 + \bar{\lambda}_{tu}^1(2 + 3\bar{\lambda}_{tu}^2 + \theta_1)]^2} = k_1. \]  
(23)

Totally differentiating this condition, we obtain

\[ \frac{\partial \bar{\lambda}_{tu}^1}{\partial k_1} |_{k_1 = \bar{k}_1} = \frac{1}{\omega_1 + \frac{\omega_s \theta_1^2}{4} \frac{\partial^2 \bar{\lambda}_{tu}^1}{\partial \bar{\lambda}_{tu}^1^2} |_{k_1 = \bar{k}_1}}, \]  
(24)

\[ = \frac{1}{\omega_1 + \frac{\omega_s \theta_1^2}{4} \left[ 1 - \frac{\omega_s (1 - \theta_1)^2}{4 \omega_2 + \omega_s (1 - \theta_1)^2} \right]}. \]  
(25)
Given the previous considerations, and using (22) and (25), tedious but straightforward computations lead to

\[
\frac{d \Delta SW}{dk_1} |_{k_1 = k_2} \propto -4\omega_2 + (1 - \theta_1) \omega_s.
\]

We conclude with the following lemma.

**Lemma 3** Assume that demands are linear, \( c_d = 0 \), \( \lambda_{pf} = 0 \) and that country 2 adopts the users-pay system. For \( k_2 \) sufficiently close to \( \hat{k}_2 \), in a neighborhood of \( \hat{k}_1 \), country 1 prefers to adopt the users-pay system than the taxpayers-pay one if \( \omega_s \theta_2 - 4\omega_2 < 0 \) and conversely.

Finally, notice that whatever the parameters values we have \( k_1 \geq \hat{k}_1 (\hat{\lambda}_{ad}^2) \). Our results can be extended to situation in which the infrastructure deficit in country 2 is not close to 0. In this case, the condition stated in Proposition 3 will be different and will depend on \( \hat{\lambda}_{ad}^2 \) and \( \hat{\lambda}^2_{uu} \).

**Numerical illustrations** We propose three examples that illustrate the possible equilibria of the two-stage game. We always consider \( \lambda_{pf} = 0 \), \( \alpha_i = 2 \), \( \gamma_i = \gamma_s = 1 \), \( c_u = 1 \), \( \alpha_u \) such that \( \omega_u = 8\omega + \epsilon_1 \), \( k_i = \frac{8\omega}{\alpha_u} + \epsilon_2 \) and \( \theta_1 = \theta_2 = \frac{1}{2} \). Welfare are approximated to the fourth decimal.

- \( \epsilon_1 = -\frac{1}{2} \) and \( \epsilon_2 = 0.01 \). The matrix of social welfare is (country 1 chooses the row, country 2 chooses the column)

<table>
<thead>
<tr>
<th></th>
<th>( t )</th>
<th>( u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>(0.8288,0.8288)</td>
<td>(0.825,0.83)</td>
</tr>
<tr>
<td>( u )</td>
<td>(0.83,0.825)</td>
<td>(0.8253,0.8253)</td>
</tr>
</tbody>
</table>

In this first example, both infrastructure managers have the incentive to adopt the users-pay system whatever the choice in the other country. As a result the unique Nash equilibrium is \( (u, u) \).

- \( \epsilon_1 = +\frac{1}{2} \) and \( \epsilon_2 = 0.01 \). The matrix of social welfare is (country 1 chooses the row, country 2 chooses the column)

<table>
<thead>
<tr>
<th></th>
<th>( t )</th>
<th>( u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>(1.1824,1.1824)</td>
<td>(1.1762,1.1844)</td>
</tr>
<tr>
<td>( u )</td>
<td>(1.1844,1.1762)</td>
<td>(1.1752,1.1752)</td>
</tr>
</tbody>
</table>

In this second example, an infrastructure manager prefers to use the users-pay system when the other infrastructure manager adopts the taxpayers-pay system only. There exist then two pure-strategy asymmetric equilibria \( (u,t) \) and \( (t,u) \) and one mixed-strategy equilibrium.
\cdot \epsilon_1 = -2 \text{ and } \epsilon_2 = 0.15. \text{ The matrix of social welfare is (country 1 chooses the row, country 2 chooses the column)}

<table>
<thead>
<tr>
<th></th>
<th>t</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>(0.3928, 0.3928)</td>
<td>(0.3805, 0.3818)</td>
</tr>
<tr>
<td>u</td>
<td>(0.3818, 0.3850)</td>
<td>(0.3665, 0.3665)</td>
</tr>
</tbody>
</table>

Finally, in this case, both infrastructure managers prefer to adopt the taxpayers-pay financing system and the unique Nash equilibrium is \((t, t)\).

### 8.3.2 Strategic complementarity and iso-elastic demands

Let us assume that the domestic demand functions are given by \(q_i(p_i) = p_i^{-\eta_i}\), \(i = 1, 2\). The international demand is given by \(q_s(p_s) = p_s^{-\eta_s}\). To have all the optimality conditions satisfied, we assume that the elasticity parameters are strictly greater than 1.

The methodology is identical and we content ourselves with the presentation of the main results.

**Taxpayers-pay in country 2** Assume that country 1 adopts the users-pay system. If the fixed cost of the infrastructure in country 1 is such that

\[
k_1 = \tilde{k}_1 \equiv (1 - \theta_1) \frac{(\eta_s - 1) \eta_s^{-1} \eta_s}{(2c_n \eta_s)^{\eta_s-1} \eta_s},
\]

then one can check that \(\tilde{\lambda}_{1d} = 0\). Differentiating the budget-balance constraint in country 1 under a users-pay system, we obtain

\[
\frac{\partial \tilde{\lambda}_{1d}}{\partial k_1} \bigg|_{k_1 = \tilde{k}_1} = \frac{1}{\eta_s \eta_s^{-1} + \frac{1}{(2c_n \eta_s)^{\eta_s-1} \eta_s}}.
\]

After tedious but straightforward computations, we find that

\[
\frac{d \Delta SW_1}{dk_1} \bigg|_{k_1 = \tilde{k}_1} = \left[ \frac{\partial \Delta SW_1}{\partial k_1} + \frac{\partial \Delta SW_1}{\partial \tilde{\lambda}_{1d}} \frac{\partial \tilde{\lambda}_{1d}}{\partial k_1} \right] \bigg|_{k_1 = \tilde{k}_1}
\]

\[
\propto \frac{\theta_2^2 c_n (\eta_s - 1)^{\eta_s}}{2^{\eta_s-1} (c_n \eta_s)^{\eta_s} (\eta_s - 1)^2} > 0.
\]

This enables us to state the following lemma.

**Lemma 4** Assume that demands are iso-elastic, \(c_d = 0\), \(\lambda_{pf} = 0\) and that country 2 adopts the taxpayers-pay system. In a neighborhood of \(\tilde{k}_1\), country 1 prefers to adopt the taxpayers-pay system than the users-pay one.
Users-pay in country 2 First assume that country 1 adopts the taxpayers-pay financing system. Then one can check that when the fixed cost of the infrastructure in country 2 is such that

\[ k_2 = k_2 \equiv \theta_1 \left( \frac{(\eta_a - 1)^{\gamma_a - 1}}{(2c_u \eta_a)^{\gamma_a - 1} \eta_a} \right), \]

then \( \hat{\lambda}_{2u}^2 = 0 \). From now on, we assume that \( k_2 \) is in a neighborhood of \( k_2 \). Moreover, when \( k_1 = k_2 \) then country 1 becomes indifferent between choosing the taxpayers-pay or the users-pay system, implying that \( \hat{\lambda}_{1u}^1 = 0 \).

Differentiating the budget-balance condition in country 2 when both countries adopt the users-pay financing system we obtain

\[ \frac{\partial \hat{\lambda}_{2u}^2}{\partial \hat{\lambda}_{1u}^1} |_{k_1 = k_2} = \frac{2\eta_2 \theta_1^2 c_{2u}^{\gamma_2}}{2^{\gamma_a} (\eta_a - 1) \left( \frac{c_{2u} \eta_a}{\eta_a - 1} \right)^{\gamma_a} + 2c_{2u} \eta_2 (1 - \theta_1)^2}. \]  

(30)

Differentiating the budget-balance condition in country 1 when both countries adopt the users-pay financing system and using (30) we obtain

\[ \frac{\partial \hat{\lambda}_{1u}^1}{\partial k_1} |_{k_1 = k_2} = \frac{c_{1u}^{\gamma_1 - 1} \eta_1 \left[ 2^{\gamma_2} (\eta_a - 1) \left( \frac{c_{2u} \eta_a}{\eta_a - 1} \right)^{\gamma_a} + 2c_{2u} \eta_2 (1 - \theta_1)^2 \right]}{2^{\gamma_a} (\eta_a - 1) \left( \frac{c_{2u} \eta_a}{\eta_a - 1} \right)^{\gamma_a} + 2c_{2u} \eta_2 (1 - \theta_1)^2 + 2\eta_1 c_{1u}^{\gamma_1} \theta_1^2}. \]  

(31)

This enables us to obtain finally

\[ \frac{d \Delta SW_{1u}^1}{dk_1} |_{k_1 = k_2} = \left( \frac{\partial \Delta SW_{1u}^1}{\partial k_1} + \frac{\partial \Delta SW_{1u}^1}{\partial \hat{\lambda}_{1u}^1} \frac{\partial \hat{\lambda}_{1u}^1}{\partial k_1} \frac{\partial \lambda_{1u}^1}{\partial \hat{\lambda}_{1u}^1} \right) |_{k_1 = k_2} \]  

(32)

\[ \propto \frac{c_{2u} \eta_1 \left( \frac{c_{2u} \eta_a}{\eta_a - 1} \right)^{\gamma_a} \left[ 2^{\gamma_2} (\eta_a - 1) \left( \frac{c_{2u} \eta_a}{\eta_a - 1} \right)^{\gamma_a} + 2c_{2u} \eta_2 (1 - \theta_1)^2 \right]}{(\eta_a - 1)^2 \left( \frac{c_{2u} \eta_a}{\eta_a - 1} \right)^{\gamma_a} + 2c_{2u} \eta_2 (1 - \theta_1)^2} > 0. \]  

(33)

We conclude with the following lemma.

**Lemma 5** Assume that demands are iso-elastic, \( c_d = 0 \), \( \lambda_{2u}^2 = 0 \) and that country 2 adopts the users-pay system. For \( k_2 \) sufficiently close to \( k_2 \), in a neighborhood of \( k_2 \), country 1 prefers to adopt the taxpayers-pay system than the users-pay one.
Figure 1: Country $i$'s first stage choice of mode of regulation.