

Tax Competition Between Asymmetrically Informed Governments*

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21st December 2000

Abstract

A firm undertakes two activities for two different governments. Each government levies a tax from the firm, but only knows the cost parameter related to the local activity. There exists an informative equilibrium in which each government obtains the revelation of the unknown information. Since each government cannot affect the firm's incentives to reveal through his production decision, information revelation operates only through the substitutability of activities. Therefore, there also exists a non informative equilibrium in which each government refrains from obtaining the unknown information. The latter equilibrium Pareto-dominates the former. Moreover, we show that a government sometimes has the possibility to exclude his rival from the firm's services. *Keywords:* Intergovernmental Tax Competition, Common Agency Equilibria. *JEL classification:* L51, F13, H73, F23, D82

*We wish to express our gratitude to Bernard Caillaud and Jacques Crémer for their help and detailed comments. We also thank David Ettinger, David Martimort, Jean-Jacques Laffont and participants to the CERAS-LEI seminar.

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1 Introduction

The relation between multinational firms and countries hosting them is often paradoxical.

On the one hand, governments make a lot of efforts to attract part of the activities of those firms. Indeed, not only do these firms provide countries with foreign direct investment, job creation, technology transfer..., they may also generate important tax revenues¹. As an extreme example, according to the OECD (1999), Ireland experienced a macroeconomic boom over the last seven years, which is mainly related to the role of newly established multinational firms.

To attract multinational firms, Ireland has set lower tax rates for foreign firms than for local firms. But, such a practice is far from being an exception: The last OECD summit in Helsinki identified various preferential tax treatments² “ranging from The Netherlands’ willingness to negotiate secret advance agreements with foreign firms on how much tax they will pay, to Gibraltar’s tax exemptions for branches of non-resident companies operating there” (The Economist (2000)). Moreover, such preferential tax policies have been successful in influencing multinational firms’ investment decisions: “In analyzing corporate tax-return data for 1984-92, Altshuler and al. (1998) found that by the end of this period, the typical American multinational had become twice as likely to locate its operations where taxation was lowest as it had been at the beginning” (The Economist (2000)). Gorter and Parikh (2000) confirm this fact for the European Union.

On the other hand, these same governments do have a real problem dealing with those firms for, at least, two reasons: the lack of information about their foreign activities and the flexibility multinational firms have in shifting production and resources from one activity to another.

Indeed, multinational firms usually provide local governments only with partial accounting numbers. However, information about the foreign affiliates is needed by a government to determine how the tax policy should be optimally designed given the possibility of internal resource allocation by the firm or to assess how credible is the threat of complete relocation by the firm in the other countries.

As an illustration of the second point, common cost allocation schemes or transfer price procedures both endow the multinational firm with the capability to play one government against another and to decrease the cost of taxes. Gresik (2000) offers a detailed description of the various possibilities for multinational firms to manipulate tax policies and the consecutive responses by national governments.

If the production decisions of a multinational firm in each tax jurisdiction were perfectly independent, then the tax policy in a country would not create any externality neither on the tax policy in other countries, nor on the localization decisions

¹See Mucchielli (1998) for a descriptive analysis of the role and the impact of multinational firms.

²See also Gresik (2000) for a clear exposition of the differences in tax systems across countries (with a specific emphasis on multinational firms).

of the firm. However, the mere existence of a multi-activity firm comes from the presence of links between its production decisions. The interdependency of activities implies that any modification in one country's production leads to a readjustment in the other country's production decisions. This in turn implies that governments who fail to recognize the interdependency of their tax policies and do not succeed in overcoming their coordination problem, create externalities on each other³.

The focus of this paper is precisely on the competition between governments offering shaped tax policies to a multinational firm⁴. To this purpose, we consider a common agency game under adverse selection in which governments can use an unrestricted set of instruments (given the observable and contractible variables). This normative approach helps us to identify the best outcome of the tax competition game between governments⁵.

Let us describe the multinational firm. The parent firm produces an intermediate good (at a quadratic cost) which is then transferred to subsidiaries located in different countries. Each subsidiary transforms (at a constant marginal cost) the intermediate input into a final consumption good which is sold on the local market. The firm perfectly controls its affiliates and maximizes the sum of the subsidiaries' profits minus the cost incurred by the parent division. In each country, the firm is under the jurisdiction of a fiscal authority who imposes a tax to be paid by the multinational firm. The tax schedule imposed by a government depends on the local production and cannot be made contingent neither on the other subsidiaries' production levels nor on the firm's choice of localization.

In the standard common agency setting, the agent has one private information and both principals are uninformed about this information. By contrast, we assume that the subsidiaries' costs of transformation are different across countries. Moreover, we assume that each government knows the cost parameter related to the

³This absence of coordination between competing governments (despite recent attempts to harmonize tax policies by the OECD or the GATT) is well documented in the EU: The Netherlands Bureau for Economic Policy Analysis reports that "The Ruding report of 1992 proposed a minimum corporate income tax in the EU and harmonization of the tax base, but this proposal has never been approved."

⁴Notice that we take the existence of multinational firms as given. See Markusen and Venables (1998) for the study of endogenous multinational firms formation. Similarly, we take the existence of tax policies as given: Gresik (2000) reviews various justifications for the role of taxes in an open economy.

⁵The tax competition literature started with Oates' (1972) seminal contribution that focuses on the potential inefficiencies arising from the competition for mobile capital between governments. This literature has studied issues such as intergovernmental competition or the desirable degree of economic interaction between countries; see Wilson (1999) for a comprehensive and recent survey. In this literature, the set of instruments at the disposal of the governments is exogenously restricted. Recently, a growing literature tries to relax these assumptions and enriches the relation between the firm and the government(s) with informational asymmetries. For instance, Bond and Gresik (1996) adopt the common agency framework to study the tax competition game between uninformed governments facing a multinational firm.

activity which takes place in his country, but is uninformed on the one which takes place in the other country. Each government can condition the tax policy offered to the firm on the quantity produced for the local market, or, equivalently given our informational structure, on the profit of the local subsidiary.

Importantly, we also assume that the firm can decide to be active in only one country: Each government must ensure not only that the firm's total profit is positive, but also that the firm will not relocate its activities in the other country. The latter condition is labeled the 'no relocation' constraint.

In this setting, under complete information, the Nash game between governments yields quantity profiles that are not affected by the presence of the non relocation constraint. Nonetheless, this constraint implies that, in equilibrium, the firm earns strictly positive rents: capital mobility provides the firm with the power to threaten one government to stop producing in his country and to relocate its activity in the other country only. This drives taxes down with respect to the case in which relocation is very costly or impossible.

Under asymmetric information, we first determine an equilibrium in which each government makes the firm reveal the unknown information, that is the production cost in the other country. From the point of view of the domestic government, the acquisition of information about the foreign subsidiary can operate only in an indirect way, that is, through the substitutability of activities at the firm level (since the domestic production does not directly affect the cost of production in the foreign country). From the point of view of the domestic government, the larger is the cost of the foreign subsidiary, the stronger is the firm's incentive to substitute foreign production by domestic production, and the more the firm appears as efficient to this government. This is the information effect that leads each government to decrease the local production, thereby triggering an increase in the other production through the substitutability, in order to provide the firm with the incentive to reveal the unknown information on the non-resident affiliate. However, the larger the cost parameter in the foreign country, the smaller the quantity requested by the foreign government because the firm appears to him as inefficient: This is the efficiency effect which impacts in equilibrium the domestic production through the substitutability of activities. Depending on the magnitude of each effect, the equilibrium quantities in one country can be either above or below the full information ones.

Another implication of the fact that information acquisition operates only in an indirect way is that if one government decides to refrain from obtaining the unknown information, then the other government can no longer obtain the revelation of the unknown information. Indeed, if the firm can no longer substitute the domestic quantity to the foreign one, that is, if the foreign government offers a non informative contract, then the domestic government can no longer affect the firm's rent via the interaction of production decisions at the firm level and therefore offers in turn a non informative contract. This is the non informative equilibrium.

We then compare the welfare for both equilibrium configurations. Despite its

inadequacy to all the relevant information, the non informative equilibrium is shown to Pareto-dominate the informative equilibrium since it softens the competition between governments and leaves a smaller rent to the multinational firm. Therefore, the non informative equilibrium exhibits a new reason for strategic ignorance: Each government prefers not to know all the relevant information because the process of information acquisition intensifies competition between tax institutions.

Finally, we explore a third type of equilibria. In an asymmetric equilibrium, a government designs his tax policy with the aim of excluding the other government from the multinational firm's services. In our setting, when the firm decides to stop the production in the foreign country, the domestic country becomes under complete information vis-à-vis the firm. Therefore, there exists an incentive for the domestic government to exclude the foreign one in order to reach the full benefit associated to taxation under complete information. When the firm produces only in the domestic country, the domestic government can use a two-part tax. However, he could as well offer a nonlinear tax to obtain some additional degrees of freedom. Under competition between tax authorities, these degrees of freedom can be used strategically to modify the firm's out-of-equilibrium behavior with respect to the other government. Indeed, we show that such nonlinear taxes can be used by a government to exclude his rival.

Our paper borrows from the common agency under adverse selection literature pioneered by Martimort (1992) and Stole (1991)⁶. In this framework, when the production activities undertaken by the firm are substitutes the equilibrium quantities under asymmetric information are usually lower than under complete information. This feature does not carry over to our setting. Moreover, to the best of our knowledge, this literature usually considers symmetric equilibria only. Biglaiser and Mezzetti (1993) study a setting of principals competing for the exclusive service of an agent. Our setting is different as the firm always has the possibility to produce for both countries. Finally, this paper also belongs to the growing literature on the taxation of multinational firms as in Bond and Gresik (1996).

Our paper is closer to Ivaldi and Martimort (1996) in which the agent has two pieces of private information and the principals are uninformed on both pieces. Our informational structure is different and yields other qualitatively different equilibria. Similarly, in a model of asymmetrically informed principals, Bond and Gresik (1997) show that there exist different types of equilibria. They focus on two-part tax schedules, whereas the existence of asymmetric equilibria in our setting strongly relies on the use of quadratic instead of two-part taxes.

The outline of the paper goes as follows. In the next section (Section 2) we present the model. Section 3 studies the complete information benchmark. In Section 4, we consider the unilateral taxation case in which the firm is regulated only in

⁶See also Martimort and Stole (1998) for recent developments of the analysis of those games when there are direct externalities between governments.

one country. This enables us to derive some intuitions on the impact of the competition between tax authorities and on the occurrence of the different types of equilibria under bilateral taxation. In Section 5, we exhibit the informative equilibrium and in Section 6 we study the non informative equilibrium. Section 7 is devoted to the welfare analysis of these equilibria. Section 8 studies asymmetric equilibria. Section 9 concludes. All proofs are gathered into an Appendix.

2 The model

The firm We consider a multinational firm potentially active in two countries: the domestic country, denoted by ‘ d ’, and the foreign country, denoted by ‘ f ’. The firm produces an amount $Q = q_d + q_f$ of an intermediate good at cost $C(Q) = Q^2$. Input q_i is then transferred to the subsidiary division located in country $i = d, f$. Final stage productions transform the intermediate input into a final output⁷ sold on the local market characterized by an inverse demand function⁸ $p_i(q_i) = a_i - q_i$, $i = d, f$. The transformation takes place at marginal cost θ_i . The total cost to obtain quantities q_d and q_f of final outputs, $\theta_d q_d + \theta_f q_f + Q^2$, is then composed by separable (the first two terms) and non separable or common costs (the third term).

In this setting, the net profit of the firm is given by

$$\pi(\theta_d, \theta_f, q_d, q_f) = \sum_{i=d,f} [p_i(q_i) - \theta_i] q_i - (q_d + q_f)^2 - T, \quad (1)$$

where T is the total tax (see below) to be paid by the firm to the governments. The term $[p_i(q_i) - \theta_i] q_i$ is what we shall refer to as the local profit in country i . Notice that the cross-derivative of the gross profit with respect to the two outputs is negative implying that outputs are substitutes at the firm level.

The informational structure In each country, the local government has perfect information about the local activity of the firm but not about the one abroad. In other words, the government in country i , denoted by G_i , perfectly knows θ_i but only knows (as is common knowledge) that θ_j is distributed according to a density $h_j(\theta_j)$ with cdf $H_j(\theta_j)$ on $\Theta_j \equiv [\underline{\theta}_j, \bar{\theta}_j]$. These distributions satisfy the monotone hazard rate assumption, namely $\frac{d}{d\theta_j} \frac{1-H_j(\theta_j)}{h_j(\theta_j)} \leq 0$. The cost parameters θ_d and θ_f are assumed to be independently distributed⁹.

⁷We implicitly assumed that one unit of intermediate input yields one unit of final output. More sophisticated technologies could be used without altering the nature of our arguments.

⁸Our results extend qualitatively to differentiable, decreasing and concave inverse demand functions. The linearity assumption alleviates substantially the presentation and simplifies the determination of the quantity profiles in the uniform case.

⁹Our results could be extended to imperfect correlation between the private information parameters. This would only change the prior distribution of a government on the unknown private information parameter.

This informational structure is the main theoretical novelty with respect to the common agency literature in which it is typically assumed that the information of the common agent is unidimensional and privately known by this agent. In our model, governments possess different pieces of information but remain uninformed along a particular dimension of the firm's private information.

The governments We assume that government G_i observes the production undertaken in his country and the one realized ex post in the other country but cannot condition¹⁰ the tax proposed to the firm on the production decision in country j . Since local profit and local production are one-to-one related (given the knowledge of the cost of the local subsidiary), a quantity-related tax imposed in country i is equivalent to a tax based on local profit.

We finally make the following assumption: Government G_i cannot make the firm pay when that latter decides to shut down production in his country. Otherwise, the threat of relocation that we describe in Section 3.2 could be easily fought by heavily punishing the firm when it decides to shut down the production.

Throughout the paper, we shall focus on the domestic country. The objective of government G_d is to maximize the net surplus of the consumers in his country plus the tax revenue earned from the firm¹¹, or

$$SW_d = \int_0^{q_d} p_d(x) dx - p_d(q_d)q_d + T_d(\theta_d, q_d). \quad (2)$$

The game To summarize, we consider the following game:

1. Nature draws $\theta \equiv (\theta_d, \theta_f)$ according to the common knowledge probability distribution $h_d \times h_f$ on $\Theta_d \times \Theta_f$. The firm learns θ whereas government G_i only learns θ_i , $i = d, f$.
2. Governments offer simultaneously and secretly their tax policy to the firm.
3. The firm simultaneously chooses its production levels for both countries.

¹⁰This restriction on the scope of taxation (which is always made in common agency games under adverse selection) can be loosely interpreted as a way to avoid double taxation phenomena that could arise had we assumed that government G_i could condition his tax on the firm's production decision in country j . Moreover, many countries have by now tax exemption agreements. In Europe for example, Austria, Denmark, Finland, France, Germany, The Netherlands and Portugal have agreed not to tax the profit of multinationals' subsidiaries which have already been taxed in one of those countries.

¹¹There would still exist different types of equilibria if the governments evaluated also the firm's profit according to country specific weights.

3 Complete information

In this section only, we assume that each government knows both information parameters of the firm. Such informational structure is very unlikely to happen in reality but will serve as a useful benchmark to the rest of the paper. In particular, we shall see that, even in the absence of any informational considerations, the fear of relocation induces governments to set lower taxes with respect to the case where the firm cannot relocate its activities.

3.1 Equilibrium when relocation is not possible

We analyze here the case in which a government does not face any constraints on the fact that the firm can relocate its activities in the other country. We will determine the equilibrium in quantities and tax schedules. With a slight abuse of notations, we define the rent earned by the firm as follows

$$\pi(\theta) \equiv \max_{q_d > 0, q_f > 0} \{\pi(\theta, q_d, q_f)\}. \quad (3)$$

Under complete information, government G_i can restrict to a direct contract specifying a quantity $q_i(\theta)$ to be produced against the payment of a lump sum tax $T_i(\theta)$. This contract must ensure that the firm is effectively willing to be active in both countries in equilibrium¹², or (after having redefined the rent of the firm accordingly)

$$\pi(\theta) \geq 0. \quad (GP)$$

We call hereafter this constraint the ‘global participation’ constraint. Consequently, government G_i solves

$$\begin{aligned} & \max_{T_i(\theta), q_i(\theta) > 0} \{SW_i\} \\ & \text{subject to } (GP). \end{aligned}$$

As the firm’s rent is socially costly¹³, the participation constraint will be binding in equilibrium. Solving the system corresponding to the two first-order conditions¹⁴ of the previous program for both countries yields the following equilibrium quantities

$$\begin{cases} q_d^*(\theta) &= \frac{1}{5} [3(a_d - \theta_d) - 2(a_f - \theta_f)], \\ q_f^*(\theta) &= \frac{1}{5} [3(a_f - \theta_f) - 2(a_d - \theta_d)]. \end{cases} \quad (4)$$

However, it might be possible that one of those equilibrium quantities is negative, implying that an asymmetric equilibrium prevails in which the firm shuts down the

¹²This is the usual intrinsic common agency setting, as coined by Bernheim and Whinston (1986).

¹³Indeed, replacing the tax $T_d(\theta)$ by its expression as function of the firm’s rent in SW_d shows that the firm’s rent enters negatively in the domestic government’s objective.

¹⁴Second-order conditions are trivially satisfied.

production in one country. In the case where the firm stops producing, say, in country j , the quantities are given by

$$\begin{cases} q_i^{*a}(\theta) &= \frac{1}{3}(a_i - \theta_i), \\ q_j^{*a}(\theta) &= 0, \end{cases} \quad (5)$$

where superscript ‘ a ’ stands for asymmetric equilibrium (in the sense that one of the quantities is null in equilibrium). Summarizing, we obtain the following proposition.

Proposition 1 *Under complete information, the equilibrium quantity profiles are characterized as follows:*

- *If $\frac{3}{2} \geq \frac{a_d - \theta_d}{a_f - \theta_f} \geq \frac{2}{3}$, then both quantity profiles are strictly positive and given by (4).*
- *Otherwise, if $\frac{a_i - \theta_i}{a_j - \theta_j} > \frac{3}{2}$, then the quantity profiles are given by (5).*

Proof. See Appendix 11.1.1. ■

An illustration of this result can be obtained from the reaction functions of the governments that we draw in Figure 1.

Insert Figure 1 here

Finally, from the binding participation constraint, one can immediately see that, in equilibrium, $T_d(\theta)$ and $T_f(\theta)$ are undetermined. Only the sum of these taxes is determined in equilibrium, a usual feature of intrinsic common agency games.

Note that, so far, we have restricted ourselves to direct mechanisms to determine the quantity profiles that result from the competition between governments under complete information. We want now to recover the instruments used by the governments in our original game, i.e., the tax schedules as functions of quantities which induce the firm to optimally choose the equilibrium quantity profiles.

Under complete information, the domestic government can use a two-part tax. Indeed, the fixed part is set so as to leave the firm with no rent in equilibrium, while the variable part ensures that the firm has the correct incentives, i.e., that its production decision in the domestic country coincides with the quantity profile determined previously. We thus obtain the following lemma.

Lemma 1 *Under complete information, each government offers a two-part tax schedule to the firm.*

Proof. See Appendix 11.1.2. ■

That, in each country, a two-part tax schedule is sufficient to replicate the equilibrium allocation is not surprising and is standard in one-principal one-agent models. As explained in Martimort and Stole (1998), this feature holds also in common

agency settings under complete information provided that there are no direct externalities¹⁵ between governments. Without direct externalities, the tax offered by the domestic government to the firm takes into account that the latter optimally adapts its production decision in the foreign country to any change in the production decision in the domestic one. In the end, under complete information, for each country, there is no gain to use an ‘aggressive’ tax policy (that is, a quadratic tax schedule instead of a two-part one for instance).

3.2 The fear of relocation

From now on, we will introduce the possibility for the firm to relocate its activities in one country only¹⁶. This is the case if, for instance, capital is sufficiently mobile so that the firm can decide to shut down the production in one country at no cost. As we shall see, the potential for the firm to relocate its activities uniquely and unambiguously determine the way equilibrium tax revenues are shared between governments.

From the viewpoint of the government in the domestic country, this new constraint, that we call the ‘non relocation’ constraint, is written as follows

$$\pi(\theta) \geq \pi_d^{nr}(\theta_f) \equiv \max_{q_f > 0} \{\pi(\theta_d, \theta_f, q_d = 0, q_f)\}. \quad (NR_d)$$

We can prove the following lemma¹⁷, which is valid whatever the informational structure.

Lemma 2 *If the non relocation constraints are satisfied, then the global participation constraint is also satisfied.*

Proof. See Appendix 11.2. ■

This result mainly relies on the substitutability of the goods produced by the firm. With substitutability, the threat to leave one country is credible because the firm is then able to reduce its production cost¹⁸. Thus, both countries will be obliged to increase the firm’s rent in equilibrium in order to prevent a complete relocation of the firm’s activities in the other country.

As a consequence of this lemma, for the rest of the analysis we will ignore the global participation constraint. Adding the non relocation constraint in the program of the domestic government does not alter the equilibrium quantity profiles.

¹⁵The term ‘direct externality’ refers to the fact that the action undertaken by one government affects directly another government’s welfare, not just through the effect on the firm’s rent.

¹⁶This is the so-called ‘delegated’ common agency framework.

¹⁷The proof is borrowed from Ivaldi and Martimort (1996).

¹⁸On the contrary, with complementarity this threat would be less credible. Indeed, in this case (but in a slightly different setting), Ivaldi and Martimort (1996) show that (*GP*) might be the most demanding participation constraint in equilibrium.

Indeed, for the domestic government, the outside opportunity of the firm (when that latter produces only in the foreign country) does not depend on the quantity produced for the domestic market in an equilibrium with positive production levels in both countries. Therefore, production levels are unchanged, but each government has to reduce the tax it imposes to the firm to prevent that latter from shutting down the production in its country. Moreover, the non relocation constraints now define uniquely the set of taxes offered by the governments, allowing us to perform unambiguous welfare analysis.

Loosely speaking, the potential for relocation defines the ‘bargaining power’ of each country. This bargaining power in sharing tax revenues depends, in our model, on the gain of the firm following a potential relocation of its activities. Therefore, for a given country, it depends directly on market conditions prevailing in the other country: for instance, a country with a much larger market size than the other will be able so set a higher tax since the threat of relocation is less important. By contrast, when that threat is very credible, a government is forced to set a low tax.

4 Unilateral taxation

In this section, we consider the situation in which the firm is free from any taxation in the foreign country. Government G_d is assumed to be uninformed on θ_f . This exercise will prove to be useful to understand the distortions on the equilibrium quantity profiles as well as the reasons for the emergence of the non informative equilibrium under bilateral taxation.

Under unilateral taxation, the firm’s profit and rent are defined respectively by

$$\pi^{ut}(\theta, q_d, q_f) = \sum_{i=d,f} [p_i(q_i) - \theta_i] q_i - (q_d + q_f)^2 - T_d(\theta_d, q_d), \quad (6)$$

$$\pi^{ut}(\theta) = \max_{q_d > 0, q_f > 0} \{ \pi^{ut}(\theta, q_d, q_f) \}, \quad (7)$$

where ‘*ut*’ stands for unilateral taxation.

4.1 The non relocation constraint

Under unilateral taxation, the outside opportunity of the firm with respect to government G_d becomes

$$\pi_d^{nr,ut}(\theta_f) = \max_{q_f > 0} \{ [p_f(q_f) - \theta_f] q_f - q_f^2 \} = \frac{1}{8}(a_f - \theta_f)^2.$$

Obviously, this outside opportunity corresponds to the monopoly profit in the foreign country (when the production in the domestic country is null). Under unilateral taxation, the non relocation constraint for a firm with private information θ is

$$\pi^{ut}(\theta) \geq \pi_d^{nr,ut}(\theta_f). \quad (NR_d^{ut})$$

4.2 Unilateral taxation under complete information

The domestic government maximizes the social welfare of his country, but must also ensure the participation of the firm. Direct computations show that the equilibrium quantity profiles under unilateral taxation and complete information are characterized by¹⁹

$$\begin{cases} q_d^{ut*}(\theta) = \frac{1}{8} [4(a_d - \theta_d) - 2(a_f - \theta_f)], \\ q_f^{ut*}(\theta) = \frac{1}{8} [3(a_d - \theta_d) - 2(a_f - \theta_f)]. \end{cases}$$

Under unilateral taxation, the firm is free to choose its quantity in the foreign country. However, in the domestic country, government G_d forces the firm to produce a larger quantity. Therefore, via the substitutability of productions at the firm level, the quantity produced in the foreign country decreases (with respect to the case where the firm would be unregulated in both countries). Notice finally that this outcome corresponds to the Stackelberg equilibrium.

4.3 Government G_d 's mechanism design problem under unilateral taxation

We consider now the situation in which the domestic government is uninformed on the production cost of the foreign affiliate. Let us focus on to the incentive problem faced by government G_d . Define

$$\hat{q}_f^{ut}(\theta_f, q_d) \equiv \arg \max_{q_f > 0} \{ [p_f(q_f) - \theta_f] q_f - (q_d + q_f)^2 \},$$

the optimal output choice of the firm in the foreign country, for any given domestic production q_d . This production decision is characterized by

$$\hat{q}_f^{ut}(\theta_f, q_d) = \frac{1}{4} (a_f - \theta_f - 2q_d). \quad (8)$$

Define also

$$\hat{\pi}_d^{ut}(\theta_f, q_d) \equiv \max_{q_f > 0} \{ [p_f(q_f) - \theta_f] q_f - (q_d + q_f)^2 \}, \quad (9)$$

the indirect utility function of the firm with respect to the domestic government. This function represents the gain of the firm in the foreign market when choosing optimally its production level in the foreign country, for a given quantity q_d produced in the domestic country.

¹⁹We do not consider the possibility that one of these quantity profiles is negative in equilibrium. The purpose of this section is to understand the distortions in the domestic quantity profile induced by asymmetric information on the cost parameter in the foreign country.

Equation (8) indicates that, because production decisions are linked at the firm level through the common cost, government G_d has to take into account that the firm may substitute one production to the other. Therefore, from government G_d 's perspective the rent of the firm must be rewritten as

$$\pi^{ut}(\theta) = \max_{q_d > 0} \{ [p_d(q_d) - \theta_d] q_d + \hat{\pi}_d^{ut}(\theta_f, q_d) - T_d(\theta_d, q_d) \}. \quad (10)$$

According to the Revelation Principle²⁰ there is no loss of generality in restricting attention to direct contracts that ensure the truthful revelation of the private information held by the firm. These contracts take the form $\{q_d(\theta_d, \tilde{\theta}_f), T_d(\theta_d, \tilde{\theta}_f)\}$ where $\tilde{\theta}_f$ is the report sent by the firm to government G_d . In the next lemma, we derive the requirements of incentive compatibility expressed in terms of rent-production pair instead of tax-production pair.

Lemma 3 *Under unilateral taxation in the domestic country, a pair $\{\pi^{ut}(\theta_d, \cdot), q_d(\theta_d, \cdot)\}$ is implementable by government G_d if and only if for all $\theta \in \Theta_d \times \Theta_f$ the following conditions are satisfied:*

- *First-order condition,*

$$\frac{\partial \pi^{ut}}{\partial \theta_f}(\theta) = \frac{\partial \hat{\pi}_d^{ut}}{\partial \theta_f}(\theta_f, q_d(\theta)); \quad (11)$$

- *Second-order condition,*

$$\frac{\partial^2 \hat{\pi}_d^{ut}}{\partial q_d \partial \theta_f}(\theta_f, q_d(\theta)) \frac{\partial q_d}{\partial \theta_f}(\theta) \geq 0. \quad (12)$$

Proof. See Appendix 11.3.1. ■

Using the definition of $\hat{\pi}_d^{ut}(\theta_f, q_d)$ we find that under unilateral taxation the so-called Spence-Mirrlees condition amounts to

$$\frac{\partial^2 \pi^{ut}}{\partial q_d \partial \theta_f}(\theta) = \frac{\partial^2 \hat{\pi}_d^{ut}}{\partial q_d \partial \theta_f}(\theta_f, q_d(\theta)) \quad (13)$$

$$= -\frac{\partial \hat{q}_f^{ut}}{\partial q_d}(\theta_f, q_d(\theta)) = \frac{1}{2} > 0. \quad (14)$$

Hence, the slope of the firm's rent, with respect to government G_d , increases with the quantity produced in the domestic country. This implies that the local second-order condition for incentive compatibility (12) will be satisfied if and only if

$$\frac{\partial q_d}{\partial \theta_f}(\theta) \geq 0.$$

²⁰See Green and Laffont (1977) or Myerson (1979) among others.

This particular feature deserves some comments. For a given θ_d (known by government G_d), a firm with a large θ_f will be more willing to substitute production in the foreign country for the production in the domestic country than a firm with a small θ_f . Hence, the larger θ_f is, the more the firm appears as efficient with respect to government G_d . The role of the substitutability is therefore crucial to determine government G_d 's perception of the firm's efficiency²¹. Moreover, it is easy to show that

$$\frac{\partial}{\partial \theta_f} \left[\pi^{ut}(\theta) - \pi_d^{nr,ut}(\theta_f) \right] = \frac{1}{2} q_d(\theta) > 0. \quad (15)$$

Given that we require that the non relocation constraint be satisfied by government G_d , the net rent $\pi^{ut}(\theta) - \pi_d^{nr,ut}(\theta)$ characterizes the rent that must be given up by the domestic government, beyond that necessary to ensure the participation of the firm, in order to obtain the revelation of the private information by the firm. From (15), we see that this net rent increases in the efficiency parameter θ_f as a large efficiency parameter in the foreign country is synonymous of an efficient firm from the domestic government's point of view. This is a crucial difference with usual adverse selection models à la Baron-Myerson²² in which the rent of the firm is decreasing in the cost parameter of the firm. This finally implies that if the non relocation constraint is satisfied in $\underline{\theta}_f$, then it is satisfied for all θ_f .

Another way to understand this peculiarity is to look directly at the firm's net rent. Substituting the quantity produced in the foreign country defined by (8) into the firm's profit function, we can rewrite the net rent as follows

$$\pi(\theta) - \pi_d^{nr,ut}(\theta_f) = \max_{q_d > 0} \left\{ \Omega(\theta_d, a_d, a_f, q_d) + \frac{1}{2} \theta_f q_d - T_d(\theta_d, q_d) \right\}.$$

where $\Omega(\theta_d, a_d, a_f, q_d) = [p_d(q_d) - \theta_d]q_d - \frac{1}{2}q_d(a_f + q_d)$.

Hence, from the point of view of government G_d everything happens as if he were facing a firm with private information on its marginal cost of production equal to $-\theta_f$ and deriving a gross profit from this production equal to $\Omega(\theta_d, a_d, a_f, q_d)$. Consequently, the more inefficient the firm is in the foreign country, the more this firm appears as efficient to the government in the domestic country and the larger the rent it will receive from government G_d is.

Finally, notice that these features do not hinge on the particular participation constraint to be satisfied by government G_d . Indeed, from the point of view of government G_d the rent of the firm $\pi^{ut}(\theta)$ exhibits the same features: The part of the rent that is affected by the quantity in the domestic country increases with the information parameter related to the production in the foreign country. As in the complete information case, the non relocation constraint will nonetheless force governments to give up larger rents to the firm.

²¹We would have obtained an opposite conclusion with complementary productions.

²²Including common agency models.

Under unilateral taxation, the program of the domestic government can therefore be stated as follows

$$(\mathcal{P}_d^{ut}) \left\{ \begin{array}{l} \max_{\{q_d(\theta_d), \pi^{ut}(\theta_d)\}} \mathbf{E}_{\theta_f} \left\{ \int_0^{q_d(\theta)} p_d(x) dx - \theta_d q_d(\theta) + \hat{\pi}_d^{ut}(\theta_f, q_d(\theta)) - \pi^{ut}(\theta) \right\} \\ \text{subject to } \forall \theta_f, (NR_d^{ut}), (11), (12). \end{array} \right.$$

This problem is a standard Principal-Agent problem under adverse selection. The resolution is provided in the Appendix.

4.4 Equilibrium quantity profiles under unilateral taxation

In Appendix 11.3.2, we show that the equilibrium quantity profiles for the domestic and the foreign country are given by²³

$$\begin{cases} q_d^{ut}(\theta) &= \frac{1}{8} \left[4 \left(a_d - \theta_d - \frac{1}{2} \frac{1-H_f(\theta_f)}{h_f(\theta_f)} \right) - 2(a_f - \theta_f) \right], \\ q_f^{ut}(\theta) &= \frac{1}{8} \left[3(a_f - \theta_f) - 2 \left(a_d - \theta_d - \frac{1}{2} \frac{1-H_f(\theta_f)}{h_f(\theta_f)} \right) \right] \end{cases}$$

This enables to state the following proposition.

Proposition 2 *Under unilateral taxation in the domestic country, the production level in the domestic (foreign) country is smaller (larger) than in the complete information situation.*

Proof. See Appendix 11.3.2 ■

Let us comment this proposition. First, note that government G_d wants to obtain the revelation of parameter θ_f . However, he cannot alter directly the firm's incentives to reveal or to conceal θ_f : We can rewrite the first-order incentive compatibility constraint (11) as

$$\frac{\partial \pi^{ut}}{\partial \theta_f}(\theta) = -\hat{q}_f(\theta_f, q_d(\theta)).$$

This directly implies that the rent of the firm is given by

$$\pi^{ut}(\theta) = \pi_d^{nr, ut}(\theta_f) - \int_{\underline{\theta}_f}^{\theta_f} \hat{q}_f(x, q_d(x, \theta_d)) dx.$$

Therefore, the government in the domestic country can obtain the revelation of θ_f only in an indirect way, that is, by playing on the substitutability of the production decisions at the firm level: By decreasing the quantity produced in the domestic

²³Under the monotone hazard rate assumption all the implementability conditions for the domestic government are satisfied.

country, government G_d triggers an increase in the quantity produced in the foreign country (due to the substitutability), which leads to a reduction in the firm's rent from the viewpoint of the domestic government.

On the contrary, had we assumed that θ_f also affects directly the cost of the domestic production we would have obtained the standard one principal-one agent setting²⁴, in which a government can affect the firm's rent both in a direct and in an indirect way.

This feature naturally raises the following question: What happens if the firm decides to produce in the foreign country a quantity that does not depend on the production in the domestic country? In such a case, government G_d could no longer obtain the revelation of the firm's private information because $\frac{\partial^2 \pi^{ut}}{\partial q_d \partial \theta_f}(\theta) = 0$. Hence, he will be forced to offer a contract that does not depend on the unknown information θ_f . Obviously, under unilateral taxation this situation cannot be an equilibrium because faced with such a contract, the firm always chooses a production level for the foreign country that depends on its production for the domestic country. However, we shall show that such an equilibrium can emerge under bilateral taxation.

Finally, notice that government G_d cannot design his contract in such a way that the firm shuts down the production in the foreign country. We shall show that with bilateral taxation such asymmetric equilibria could also arise.

5 Informative equilibrium

From now on, we return to the hypothesis that the firm is taxed in both countries.

5.1 Government G_d 's mechanism design problem

As in the unilateral taxation case, the key point to solve this common agency game is to recognize that for a given contract offered by one government, one can apply the Revelation Principle to find the optimal best response contract for the other government. Indeed, for a given $T_f(\cdot)$, any payoff that government G_d can achieve using a nonlinear transfer $T_d(\theta_d, q_d)$ can be replicated with a menu of direct contracts of the form $\{q_d(\theta), T_d(\theta)\}$ ²⁵. However, because production decisions are linked at the firm level, different taxes offered by government G_f affect differently the firm's incentives vis-à-vis government G_d for informational reasons. We must then define the firm's indirect utility function, which captures the way the firm's behavior with respect to government G_d is affected by government G_f 's tax proposal, as follows

$$\hat{\pi}_d(\theta_f, q_d) \equiv \max_{q_f > 0} \{ [p_f(q_f) - \theta_f] q_f - (q_d + q_f)^2 - T_f(\theta_f, q_f) \}. \quad (16)$$

²⁴The same remark holds under bilateral taxation. See Martimort (1992) for instance.

²⁵Once again, we refer the reader to Martimort and Stole (1998) for a more detailed analysis of common agency games.

This function gives the gain of the firm in the foreign market when choosing optimally its production level for the foreign country, for a given quantity q_d produced in the domestic country. For further reference, this latter quantity is denoted by $\hat{q}_f(\theta_f, q_d)$ and is characterized by²⁶

$$4\hat{q}_f(\theta_f, q_d) + \frac{\partial T_f}{\partial q_f}(\theta_f, \hat{q}_f(\theta_f, q_d)) = a_f - \theta_f - 2q_d. \quad (17)$$

The total rent of the firm with respect to government G_d can now be rewritten as

$$\pi(\theta) = \max_{q_d > 0} \{ [p_d(q_d) - \theta_d]q_d + \hat{\pi}_d(\theta_f, q_d) - T_d(\theta_d, q_d) \}. \quad (18)$$

Standard computations enable us to derive the local first- and second-order conditions for incentive compatibility. In the next lemma, we express these conditions in terms of rent-production pair $\{\pi(\theta_d, \cdot), q_d(\theta_d, \cdot)\}$.

Lemma 4 *A pair $\{\pi(\theta_d, \cdot), q_d(\theta_d, \cdot)\}$ is implementable by government G_d if and only if for all $\theta \in \Theta_d \times \Theta_f$ the following conditions are satisfied:*

- *First-order condition,*

$$\frac{\partial \pi}{\partial \theta_f}(\theta) = \frac{\partial \hat{\pi}_d}{\partial \theta_f}(\theta_f, q_d(\theta)); \quad (19)$$

- *Second-order condition,*

$$\frac{\partial^2 \hat{\pi}_d}{\partial q_d \partial \theta_f}(\theta_f, q_d(\theta)) \frac{\partial q_d}{\partial \theta_f}(\theta) \geq 0. \quad (20)$$

Proof. Once the indirect utility function is modified to account for the effect of the foreign government's tax schedule on the firm's incentive with respect to the domestic government, the local necessary and sufficient conditions for incentive compatibility are obtained exactly as for the case of unilateral taxation (Lemma 3). ■

As is usual in common agency games, and contrary to the unilateral taxation situation, the equivalent to the Spence-Mirrlees condition

$$\frac{\partial^2 \hat{\pi}_d}{\partial q_d \partial \theta_f}(\theta_f, q_d(\theta)) \geq 0$$

cannot be postulated a priori because it depends endogenously on the tax proposed to the firm in the foreign country. This condition has to be checked in equilibrium.

²⁶To ensure that the production in the foreign country is still characterized by the first-order condition (17) even for an out-of-equilibrium production level in the domestic country, the tax $T_f(\cdot, \theta_f)$ must be extended to allow for out of equilibrium quantities. In a slightly different setting, Martimort (1992) constructs explicitly such extensions.

Finally, we can restate the mechanism design problem of government G_d as follows

$$(\mathcal{P}_d^{ie}) \left\{ \begin{array}{l} \max_{\{q_d(\theta_{d,\cdot}), \pi(\theta_{d,\cdot})\}} \mathbf{E}_{\theta_f} \left\{ \int_0^{q_d(\theta)} p_d(x) dx - \theta_d q_d(\theta) + \hat{\pi}_d(\theta_f, q_d(\theta)) - \pi(\theta) \right\} \\ \text{subject to } \forall \theta_f, (NR_d), (19), (20). \end{array} \right.$$

As is usual in common agency models, we have expressed the firm's optimization behavior with respect to each government. However, it remains to check that it defines a global optimum for the firm, i.e., that the firm is effectively willing to take both contracts in equilibrium.

5.2 Quantity profiles in an informative equilibrium

Solving (\mathcal{P}_d^{ie}) and the corresponding program for G_f yields the following proposition.

Proposition 3 *Under incomplete information, in an informative equilibrium of the game between governments the quantity profiles satisfy the following necessary conditions*

$$\left\{ \begin{array}{l} 3q_d^{ie}(\theta) + 2q_f^{ie}(\theta) = a_d - \theta_d + 2 \frac{1-H_f(\theta_f)}{h_f(\theta_f)} \frac{\frac{\partial q_d^{ie}}{\partial \theta_d}(\theta) \frac{\partial q_f^{ie}}{\partial \theta_f}(\theta) - \frac{\partial q_d^{ie}}{\partial \theta_f}(\theta) \frac{\partial q_f^{ie}}{\partial \theta_d}(\theta)}{\frac{\partial q_d^{ie}}{\partial \theta_d}(\theta)}, \\ 3q_f^{ie}(\theta) + 2q_d^{ie}(\theta) = a_f - \theta_f + 2 \frac{1-H_d(\theta_d)}{h_d(\theta_d)} \frac{\frac{\partial q_d^{ie}}{\partial \theta_d}(\theta) \frac{\partial q_f^{ie}}{\partial \theta_f}(\theta) - \frac{\partial q_d^{ie}}{\partial \theta_f}(\theta) \frac{\partial q_f^{ie}}{\partial \theta_d}(\theta)}{\frac{\partial q_f^{ie}}{\partial \theta_f}(\theta)}, \end{array} \right. \quad (21)$$

with initial conditions $q_i^{ie}(\bar{\theta}_d, \bar{\theta}_f) = q_i^*(\bar{\theta}_d, \bar{\theta}_f)$, $i = d, f$.

Proof. See Appendix 11.4.1. ■

Superscript 'ie' stands for 'informative equilibrium'. To contrast with the next section, notice that in this equilibrium both governments extract the unknown information. Information acquisition necessitates some distortions on the production profiles to limit the rent the firm can command from its private information.

As one can see, the equilibrium quantity profiles are characterized by a system of partial differential equations. The most inefficient firm produces the optimal quantity in both countries. Recall that the most inefficient firm is the firm with cost parameters $(\bar{\theta}_d, \bar{\theta}_f)$. Therefore 'no distortion at the bottom' arises naturally as, given our information structure, a firm with type $\bar{\theta}_f$ is perceived as the most efficient firm by government G_d , and a firm with type $\bar{\theta}_d$ is perceived as the most efficient firm by government G_f .

As it is often the case in asymmetric common agency under adverse selection models, it is quite difficult to check that all the optimality conditions are satisfied in equilibrium. Under the assumption that these conditions are satisfied in equilibrium, as concerns the comparison with the complete information benchmark, we obtain the following corollary.

Corollary 1 *Assume that the Spence-Mirrlees and the second-order conditions are satisfied in equilibrium. In an informative equilibrium (i) there is never under-production in both countries simultaneously and (ii) the sum of the quantities produced in both countries is always smaller than the sum of the quantities under complete information.*

Proof. See Appendix 11.4.2. ■

In the next subsection, we illustrate and comment on the equilibrium distortions stated in Corollary 1 in a particular case in which all the optimality conditions are satisfied in equilibrium.

5.3 An illustration

In this section, we go one step further and compute the solutions to the partial differential equations characterizing the optimal quantities in a simple case. In this situation, all the optimality conditions are satisfied in equilibrium.

Corollary 2 *Assume that the private information parameters are independently and uniformly distributed on $[0, 1]$. In an informative equilibrium, with respect to the complete information situation*

- *there is under-production in country i if and only if $1 + 2\theta_i - 3\theta_j \geq 0$,*
- *the total quantity produced is always smaller.*

Proof. See Appendix 11.4.3. ■

Let $\Delta q_i^i(\theta) \equiv q_i^*(\theta) - q_i^{ie}(\theta)$ be the difference between the complete information and the informative equilibrium quantity profiles. Figure 2 represents those differences for both countries.

Insert Figure 2 here

There are basically two driving forces at work in our model. For informational reasons, government G_d is led to decrease the quantity produced in the domestic country to trigger an increase in the production in the foreign country via the substitutability of outputs at the firm level. The more efficient the firm is in the foreign country, the more it appears as inefficient to the government in the domestic country, and the larger is the distortion on the domestic quantity: This is the *information effect*. However, the more efficient the firm is in the foreign country, the larger is the quantity it will be required to produce by government G_f : This is the *efficiency effect*.

More precisely, let us consider the following polar cases. First, consider the case in which θ_f is very low. From the point of view of government G_d , such a firm appears as very inefficient and the distortion on the quantity produced in the

domestic country to obtain an increase in the production of the foreign country (to limit the rent of firms with larger θ_f) is large. Moreover, from the point of view of government G_f this firm appears as very efficient, implying that he will require a large production for the foreign country. Both the information and the efficiency effects are non ambiguous and lead to an under-production in the domestic country.

Secondly, consider a firm with a large θ_f . From government G_d 's perspective, this firm is very efficient and the distortion on the domestic country is weak, implying that the distortion caused on the foreign quantity is also small. However, this firm appears as very inefficient to the foreign government. Government G_f will require a low production for his country, leading (via the substitutability) to a large production for the domestic country. Those effects lead to an over-production in equilibrium.

For intermediate values of the cost parameters, the distortions come from the confrontation of those (sometimes conflicting) effects since each government tries to elicit a particular dimension of the private information.

6 Non informative equilibrium

We show that another type of equilibria may emerge as an outcome of our common agency game. In this equilibrium, each government offers a non informative contract that depends only on the piece of information he knows.

Proposition 4 *There exists a non informative equilibrium of the game between governments in which each government does not make the firm reveal the relevant information.*

Proof. See Appendix 11.5.1. ■

The intuition for this result is very simple. Indeed, let us assume that government G_f offers a non informative contract²⁷ $\{q_f^{nie}(\theta_f), T_f^{nie}(\theta_f)\}$ stipulating to the firm a (strictly positive) quantity level to be produced and a transfer to be paid whatever the information of the firm unknown to this government, θ_d . Note that this contract still depends on the information at the disposal of G_f , namely θ_f . Faced with such a contract, the firm can no longer trade off the production in the foreign country against the production in the domestic country. In its relation with government G_f , the indirect utility function of the firm becomes therefore

$$\hat{\pi}_d(\theta_f, q_f^{nie}(\theta_f)) = [p_f(q_f^{nie}(\theta_f)) - \theta_f]q_f^{nie}(\theta_f) - [q_f^{nie}(\theta_f) + q_d]^2 - T_f^{nie}(\theta_f),$$

implying that

$$\frac{\partial \pi}{\partial \theta_f}(\theta) = -q_f^{nie}(\theta_f) - \frac{dT_f^{nie}}{d\theta_f}(\theta_f) \quad \text{and} \quad \frac{\partial^2 \pi_d}{\partial q_d \partial \theta_f}(\theta_f, q_d(\theta)) = 0.$$

²⁷We use the term 'non informative' instead of 'pooling' because such contracts remain conditional on the government's information.

From the point of view of government G_d the rent of the firm becomes constant with respect to the quantity q_d . This leads government G_d to be unable to screen the different types of firm and to offer in turn a non informative contract. Hence, because initially government G_d can only modify the firm's rent in an indirect way, he can loose his screening ability as soon as the government of the foreign country refrains from making the firm reveal its information²⁸.

We can then easily determine the optimal quantities in a non informative equilibrium. Assuming that government G_d wants to make all types of firm produce, the following participation constraint must be satisfied

$$\pi(\theta_d, \underline{\theta}_f) \geq \pi_d^{nr}(\underline{\theta}_f).$$

This constraint will be binding in equilibrium. Standard computations enable to derive the equilibrium quantity profiles

$$\begin{cases} q_d^{nie}(\theta_d) = \frac{1}{3} [(a_d - \theta_d) + \frac{4}{5}(a_d - \underline{\theta}_d) - \frac{6}{5}(a_f - \underline{\theta}_f)], \\ q_f^{nie}(\theta_f) = \frac{1}{3} [(a_f - \theta_f) + \frac{4}{5}(a_f - \underline{\theta}_f) - \frac{6}{5}(a_d - \underline{\theta}_d)]. \end{cases}$$

Then, we obtain the following proposition.

Proposition 5 *In a non informative equilibrium, with respect to the complete information situation*

- *there is under-production in country i if and only if $3\theta_j - 2\theta_i \geq 0$,*
- *the total quantity produced is always smaller.*

Proof. See Appendix 11.5.2. ■

Let $\Delta q_i^{ni}(\theta) \equiv q_i^*(\theta) - q_i^{nie}(\theta)$ be the difference between the complete information and the non informative equilibrium quantity profiles. Figure 3 represents those differences for both countries.

Insert Figure 3 here

With respect to the quantities in an informative equilibrium, the pattern of distortions is quite different. In a non informative equilibrium, the government in the domestic country considers that the firm's cost parameter in the foreign country is equal to $\underline{\theta}_f$ (since the non relocation constraint is binding for this type of firm in the domestic country). Therefore, for a fixed quantity produced in the foreign country, the quantity required by the domestic government tends to be smaller than the complete information one. This effect is reinforced if the true cost parameter θ_f is low as the government in the foreign country will indeed implement a large quantity

²⁸Laffont and Tirole (1991) also exhibit an equilibrium in pooling contracts in a common agency framework with perfectly complementary activities.

in his country. On the contrary, if θ_f is sufficiently large, then a low production in the foreign country tends to increase the production in the domestic country, via the substitutability of activities at the firm level. Both effects combine in equilibrium to yield the pattern of distortions illustrated in Figure 3.

7 Welfare analysis of informative and non informative equilibria

We assume that the private information parameters of the firm are independently and uniformly distributed on $[0, 1]$. We also assume that market sizes are identical, $a_d = a_f \equiv a$. In order to ensure that in each equilibrium the optimal quantity profiles are strictly positive, we must assume that a is sufficiently large. We perform the welfare comparisons from stage 1 of our game (i.e., in expectation over both private information parameters).

Proposition 6 *The non informative equilibrium Pareto-dominates the revelation equilibrium.*

Proof. See Appendix 11.6.1. ■

Notice first that, given the symmetry of the model, this result holds both from the perspective of the individual welfare of a country and also from the point of view of the sum of the countries' welfare.

This quite surprising proposition shows that the competition between tax authorities is so strong that the distortions needed for informational reasons under an informative equilibrium become very large, leading each government to prefer the non informative equilibrium. Despite the fact that the non informative equilibrium disregards some information which is relevant, it dominates the informative equilibrium. In the end, if governments had the choice or the possibility to coordinate on a particular equilibrium of our tax competition game, they would choose to tax the firm according to the available information only. By sticking to a non informative contract, governments have no longer to compete each against the other and to suffer from the distortions needed to ensure the revelation of information.

This is not to say that non informative tax systems (i.e., tax systems based only on local information) are the panacea. We could even conjecture that, in an idealized world of perfect cooperation between governments, an optimal tax system would make the firm reveal its information. However, when governments act non cooperatively, a combination of lump sum taxes and fixed productions perform better than incentive-based taxes. since they reduce the competition for information acquisition between governments.

Finally, we also obtain the following proposition²⁹.

²⁹Whether the results of Propositions 6 and 7 hold for more general parameterizations remains an open question.

Proposition 7 *The firm's rent is larger in an informative equilibrium than in a non informative equilibrium.*

Proof. See Appendix 11.6.2. ■

This proposition is obtained by comparing the firm's rent from stage 2 of our game, that is for all possible cost parameters. In an informative equilibrium, because the firm can play each government against each other, the firm obtains a large rent. This proposition shows (for our particular example) that one of the advantages of the non informative equilibrium is to provide the firm with less rent. It also shows that the firm has always an incentive to induce one government to compete against another.

8 Asymmetric equilibria

So far, we considered situations in which both equilibrium quantities were positive. However, as exemplified in the complete information case, for instance when market sizes are too different, a government might sometimes prefer to shut down the production in his country.

The purpose of this section is twofold. First, we show that the same kind of asymmetric equilibria emerge under incomplete information. Second, we show that incomplete information also generates the incentive and the possibility for a government to try to exclude the other government from the firm's services through an 'aggressive' tax system.

8.1 Standard exclusion

Let us assume for the moment that in equilibrium the quantity profile of the foreign country computed in Section 5.2 is negative for all parameters values. We have then to determine an equilibrium in which $q_f(\theta) = 0$ for all possible cost parameters.

In such a case, that is, when the government in the foreign country does not make the firm produce at all, the government in the domestic country is under complete information with respect to the firm. Consequently, he will implement the quantity $q_d^{*a}(\theta) = \frac{1}{3}(a_d - \theta_d)$ and designs his tax schedule in such a way that (i) faced with this tax the firm with type θ chooses the quantity $q_d^{*a}(\theta)$ and (ii) the firm gets no rent in equilibrium. A two-part tax schedule is therefore sufficient.

Finally, to ensure that such a situation is an equilibrium, it just remains to check that when government G_d offers this two-part tax schedule government G_f is effectively willing to shut down the production in his country. This enables us to state the following proposition.

Proposition 8 *Under incomplete information, if $\frac{a_i - \bar{\theta}_i}{a_j - \bar{\theta}_j} > \frac{3}{2}$, then there exists an asymmetric equilibrium with quantities $q_i^{*a}(\theta) = \frac{1}{3}(a_i - \theta_i)$ and $q_j^{*a}(\theta) = 0$, $i, j = d, f$ and $i \neq j$.*

Proof. See Appendix 11.7.1. ■

In view of the results obtained under complete information this proposition is not very surprising. However, as we shall demonstrate in the next subsection, incomplete information also gives rise to other asymmetric equilibria.

8.2 Voluntary exclusion

One issue to implement and enforce coordinated taxation from governments is probably the resistance by smaller countries which can, by mean of ‘machiavellian’ taxation, simply induce the firm to relocate all its activities in their own country, whatever other countries do. We illustrate this point in a simple example

One of the characteristics of a common agency game is that what is proposed by one government affects what can be achieved by the other government. In this subsection, we show that the government in the domestic country can offer such an incentive contract to the firm that the other government can no longer tax the firm efficiently.

Proposition 9 *Under incomplete information, if $\frac{a_i - \theta_i}{a_j - \theta_j} > \frac{3}{2}$ then there exists an asymmetric equilibrium with quantities $q_i^{*a}(\theta) = \frac{1}{3}(a_i - \theta_i)$ and $q_j^{*a}(\theta) = 0$, $i, j = d, f$ and $i \neq j$: Government G_i can enlarge the set of parameters for which government G_j is excluded through the use of a quadratic tax instead of a linear one.*

Proof. See Appendix 11.7.2. ■

The intuition for this result goes as follows. If government G_f decides to shut down the production in his country, then government G_d is under complete information with respect to the firm. As explained earlier on, this implies that a two-part tax schedule is enough to decentralize the corresponding production decision to the firm.

However, suppose that government G_d decides to use a quadratic tax schedule. The quadratic part becomes a free parameter³⁰ and has a strong influence on the firm’s behavior with respect to the other government. In particular, we show that this free parameter can be chosen in such a way that the monotonicity condition of the optimal quantity profile in the foreign country is violated! Hence, the government in this country loses his screening ability and is forced to offer a non informative contract to the firm. In contrast to the non informative equilibrium determined in Section 6, it turns out that the choice of the quadratic part of the transfer in the domestic country also affects the objective of the government in the foreign country, which becomes a convex function of the quantity implemented in this country. We then show that faced with such a contract, under the condition stated in the previous proposition, government G_f prefers to shut down the production in his country.

³⁰Actually, it is only restricted to satisfy the second-order conditions of the firm’s maximization problem.

The interest of the last proposition lies in the fact that by proposing a quadratic tax schedule instead of a two-part one, government G_d modifies the firm's incentives with respect to the other government and can finally enlarge the zone of parameters values such that the foreign country is forced to shut down the production. Indeed, we explained previously that if government G_i were using a two-part tax schedule, then government G_j would be excluded from the firm's services under the condition stated in Proposition 8, i.e., $\frac{a_i - \bar{\theta}_i}{a_j - \bar{\theta}_j} > \frac{3}{2}$. With a quadratic tax, G_i modifies the firm's behavior in its relation with the other government, and the zone of parameters such that G_j is excluded is enlarged and given by $\frac{a_i - \bar{\theta}_i}{a_j - \bar{\theta}_j} > \frac{3}{2}$. Notice finally that the qualitative nature of these two types of exclusion is substantially different.

Actually, what the excluding government does is just to offer a very high discount (see Appendix 11.7.2, Lemma 6)³¹. Notice that a condition for one government to exclude another is to have a much bigger market size in his country. More generally, the condition bears on the relative valuation of the countries for the goods produced by the multinational firm. A country with a large valuation can attract the multinational by offering a nonlinear tax schedule with a large quantity discount. Such a nonlinear tax schedule is not needed in equilibrium since the domestic government is under complete information with respect to the firm; but, out of equilibrium, it enables to alter the firm's incentive in its relation with the other government, leading finally that latter to stop the production in his country. This is typically an instance of a harmful tax practice pointed out within the EU. A country offers a tax system with a large quantity discount, which a priori would reduce the tax revenues. However, the tax base becomes much larger as the firm in the end substitutes the whole production to relocate it only in the country offering the 'machievellian' tax policy. That latter country can then totally benefit from his knowledge of the local production condition since the firm produces only in this country.

Finally, notice that in the zone of voluntary exclusion, the excluded government obviously prefers not to contract at all with the firm. Had we added to our game a first stage in which each government decides to contract or not with the firm we would have obtained that the excluded government prefers not to tax the firm. In this case, we are back to the unilateral taxation situation.

9 Conclusion

Throughout the paper, we have focused on the taxation of a multinational firm by non cooperative governments. The role of local informational advantage has been shown to create perverse incentives when the firm can relocate its production at no

³¹The quadratic part of the tax schedule in an equilibrium with voluntary exclusion is equal to $-\frac{8}{3}$ and is smaller than $\frac{-13+\sqrt{41}}{4}$ which is the quadratic part of the tax schedule in the informative equilibrium (with uniform distributions).

cost: A government can try to exclude his rival in order to reach all the benefits associated to his knowledge on some dimension of the firm's private information.

In this section, we would like to convince the reader that our insights could be applied to other economic settings, in particular to the debate about decentralization versus centralization.

Indeed, in the political economy arena, it has often been argued that one basic trade-off between those two constitutions is the following: On the one hand, centralization enables to internalize the externalities between countries. On the flip side, decentralization enables to observe more precisely some policy-relevant variables that remain unobservable at the central level. This is the case for instance in Seabright (1996) (although the focus is more on the impact of decentralization on the accountability of politicians) or in Caillaud, Jullien and Picard (1996).

One could model this problematic as a common agency game in which under decentralization the decisions are undertaken by two noncooperative principals, whereas under centralization both decisions are reached by a unique principal³².

The basic message conveyed by our results would lead us to conjecture that the informational superiority of each principal might not be such a strong argument in favor of decentralization. Indeed, noncooperative principals may engage in a process of information acquisition that drives them to a Pareto-dominated equilibrium. Moreover, this also gives the possibility and the incentive for a principal to engage in actions aiming at excluding the other government, a situation which may not be satisfactory from the point of view of total social welfare.

Although we think that it would be interesting to compare our model of decentralization with the case of centralization, that is, when both activities are under the jurisdiction of a unique government (with the caveat that it would involve multi-dimensional screening techniques...), we also think that an interesting research area concerns the design of rules that implement a certain level of cooperation between principals, either through information sharing or through the coordination on a Pareto-non dominated equilibrium. For instance, Bond and Gresik (1998) have shown, in a related setting of principals with different information structures, that allowing governments to communicate and exchange some information on the firm's cost parameter might not be welfare-improving. The basic reason is that information sharing needs to satisfy some incentive constraints. Future research should extend our model to allow for information transmission between governments before the tax competition game.

³²See Laffont and Pouyet (2000) for a model along those lines.

10 References

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11 Appendix

11.1 Complete information

11.1.1 Proof of Proposition 1

We consider direct contracts of the form $\{T_i(\theta), q_i(\theta)\}$, $i = d, f$. Since the rent of the firm is socially costly for the domestic government, the (GP) constraint will be binding in equilibrium, or

$$T_d(\theta) = \sum_{i=d,f} [p_i(q_i(\theta)) - \theta_i] q_i(\theta) - [q_d(\theta) + q_f(\theta)]^2 - T_f(\theta).$$

Replacing this value of $T_d(\theta)$ in the domestic government's objective and optimizing with respect to $q_d(\theta)$ yields³³

$$3q_d(\theta) + 2q_f(\theta) = a_d - \theta_d.$$

Performing a similar exercise for the foreign government yields the following first-order condition

$$3q_f(\theta) + 2q_d(\theta) = a_f - \theta_f. \quad (22)$$

Solving these conditions yields the full information quantity profiles (4). However, we must check that in equilibrium both quantities are positive. Direct computations show that this will be the case if

$$\frac{3}{2} \geq \frac{a_d - \theta_d}{a_f - \theta_f} \geq \frac{2}{3}.$$

If, say, $\frac{a_i - \theta_i}{a_j - \theta_j} > \frac{3}{2}$, then government G_j prefers to shut down the production in his country. When $q_j(\theta) = 0$, we can perform a similar exercise and find government G_i 's optimal production level³⁴. Finally, using the foreign government's best-response (22), we can check that, indeed, the foreign government is not willing to make the firm produce a positive quantity in its country.

11.1.2 Proof of Lemma 1

We first show that two-part taxes enable the governments to decentralize the equilibrium quantity profiles. Then, we show that 'augmented' instruments, in the form of quadratic tax schedules instead of two-part ones, are useless under complete information.

³³Notice that the second-order conditions of the program of each government are trivially satisfied.

³⁴Notice that the second-order condition of the program of government G_i is trivially satisfied.

Equilibrium tax schedules under complete information So far, we considered direct mechanisms. We could have instead considered indirect mechanisms. In equilibrium, government G_i must ensure that

- the firm's participation constraint (GP) is binding,
- the firm's choice of production in his country coincides with the optimal quantity for this country.

Assume that the government in country i imposes a two-part tax schedules of the form $T_i(\theta, q_i) = \gamma_i(\theta) + \alpha_i(\theta)q_i$. The fixed part must be such that the participation constraint of the firm is binding. We can then compute the corresponding choice of quantities by the firm, replace these quantities in the governments' objectives, and optimize government G_i 's objective with respect to $\alpha_i(\theta)$. This gives two first-order conditions. Solving for this system gives the variable parts of the taxes

$$\begin{cases} \alpha_d(\theta) = -\frac{1}{5} [3(a_d - \theta_d) - 2(a_f - \theta_f)], \\ \alpha_f(\theta) = -\frac{1}{5} [3(a_f - \theta_f) - 2(a_d - \theta_d)]. \end{cases}$$

For the fixed parts, we obtain

$$\gamma_d(\theta) + \gamma_f(\theta) = \frac{2}{25} [7(a_d - \theta_d)^2 + 7(a_f - \theta_f)^2 - 11(a_d - \theta_d)(a_f - \theta_f)].$$

Finally, we must check that, in equilibrium, the firm is effectively willing to accept both contracts simultaneously. The firm's maximization problem is

$$\max_{q_d > 0, q_f > 0} \left\{ \sum_{i=d,f} \{ [p_i(q_i) - \theta_i]q_i - T_i(\theta_i, q_i) \} - (q_d + q_f)^2 \right\}.$$

The Hessian associated to the firm's maximization problem must be definite semi-negative. This implies that the following conditions must hold in equilibrium

$$\begin{cases} 4 + \frac{\partial^2 T_i(\theta_i, q_i)}{\partial q_i^2} \geq 0, i = d, f, \\ \left[4 + \frac{\partial^2 T_d(\theta_d, q_d)}{\partial q_d^2} \right] \left[4 + \frac{\partial^2 T_f(\theta_f, q_f)}{\partial q_f^2} \right] \geq 4. \end{cases} \quad (23)$$

Under complete information, since two-part tax schedules are used to decentralize the choice of the equilibrium production decisions at the firm level, these conditions are trivially satisfied.

Quadratic tax schedules are useless We want to confirm the fact that nonlinear tax schedules are useless under complete information. To this purpose, assume that government G_i , $i = d, f$, offers the firm a quadratic tax schedule $T_i(\theta, q_i) = \gamma_i(\theta) + \alpha_i(\theta)q_i + \frac{1}{2}\beta_i(\theta)q_i^2$.

Faced with these tax systems, the firm's production decisions in both countries are given by

$$\begin{cases} q_d(\theta) &= \frac{(a_d - \theta_d - \alpha_d)(4 + \beta_d) - 2(a_f - \theta_f - \alpha_f)}{4(3 + \beta_f) + \beta_d(4 + \beta_f)}, \\ q_f(\theta) &= \frac{(a_f - \theta_f - \alpha_f)(4 + \beta_f) - 2(a_d - \theta_d - \alpha_d)}{4(3 + \beta_d) + \beta_f(4 + \beta_d)}. \end{cases}$$

Let us come back to the governments' optimization problems. Let us first replace the quantities by the previous production profiles. As previously, the (GP) constraint will be binding in equilibrium and will define the sum of the fixed parts of the tax schedules.

Then, we must find the optimal coefficients $\alpha_i(\theta)$; optimizing government G_i 's objective with respect to $\alpha_i(\theta)$, $i = d, f$, and solving the system formed by these two equations, we obtain

$$\begin{cases} \alpha_d(\theta) &= -\frac{1}{5}(1 + \beta_d) [3(a_d - \theta_d) - 2(a_f - \theta_f)], \\ \alpha_f(\theta) &= -\frac{1}{5}(1 + \beta_f) [3(a_f - \theta_f) - 2(a_d - \theta_d)]. \end{cases}$$

Replace these values in government G_i 's objective. Then, simple computations show that this objective does no longer depend on $\beta_i(\theta)$. Therefore, adding a quadratic term to the tax schedule is useless under complete information.

11.2 Proof of Lemma 2

The proof is entirely borrowed from Ivaldi and Martimort (1996). We decompose the tax schedule offered by government G_i as the sum of a fixed and a variable part

$$T_i(\theta_i, q_i) = \gamma_i(\theta_i) + t_i(\theta_i, q_i).$$

From the point of view of government G_d , we want to show that $\pi_d^{nr}(\underline{\theta}_f) \geq 0$. For the proof of this lemma, in order to alleviate the presentation, we denote by π^g the gross profit of the firm. When the (NR_f) constraint is binding in the program of the foreign government, we obtain

$$\begin{aligned} \gamma_f(\theta_f) &= \max_{q_d > 0, q_f > 0} \{ \pi^g(\underline{\theta}_d, \theta_f, q_d, q_f) - T_d(\underline{\theta}_d, q_d) - t_f(\theta_f, q_f) \} \\ &\quad - \max_{q_d > 0} \{ \pi^g(\underline{\theta}_d, \theta_f, q_d, q_f = 0) - T_d(\underline{\theta}_d, q_d) \}. \end{aligned}$$

From the point of view of government G_d , the outside opportunity of the firm is

$$\pi_d^{nr}(\theta_f) = -\gamma_f(\theta_f) + \max_{q_f > 0} \{ \pi^g(\theta_d, \theta_f, q_d = 0, q_f) - t_f(\theta_f, q_f) \}.$$

Hence, $\pi_d^{nr}(\underline{\theta}_f) \geq 0$ is equivalent to

$$\begin{aligned} \max_{q_d > 0} \{ \pi^g(\underline{\theta}_d, \underline{\theta}_f, q_d, q_f = 0) - T_d(\underline{\theta}_d, q_d) \} + \max_{q_f > 0} \{ \pi^g(\underline{\theta}_d, \underline{\theta}_f, q_d = 0, q_f) - T_f(\underline{\theta}_f, q_f) \} \\ \geq \max_{q_d > 0, q_f > 0} \{ \pi^g(\underline{\theta}_d, \underline{\theta}_f, q_d, q_f) - T_d(\underline{\theta}_d, q_d) - T_f(\underline{\theta}_f, q_f) \}, \end{aligned}$$

which can be rewritten as follows

$$\begin{aligned} \max_{q_d > 0, q_f > 0} \{ \pi^g(\underline{\theta}_d, \underline{\theta}_f, q_d, q_f = 0) + \pi^g(\underline{\theta}_d, \underline{\theta}_f, q_d = 0, q_f) - T_d(\underline{\theta}_d, q_d) - T_f(\underline{\theta}_f, q_f) \} \geq \\ \max_{q_d > 0, q_f > 0} \{ \pi^g(\underline{\theta}_d, \underline{\theta}_f, q_d, q_f = 0) + \pi^g(\underline{\theta}_d, \underline{\theta}_f, q_d = 0, q_f) - 2q_d q_f - T_d(\underline{\theta}_d, q_d) - T_f(\underline{\theta}_f, q_f) \}. \end{aligned}$$

Denote by (q_d^*, q_f^*) and (q_d^{**}, q_f^{**}) the optimal values of the maximization problem in the left-hand side and the right-hand side respectively. We have $q_i^* \geq q_i^{**}$, $i = d, f$. Because the maximand of each problem is increasing and concave, the value function of the problem in the left-hand side is larger than the value function in the right-hand side.

If the (NR_i) participation constraint is added in government G_i 's problem, then the equilibrium quantity profiles remain unchanged. Indeed, for, say, the domestic government, the outside opportunity of the firm when that latter decides to produce only in the foreign country does not depend on the production level in the domestic country. Therefore, the equilibrium quantity profiles are not affected by this additional constraint, but each government must provide the firm with a positive rent to ensure that the firm will not produce for the other country only.

11.3 Unilateral taxation

11.3.1 Proof of Lemma 3

Let us consider that government G_d uses a direct contract. The rent of a firm with private information θ and that announces $\tilde{\theta}_f$ to government G_d is then

$$\pi(\tilde{\theta}_f, \theta) = [p_d(q_d(\theta_d, \tilde{\theta}_f)) - \theta_d]q_d(\theta_d, \tilde{\theta}_f) + \hat{\pi}_d^{ut}(\theta_f, q_d(\theta_d, \tilde{\theta}_f)) - T_d(\theta_d, \tilde{\theta}_f). \quad (24)$$

The first-order condition for incentive compatibility is

$$\frac{\partial \pi}{\partial \tilde{\theta}_f}(\tilde{\theta}_f, \theta) \Big|_{\tilde{\theta}_f = \theta_f} = 0$$

which can be rewritten as

$$\frac{\partial \pi}{\partial \theta_f}(\theta) = \frac{\partial \hat{\pi}_d^{ut}}{\partial \theta_f}(\theta_f, q_d(\theta)).$$

Differentiating the first-order condition (19) (that must hold as an equality in equilibrium), the second-order condition for implementability

$$\frac{\partial^2 \pi}{\partial \tilde{\theta}_f^2}(\tilde{\theta}_f, \theta)|_{\tilde{\theta}_f = \theta_f} \leq 0$$

reduces to (20).

11.3.2 Proof of Proposition 2

Given that

$$\pi_d^{nr,ut}(\theta_f) = \frac{1}{8}(a_f - \theta_f)^2$$

and that

$$\frac{\partial \pi^{ut}}{\partial \theta_f}(\theta) = \frac{\partial \hat{\pi}_d^{ut}}{\partial \theta_f}(\theta_f, q_d) = -\hat{q}_f^{ut}(\theta_f, q_d),$$

we obtain

$$\frac{\partial}{\partial \theta_f} \left[\pi^{ut}(\theta) - \pi_d^{nr,ut}(\theta_f) \right] = \frac{1}{2} q_d(\theta) \geq 0.$$

Therefore, if the (NR_d) constraint is satisfied for $\theta_f = \underline{\theta}_f$, then it is satisfied for all θ_f . Since the rent of the firm is socially costly for the domestic government, the (NR_d) constraint will be binding in $\theta_f = \underline{\theta}_f$.

Using the first-order condition for incentive compatibility, we can rewrite the firm's rent from the viewpoint of the domestic government as follows

$$\begin{aligned} \pi^{ut}(\theta) &= \pi_d^{nr,ut}(\underline{\theta}_f) + \int_{\underline{\theta}_f}^{\theta_f} \frac{\partial \hat{\pi}_d^{ut}}{\partial \theta_f}(x, q_d(\theta_d, x)) dx \\ &= \pi_d^{nr,ut}(\underline{\theta}_f) - \int_{\underline{\theta}_f}^{\theta_f} \hat{q}_f^{ut}(x, q_d(\theta_d, x)) dx. \end{aligned}$$

Using an integration by parts, we find that the expected rent of the firm from the viewpoint of the domestic government can be rewritten as follows

$$\mathbf{E}_{\theta_f} \{ \pi^{ut}(\theta) \} = \pi_d^{nr,ut}(\underline{\theta}_f) - \int_{\underline{\theta}_f}^{\bar{\theta}_f} \frac{1 - H_f(\theta_f)}{h_f(\theta_f)} \hat{q}_f^{ut}(\theta_f, q_d(\theta)) dH_f(\theta_f).$$

The expected welfare in the domestic country can therefore be rewritten as follows

$$\begin{aligned} \mathbf{E}_{\theta_f} \{ SW_d \} &= \int_{\underline{\theta}_f}^{\bar{\theta}_f} \left[\int_0^{q_d(\theta)} p_d(x) dx - \theta_d q_d(\theta) \right. \\ &\quad \left. + \hat{\pi}_d^{ut}(\theta_f, q_d(\theta)) - \pi_d^{nr,ut}(\underline{\theta}_f) + \frac{1 - H_f(\theta_f)}{h_f(\theta_f)} \hat{q}_f^{ut}(\theta_f, q_d(\theta)) \right] dH_f(\theta_f). \end{aligned}$$

Taking into account that $\hat{q}_f^{ut}(\theta_f, q_d) = \frac{1}{4}(a_f - \theta_f - 2q_d)$, pointwise optimization with respect to the domestic quantity yields the following first-order condition in the domestic country

$$3q_d^{ut}(\theta) + 2q_f^{ut}(\theta) = a_d - \theta_d - \frac{1}{2} \frac{1 - H_f(\theta_f)}{h_f(\theta_f)}.$$

In equilibrium, the firm's production decision in the foreign country satisfies the following first-order condition

$$2q_d^{ut}(\theta) + 4q_f^{ut}(\theta) = a_f - \theta_f.$$

Solving these two first-order conditions yields the following quantity profiles

$$\begin{cases} q_d^{ut}(\theta) &= \frac{1}{8} \left[4 \left(a_d - \theta_d - \frac{1}{2} \frac{1 - H_f(\theta_f)}{h_f(\theta_f)} \right) - 2(a_f - \theta_f) \right], \\ q_f^{ut}(\theta) &= \frac{1}{8} \left[3(a_f - \theta_f) - 2 \left(a_d - \theta_d - \frac{1}{2} \frac{1 - H_f(\theta_f)}{h_f(\theta_f)} \right) \right]. \end{cases}$$

Finally notice that the second-order incentive compatibility constraint for the domestic government

$$\frac{\partial q_d^{ut}}{\partial \theta_f}(\theta) \geq 0 \Leftrightarrow -2 \frac{d}{d\theta_f} \frac{1 - H_f(\theta_f)}{h_f(\theta_f)} + 2 \geq 0$$

is satisfied under the monotone hazard rate assumption.

11.4 Informative equilibrium

11.4.1 Proof of Proposition 3

We define

$$q_f^{nr}(\theta_f) \in \arg \max_{q_f} \{\pi(\theta, q_d = 0, q_f)\}, \quad (25)$$

and assume that it can always be characterized by the corresponding first-order condition

$$4q_f^{nr}(\theta_f) + \frac{\partial T_f}{\partial q_f}(\theta_f, q_f^{nr}(\theta_f)) = a_f - \theta_f. \quad (26)$$

Remember that, by definition, we have

$$4\hat{q}_f(\theta_f, q_d) + \frac{\partial T_f}{\partial q_f}(\theta_f, \hat{q}_f(\theta_f, q_d)) = a_f - \theta_f - 2q_d.$$

Define the following function

$$\phi(q_f) = 4q_f + \frac{\partial T_f}{\partial q_f}(\theta_f, q_f).$$

We have

$$\phi'(q_f) = 4 + \frac{\partial^2 T_f}{\partial q_f^2}(\theta_f, q_f) \geq 0$$

since the firm's optimality conditions (23) must be satisfied³⁵. Therefore,

$$(\phi^{-1})' = \frac{1}{\phi'} \geq 0.$$

Since q_d is positive, this implies that

$$q_f^{nr}(\theta_f) \geq \hat{q}_f(\theta_f, q_d).$$

Now, direct computations show that

$$\frac{\partial}{\partial \theta_f} [\pi(\theta) - \pi^{nr}(\theta_f)] = \int_{\hat{q}_f(\theta_f, q_d(\theta))}^{q_f^{nr}(\theta_f)} \left[1 + \frac{\partial^2 T_f}{\partial \theta_f \partial q_f}(\theta_f, x) \right] dx.$$

Because $q_f^{nr}(\theta) \geq \hat{q}_f(\theta_f, q_d(\theta))$, $\frac{\partial}{\partial \theta_f} [\pi(\theta) - \pi^{nr}(\theta_f)] \geq 0$ if and only if $1 + \frac{\partial^2 T_f}{\partial \theta_f \partial q_f}(\theta_f, q_f) \geq 0$. We shall show that, in an equilibrium where all the optimality conditions are satisfied, this must indeed be the case (see Section 11.4.3 on the proof of Corollary 1). This implies that the net rent of the firm is increasing in θ_f . The non relocation participation constraint amounts to $\pi(\theta_d, \underline{\theta}_f) \geq \pi_d^{nr}(\underline{\theta}_f)$. As the rent of the firm is socially costly, this constraint will be binding in equilibrium.

From the first-order incentive compatibility constraint (19), we can rewrite the firm's rent as follows

$$\pi(\theta) = \pi_d^{nr}(\underline{\theta}_f) + \int_{\underline{\theta}_f}^{\theta_f} \frac{\partial \hat{\pi}_d}{\partial \theta_f}(x, q_d(\theta_d, x)) dx.$$

After an integration by parts, the expected rent from government G_d 's perspective can be rewritten as

$$\mathbf{E}_{\theta_f} \{ \pi(\theta) \} = \pi_d^{nr}(\underline{\theta}_f) + \int_{\underline{\theta}_f}^{\bar{\theta}_f} \frac{1 - H_f(\theta_f)}{h_f(\theta_f)} \frac{\partial \hat{\pi}_d}{\partial \theta_f}(\theta_f, q_d(\theta)) dH_f(\theta_f).$$

³⁵More precisely, the optimality conditions for the firm's maximization problem must hold in equilibrium. As emphasized in Martimort and Stole (1998), the approach taken to compute the governments' best-responses requires that we extend the indirect taxes used by the governments to account for possible out-of-equilibrium reports lying outside the relevant intervals, or, equivalently, that we extend the indirect taxes for all possible quantities. As shown by Martimort (1992), it is possible to perform such extensions in a linear way. Therefore, for all possible q_f , we have $\phi' \geq 0$.

The expected welfare in the domestic country can therefore be rewritten as

$$\mathbf{E}_{\theta_f}\{SW_d\} = \int_{\underline{\theta}_f}^{\bar{\theta}_f} \left[\int_0^{q_d(\theta)} p_d(x) dx - \theta_d q_d(\theta) + \hat{\pi}_d(\theta_f, q_d(\theta)) - \pi_d^{nr}(\underline{\theta}_f) - \frac{1 - H_f(\theta_f)}{h_f(\theta_f)} \frac{\partial \hat{\pi}_d}{\partial \theta_f}(\theta_f, q_d(\theta)) \right] dH_f(\theta_f).$$

Pointwise optimization with respect to $q_d(\theta)$ yields

$$3q_d(\theta) + 2\hat{q}_f(\theta_f, q_d(\theta)) = a_d - \theta_d - 2 \frac{1 - H_f(\theta_f)}{h_f(\theta_f)} \frac{1 + \frac{\partial^2 T_f}{\partial \theta_f \partial q_f}(\theta_f, \hat{q}_f(\theta_f, q_d(\theta)))}{4 + \frac{\partial^2 T_f}{\partial q_f^2}(\theta_f, \hat{q}_f(\theta_f, q_d(\theta)))}, \quad (27)$$

where we used the fact that Equation (16) implies

$$\begin{cases} \frac{\partial \hat{\pi}_d}{\partial q_d}(\theta_f, q_d(\theta)) = -2 [\hat{q}_f(\theta_f, q_d(\theta)) + q_d(\theta)] \\ \frac{\partial^2 \hat{\pi}_d}{\partial q_d \partial \theta_f}(\theta_f, q_d(\theta)) = -\frac{\partial \hat{q}_f}{\partial q_d}(\theta_f, q_d(\theta)) \left[1 + \frac{\partial^2 T_f}{\partial \theta_f \partial q_f}(\theta_f, \hat{q}_f(\theta_f, q_d(\theta))) \right], \end{cases}$$

and Equation (17) implies

$$\frac{\partial \hat{q}_f}{\partial q_d}(\theta_f, q_d(\theta)) = \frac{-2}{4 + \frac{\partial^2 T_f}{\partial q_f^2}(\theta_f, \hat{q}_f(\theta_f, q_d(\theta)))}. \quad (28)$$

We will now proceed to the final simplifications. In equilibrium we have

$$\hat{q}_f(\theta_f, q_d(\theta)) \equiv q_f(\theta).$$

Substituting in (17) and differentiating with respect to θ_d and θ_f yields respectively

$$\begin{cases} \left[4 + \frac{\partial^2 T_f}{\partial q_f^2}(\theta_f, q_f(\theta)) \right] \frac{\partial q_f}{\partial \theta_d}(\theta) = -2 \frac{\partial q_d}{\partial \theta_d}(\theta), \\ \frac{\partial q_f}{\partial \theta_f}(\theta) \left[4 + \frac{\partial^2 T_f}{\partial q_f^2}(\theta_f, q_f(\theta)) \right] + 2 \frac{\partial q_d}{\partial \theta_f}(\theta) = - \left[1 + \frac{\partial^2 T_f}{\partial \theta_f \partial q_f}(\theta_f, q_f(\theta)) \right], \end{cases} \quad (29)$$

which can finally be rearranged as follows

$$\begin{cases} 1 + \frac{\partial^2 T_f}{\partial \theta_f \partial q_f}(\theta_f, q_f(\theta)) = 2 \frac{\frac{\partial q_f}{\partial \theta_f}(\theta) \frac{\partial q_d}{\partial \theta_d}(\theta) - \frac{\partial q_f}{\partial \theta_d}(\theta) \frac{\partial q_d}{\partial \theta_f}(\theta)}{\frac{\partial q_f}{\partial \theta_d}(\theta)}, \\ 4 + \frac{\partial^2 T_f}{\partial q_f^2}(\theta_f, q_f(\theta)) = -2 \frac{\frac{\partial q_d}{\partial \theta_d}(\theta)}{\frac{\partial q_f}{\partial \theta_d}(\theta)}. \end{cases} \quad (30)$$

Inserting those values into (27) gives the equilibrium quantity profile in the domestic country. The quantity profile in the foreign country is obtained in a similar way.

11.4.2 Proof of Corollary 1

Assume that the Spence-Mirrlees conditions are satisfied in equilibrium, or

$$\begin{aligned} \frac{\partial^2 \hat{\pi}_i}{\partial q_i \partial \theta_j}(\theta_j, q_i(\theta)) \geq 0 &\Leftrightarrow \frac{1 + \frac{\partial^2 T_j}{\partial \theta_j \partial q_j}(\theta_j, q_j(\theta))}{4 + \frac{\partial^2 T_j}{\partial q_j^2}(\theta_j, q_j(\theta))} \geq 0 \\ &\Leftrightarrow \frac{\frac{\partial q_j}{\partial \theta_j}(\theta) \frac{\partial q_i}{\partial \theta_i}(\theta) - \frac{\partial q_j}{\partial \theta_i}(\theta) \frac{\partial q_i}{\partial \theta_j}(\theta)}{-\frac{\partial q_i}{\partial \theta_i}(\theta)} \geq 0. \end{aligned}$$

Assume also that the monotonicity condition is satisfied, or

$$\frac{\partial q_i}{\partial \theta_j}(\theta) \geq 0.$$

To ensure that the optimality conditions of the firm's maximization problem (23) are satisfied in equilibrium, we also must have (from (30))

$$\frac{\partial q_i}{\partial \theta_i}(\theta) \leq 0.$$

This implies that in equilibrium we have

$$\frac{\partial q_i}{\partial \theta_i}(\theta) \frac{\partial q_j}{\partial \theta_j}(\theta) - \frac{\partial q_i}{\partial \theta_j}(\theta) \frac{\partial q_j}{\partial \theta_i}(\theta) \geq 0,$$

and

$$1 + \frac{\partial^2 T_f}{\partial \theta_f \partial q_f}(\theta_f, q_f(\theta)) \geq 0.$$

The system of first-order conditions (21) can be rewritten as

$$\begin{cases} q_d^{ie}(\theta) &= q_d^*(\theta) + \frac{1}{5} [3X_d(\theta) - 2X_f(\theta)], \\ q_f^{ie}(\theta) &= q_f^*(\theta) + \frac{1}{5} [3X_f(\theta) - 2X_d(\theta)], \end{cases} \quad (31)$$

where $X_i(\theta) = 2 \frac{1-H_j(\theta_j)}{h_j(\theta_j)} \frac{\frac{\partial q_i^{ie}}{\partial \theta_i}(\theta) \frac{\partial q_j^{ie}}{\partial \theta_j}(\theta) - \frac{\partial q_i^{ie}}{\partial \theta_j}(\theta) \frac{\partial q_j^{ie}}{\partial \theta_i}(\theta)}{\frac{\partial q_i^{ie}}{\partial \theta_i}(\theta)}$. Note that when the optimality conditions are satisfied, $X_i(\theta) \leq 0$.

Using (31), it is obvious that if $3X_i(\theta) - 2X_j(\theta) \geq 0$ then $3X_j(\theta) - 2X_i(\theta) \leq 0$.

11.4.3 Proof of Corollary 2

We assume that θ_i , $i = d, f$, is uniformly distributed on $[0, 1]$ ³⁶.

³⁶To obtain linear solutions of the differential equations, we only need to have a linear hazard rate, which holds when θ_i is distributed according to a Beta(1, λ_i), $\lambda_i > 0$. Moreover, one can also easily compute the quantity profiles for other supports of the private information parameters.

The optimal quantity profiles We shall restrict attention to linear solutions of (21), or

$$\begin{cases} q_d^{ie}(\theta) = x_d + x_{dd}\theta_d + x_{df}\theta_f, \\ q_f^{ie}(\theta) = x_f + x_{fd}\theta_d + x_{ff}\theta_f. \end{cases}$$

Plugging these expressions in (21) and differentiating both conditions with respect to θ_d and θ_f , we obtain the following system

$$\begin{cases} 1 + 3x_{dd} + 2x_{fd} & = 0, \\ x_{df} \left(3 - 2\frac{x_{fd}}{x_{dd}} \right) + 4x_{ff} & = 0, \\ 4x_{dd} + x_{fd} \left(3 - 2\frac{x_{df}}{x_{ff}} \right) & = 0, \\ 1 + 2x_{df} + 3x_{ff} & = 0. \end{cases}$$

There exist two sets of solutions that solve this system of equations:

$$\begin{aligned} 1. \quad x_{df} = x_{fd} = \frac{7 - 3\sqrt{41}}{40}, \quad x_{dd} = x_{ff} = \frac{-9 + \sqrt{41}}{20} \\ 2. \quad x_{df} = x_{fd} = \frac{7 + 3\sqrt{41}}{40}, \quad x_{dd} = x_{ff} = \frac{-9 - \sqrt{41}}{20}. \end{aligned}$$

However, we retain only the second set of solutions. Indeed, the first violates (among others) one of the optimality conditions of the firm's maximization problem, namely

$$4 + \frac{\partial^2 T_f}{\partial q_f^2}(\theta_f, q_f(\theta)) = -2 \frac{\frac{\partial q_d}{\partial \theta_d}(\theta)}{\frac{\partial q_f}{\partial \theta_d}(\theta)} = -2 \frac{\frac{-9 + \sqrt{41}}{20}}{\frac{7 - 3\sqrt{41}}{40}} < 0.$$

Then, the fixed part of the solutions can be immediately deduced from the initial system (21)

$$x_i = \frac{3 - \sqrt{41} + 24a_i - 16a_j}{40}.$$

The optimal quantity profiles are given by

$$q_i^{ie}(\theta) = q_i^*(\theta) - \frac{7 + 3\sqrt{41}}{20(9 + \sqrt{41})} (1 + 2\theta_i - 3\theta_j), \quad (32)$$

with $i, j = d, f, i \neq j$.

Comparative statics on the equilibrium quantity profiles Immediate computations show that

$$\begin{cases} q_i^*(\theta) - q_i^{ie}(\theta) \propto 1 + 2\theta_i - 3\theta_j, \\ \left(q_d^*(\theta) + q_f^*(\theta) \right) - \left(q_d^{ie}(\theta) + q_f^{ie}(\theta) \right) \propto 2 - (\theta_d + \theta_f). \end{cases}$$

with $i, j = d, f, i \neq j$.

Positivity of the equilibrium quantities Another condition to validate our approach is that the equilibrium quantities are strictly positive. This amounts to

$$q_i^{ie}(\bar{\theta}_i, \underline{\theta}_j) > 0 \Leftrightarrow 2(3a_i - 2a_j) > \frac{3}{4}(5 + \sqrt{41}),$$

with $i, j = d, f, i \neq j$.

The implementability conditions In equilibrium we have

$$\frac{\partial q_i^{ie}}{\partial \theta_i}(\theta) = -\frac{9 + \sqrt{41}}{20} \leq 0 \text{ and } \frac{\partial q_i^{ie}}{\partial \theta_j}(\theta) = -\frac{7 + 3\sqrt{41}}{40} \geq 0.$$

We also have

$$\frac{\partial^2 \hat{\pi}_i}{\partial q_i^{ie} \partial \theta_j}(\theta_j, q_j^{ie}(\theta)) = \frac{1 + \frac{\partial^2 T_j}{\partial \theta_j \partial q_j^{ie}}(\theta_j, q_j^{ie}(\theta))}{4 + \frac{\partial^2 T_j}{\partial q_j^{ie2}}(\theta_j, q_j^{ie}(\theta))} = \frac{\frac{\partial q_j^{ie}}{\partial \theta_j}(\theta) \frac{\partial q_i^{ie}}{\partial \theta_i}(\theta) - \frac{\partial q_j^{ie}}{\partial \theta_i}(\theta) \frac{\partial q_i^{ie}}{\partial \theta_j}(\theta)}{-\frac{\partial q_i^{ie}}{\partial \theta_i}(\theta)} = \frac{-3 + \sqrt{41}}{16} \geq 0.$$

Direct computations show that

$$1 + \frac{\partial^2 T_j}{\partial \theta_j \partial q_j^{ie}}(\theta_j, q_j^{ie}(\theta)) = 2 \frac{\frac{\partial q_d}{\partial \theta_d}(\theta) \frac{\partial q_f}{\partial \theta_f}(\theta) - \frac{\partial q_d}{\partial \theta_f}(\theta) \frac{\partial q_f}{\partial \theta_d}(\theta)}{\frac{\partial q_d}{\partial \theta_f}(\theta)} = \frac{1}{2} \geq 0.$$

As concerns the firm's optimality conditions, we have

$$4 + \frac{\partial^2 T_i}{\partial q_i^{ie2}}(\theta_i, q_i^{ie}) = -2 \frac{\frac{\partial q_j^{ie}}{\partial \theta_j}(\theta)}{\frac{\partial q_i^{ie}}{\partial \theta_j}(\theta)} = \frac{3 + \sqrt{41}}{4},$$

and

$$\left[4 + \frac{\partial^2 T_d}{\partial q_d^2}(\theta_d, q_d) \right] \left[4 + \frac{\partial^2 T_f}{\partial q_f^2}(\theta_f, q_f) \right] = \left[\frac{3 + \sqrt{41}}{4} \right]^2 \geq 4.$$

All the implementability conditions as well as the optimality conditions of the firm's maximization program are satisfied in equilibrium.

11.5 Non informative equilibrium

11.5.1 Proof of Proposition 4

Assume that government G_f decides not to make reveal the relevant information by the firm. Let $\{q_f^{nie}(\theta_f), T_f^{nie}(\theta_f)\}$, with $q_f^{nie}(\theta_f) > 0$ for all θ_f , be such a contract.

The indirect utility function of the firm with respect to government G_d is now

$$\hat{\pi}_d(\theta_f, q_f^{nie}(\theta_f)) = [p_f(q_f^{nie}(\theta_f)) - \theta_f]q_f^{nie}(\theta_f) - [q_f^{nie}(\theta_f) + q_d]^2 - T_f^{nie}(\theta_f).$$

We then have

$$\frac{\partial \hat{\pi}_d}{\partial \theta_f}(\theta_f, q_d) = -q_f^{nie}(\theta_f) - \frac{dT_f^{nie}}{d\theta_f}(\theta_f) \quad \text{and} \quad \frac{\partial^2 \hat{\pi}_d}{\partial \theta_f \partial q_d}(\theta_f, q_f^{nie}(\theta_f)) = 0.$$

This absence of the equivalent to the Spence-Mirrlees condition implies that government G_d cannot affect the rent of the firm via the quantity he proposes to obtain the revelation of the relevant information. Hence, he is also forced to offer a non informative contract of the form $\{q_d^{nie}(\theta_d), T_d^{nie}(\theta_d)\}$. Notice that we must check that, in equilibrium, $q_i^{nie}(\theta_i) > 0$, for all θ_i , $i = d, f$.

11.5.2 Proof of Proposition 5

Let us consider the problem faced by government G_d . Assuming that he wants to make produce all types of firm, the following participation constraint must be satisfied

$$\begin{aligned} [p_d(q_d^{nie}(\theta_d)) - \theta_d] q_d^{nie}(\theta_d) + [p_f(q_f^{nie}(\underline{\theta}_f)) - \underline{\theta}_f] q_f^{nie}(\underline{\theta}_f) \\ - [q_f^{nie}(\underline{\theta}_f) + q_d^{nie}(\theta_d)]^2 - T_f^{nie}(\underline{\theta}_f) - T_d^{nie}(\theta_d) \geq \pi_d^{nr}(\underline{\theta}_f). \end{aligned}$$

With non informative contracts, the outside opportunity of the firm is

$$\pi_d^{nr}(\theta_f) = [p_f(q_f^{nie}(\theta_f)) - \theta_f] q_f^{nie}(\theta_f) - [q_f^{nie}(\theta_f)]^2 - T_f^{nie}(\theta_f). \quad (33)$$

Therefore, using (33) enables to rewrite the non relocation constraint as follows

$$[p_d(q_d^{nie}(\theta_d)) - \theta_d] q_d^{nie}(\theta_d) - q_d^{nie}(\theta_d) [q_d^{nie}(\theta_d) + 2q_f^{nie}(\underline{\theta}_f)] - T_d^{nie}(\theta_d) \geq 0.$$

This constraint will be binding in equilibrium. Substituting the value of the transfer obtained, the program of G_d can be rewritten as follows

$$\max_{\{q_d^{nie}(\theta_d)\}} \int_0^{q_d^{nie}(\theta_d)} p_d(x) dx - \theta_d q_d^{nie}(\theta_d) - q_d^{nie}(\theta_d) [q_d^{nie}(\theta_d) + 2q_f^{nie}(\underline{\theta}_f)].$$

Optimizing yields the following necessary condition (the second-order condition is trivially satisfied)

$$3q_d^{nie}(\theta_d) + 2q_f^{nie}(\underline{\theta}_f) = a_d - \theta_d. \quad (34)$$

Proceeding in a similar way, we obtain the following condition for government in the foreign country

$$3q_f^{nie}(\theta_f) + 2q_d^{nie}(\underline{\theta}_d) = a_f - \theta_f. \quad (35)$$

Now, take (34) in $\theta_d = \underline{\theta}_d$ and (35) in $\theta_f = \underline{\theta}_f$ and solve to obtain

$$\begin{cases} q_d^{nie}(\underline{\theta}_d) = \frac{1}{5} [3(a_d - \underline{\theta}_d) - 2(a_f - \underline{\theta}_f)], \\ q_f^{nie}(\underline{\theta}_f) = \frac{1}{5} [3(a_f - \underline{\theta}_f) - 2(a_d - \underline{\theta}_d)]. \end{cases}$$

Replacing those values in (34) and (35) yields the optimal quantities in a non informative equilibrium.

Comparative statics on the equilibrium quantity profiles Direct computations show that

$$\begin{cases} q_i^*(\theta) - q_i^{nie}(\theta_i) \propto 3\theta_j - 2\theta_i, \\ \left(q_d^*(\theta) + q_f^*(\theta) \right) - \left(q_d^{nie}(\theta_d) + q_f^{nie}(\theta_f) \right) \propto \theta_d + \theta_f \end{cases}$$

Positivity of the equilibrium quantities Finally notice that

$$q_i^{nie}(\bar{\theta}_i) > 0 \Leftrightarrow 3(3a_i - 2a_j) > 5,$$

with $i, j = d, f, i \neq j$.

11.6 Comparisons of the different equilibria

We assume that $a_d = a_f \equiv a$. In order to ensure that the quantities in the different types of equilibria are strictly positive, we assume that $a \geq \frac{3}{8}(5 + \sqrt{41}) \approx 4.27$.

11.6.1 Proof of Proposition 6

Welfare in an informative equilibrium We determine explicitly the tax schedules used in equilibrium by each government to decentralize the choice of the production levels to the firm. We shall show that in equilibrium, government G_i uses a quadratic tax schedule of the form

$$T_i(\theta_i, q_i) = \gamma_i(\theta_i) + \alpha_i(\theta_i)q_i + \frac{1}{2}\beta_i q_i^2, \quad i = d, f,$$

where β_i does not depend on θ_i and $\alpha_i(\theta_i)$ is linear in θ_i : $\alpha_i(\theta_i) = \alpha_{i1} + \theta_i \alpha_{i2}$.

The proof proceeds in two steps and is essentially an application of the Taxation Principle stated in Rochet (1986). First, we compute the rent of the firm from the first-order incentive compatibility constraint. Second, we identify this rent with its definition to recover the parameter of the tax schedules used by the governments.

Using the definition of the indirect utility function given by Equation (16) and the first-order incentive constraint (19), the firm's rent from the viewpoint of government G_d can be rewritten as follows

$$\pi(\theta) = \pi_d^{nr}(0) - \int_0^{\theta_f} \left[q_f^{ie}(\theta_d, x) + \frac{\partial T_f}{\partial \theta_f}(x, q_f^{ie}(\theta_d, x)) \right] dx \quad (36)$$

The outside opportunity of the firm with respect to government G_d is

$$\begin{aligned}\pi_d^{nr}(\theta_f) &= \max_{q_f} \left\{ [p_f(q_f) - \theta_f] q_f - [q_f]^2 - T_f(\theta_f, q_f) \right\} \\ &= -\gamma_f(\theta_f) + \frac{[a_f - \alpha_f(\theta_f) - \theta_f]^2}{2(4 + \beta_f)}.\end{aligned}$$

Remember that, in equilibrium,

$$1 + \frac{\partial^2 T_f}{\partial \theta_f \partial q_f}(\theta_f, q_f^{ie}(\theta)) = \frac{1}{2}.$$

Therefore,

$$\begin{aligned}\int_0^{q_f^{ie}(\theta)} \left[1 + \frac{\partial^2 T_f}{\partial \theta_f \partial q_f}(\theta_f, x) \right] dx &= q_f^{ie}(\theta) + \frac{\partial T_f}{\partial \theta_f}(\theta_f, q_f^{ie}(\theta)) - \gamma_f'(\theta_f) \\ &= \frac{1}{2} q_f^{ie}(\theta).\end{aligned}$$

This last rewriting enables us to rewrite the firm's rent (36) as follows

$$\pi(\theta) = \pi_d^{nr}(0) - \int_0^{\theta_f} [q_f^{ie}(\theta_d, x) + \gamma_f'(x)] dx \quad (37)$$

$$= \frac{[a_f - \alpha_f(0)]^2}{2(4 + \beta_f)} - \gamma_f(\theta_f) - \int_0^{\theta_f} [q_f^{ie}(\theta_d, x)] dx. \quad (38)$$

Then we use Equation (32) to obtain $\theta_f \equiv \theta_f(q_d)$. Replacing θ_f by its equilibrium expression as function of q_d in Equation (38), we obtain the following expression of the firm's rent, which is quadratic in q_d ,

$$\pi(\theta) = \left\{ \frac{[a_f - \alpha_{f1}]^2}{2(4 + \beta_f)} - \gamma_f(\theta_f) - \int_0^{\theta_f} [q_f^{ie}(\theta_d, x)] dx \right\}_{\theta_f \rightarrow \theta_f(q_d)}. \quad (39)$$

Now, we use the definition of the firm's profit

$$\begin{aligned}\pi(\theta) &= [p_d(q_d) - \theta_d] q_d + [p_f(q_f^{ie}(\theta)) - \theta_f] q_f^{ie}(\theta) \\ &\quad - [q_d + q_f^{ie}(\theta)]^2 - T_d(\theta_d, q_d) - T_f(\theta_f, q_f^{ie}(\theta)).\end{aligned} \quad (40)$$

We can therefore obtain the following condition

$$\begin{aligned}T_d(\theta_d, q_d) &= \left\{ [p_d(q_d) - \theta_d] q_d + [p_f(q_f^{ie}(\theta)) - \theta_f] q_f^{ie}(\theta) - [q_d + q_f^{ie}(\theta)]^2 \right. \\ &\quad \left. - \alpha_f(\theta_f) q_f^{ie}(\theta) - \frac{1}{2} \beta_f q_f^{ie}(\theta)^2 - \frac{[a_f - \alpha_{f1}]^2}{2(4 + \beta_f)} + \int_0^{\theta_f} [q_f^{ie}(\theta_d, x)] dx \right\}_{\theta_f \rightarrow \theta_f(q_d)}. \quad (41)\end{aligned}$$

Finally, this identification equation will enable us to determine the parameters of the quadratic tax schedules used by the governments in equilibrium. Indeed, both terms of (41) are quadratic in q_d .

We can also perform a similar exercise from the point of view of government G_f . Considering symmetric solutions (i.e., $\beta_f = \beta_d$ and $\alpha_{f2} = \alpha_{d2}$), we finally obtain the following expressions for the parameters of the taxes used by the governments

$$\begin{cases} \beta_i &= \frac{-13+\sqrt{41}}{4}, \\ \alpha_i(\theta_i) &= \frac{1}{6} \left[3a_i - 2a_j + 2 - 3\theta_i + \frac{8(1-3a_i+2a_j)}{17+3\sqrt{41}} \right], \\ \gamma_i(\theta_i) &= \frac{1}{40(1887+293\sqrt{41})} \left[3(25 + 3\sqrt{41})a_i - 50a_j - 87\theta_i - 6 - \sqrt{41}(2 + 6a_j + 13\theta_i) \right]^2. \end{cases}$$

Then, in an informative equilibrium, the ex ante welfare in each country is given by

$$\mathbf{E}_\theta \{SW_i^{ie}\} = \frac{1}{400} \left[4(9 - \sqrt{41})(3a_i - 2a_j)^2 - 2(1 + \sqrt{41})(3a_i - 2a_j) + 21 + \sqrt{41} \right], \quad (42)$$

with $i, j = d, f, i \neq j$.

Welfare in a non informative equilibrium Computing the welfare in a non informative equilibrium is straightforward,

$$\mathbf{E}_\theta \{SW_i^{nie}\} = \frac{1}{18} + \frac{3(3a_i - 2a_j)^2}{50} - \frac{3a_i - 2a_j}{10}, \quad (43)$$

with $i, j = d, f, i \neq j$.

Welfare comparisons With identical market sizes, we have

$$\mathbf{E}_\theta \{SW_i^{ie} - SW_i^{nie}\} \propto a^2 4(27 - 9\sqrt{41}) + a6(57 - 3\sqrt{41}) - 11 + 9\sqrt{41},$$

which is always negative for $a \geq 2.03$ (approximation).

11.6.2 Proof of Proposition 7

The difference in the firm's rent in an informative equilibrium with the firm's rent in a non informative equilibrium is proportional to (computations are straightforward and left to the reader)

$$(144\sqrt{41} - 432)a^2 + (-1368 + 72\sqrt{41})a + (-441 + 99\sqrt{41}) + (970 - 270\sqrt{41})\theta_d\theta_f. \quad (44)$$

This defines a polynomial expression which admits two roots. The largest of these roots increases with θ_d and θ_f and is equal to 2.34 (approximation) in $\theta_d = 1$ and $\theta_f = 1$. Hence, (44) will be always positive for $a \geq 2.34$.

11.7 Asymmetric equilibria

11.7.1 Proof of Proposition 8

Assume that $q_f(\theta) = 0$ for all θ . Then the firm's profit becomes

$$\pi(\theta_d, q_d, q_f = 0) = [p_d(q_d) - \theta_d]q_d - q_d^2 - T_d(\theta_d, q_d).$$

Government G_d is then under complete information vis-à-vis the firm. Immediate computations show that the quantity chosen in the domestic country is $q_d^{*a}(\theta) = \frac{1}{3}(a_d - \theta_d)$.

To decentralize the choice of this allocation, G_d offers a tax schedule such that

$$q_d^{*a}(\theta) \in \arg \max_{q_d} \{ [p_d(q_d) - \theta_d]q_d - q_d^2 - T_d(\theta_d, q_d) \}.$$

A two-part tax schedule such that $\frac{\partial T_d}{\partial q_d}(\theta_d, q_d^{*a}(\theta_d)) = -\frac{1}{3}(a_d - \theta_d)$ and $\frac{\partial^2 T_d}{\partial q_d^2}(\theta_d, q_d) = 0$ is enough. The fixed part of the tax schedule is then computed in order to leave the firm with no rent in equilibrium.

Now let us consider the problem of government G_f . For a given tax schedule $T_d(\theta_d, q_d)$ we can go through the same steps of the general analysis and obtain

$$3q_f(\theta) + 2\hat{q}_d(\theta_d, q_f(\theta)) = a_f - \theta_f - 2 \frac{1 - H_d(\theta_d)}{h_d(\theta_d)} \frac{1 + \frac{\partial^2 T_d}{\partial \theta_d \partial q_d}(\theta_d, \hat{q}_d(\theta_d, q_f(\theta)))}{4 + \frac{\partial^2 T_d}{\partial q_d^2}(\theta_d, \hat{q}_d(\theta_d, q_f(\theta)))}. \quad (45)$$

Now assume that government G_d uses, in equilibrium, the tax system defined previously to implement the quantity $q_d^{*a}(\theta)$. In this case, Equation (45) becomes in equilibrium

$$3q_f(\theta) = a_f - \theta_f - \frac{2}{3}(a_d - \theta_d) - \frac{2}{3} \frac{1 - H_d(\theta_d)}{h_d(\theta_d)} \geq 0. \quad (46)$$

Notice that, in this case, the equivalent to the Spence-Mirrlees condition is satisfied for government G_f ,

$$\frac{1 + \frac{\partial^2 T_d}{\partial \theta_d \partial q_d}(\theta_d, q_d(\theta))}{4 + \frac{\partial^2 T_d}{\partial q_d^2}(\theta_d, q_d)} = \frac{1}{3},$$

as well as the monotonicity condition for the quantity profile produced in the foreign country,

$$\frac{\partial q_f}{\partial \theta_d}(\theta) = \frac{2}{3} - \frac{2}{3} \frac{d}{d\theta_d} \frac{1 - H_d(\theta_d)}{h_d(\theta_d)} \geq 0$$

given the monotone hazard rate assumption $\frac{d}{d\theta_d} \frac{1 - H_d(\theta_d)}{h_d(\theta_d)} \leq 0$.

Consequently, government G_f will effectively shut down the production in his country if the quantity defined by (46) is strictly negative for all possible values of θ , or

$$\frac{a_d - \bar{\theta}_d}{a_f - \underline{\theta}_f} > \frac{3}{2}.$$

11.7.2 Proof of Proposition 9

Let us assume that government G_d uses a quadratic tax schedule of the form

$$T_d(\theta_d, q_d) = \gamma_d(\theta_d) + \alpha_d(\theta_d)q_d + \frac{1}{2}\beta_d q_d^2. \quad (47)$$

As a first step to characterize the asymmetric equilibria with voluntary exclusion, let us assume that government G_f shuts down the production in his country, or $q_f(\theta) = 0$ for all θ . In this case, government G_d will implement the quantity $q_d^{*a}(\theta)$. To decentralize this allocation, the parameters of the quadratic tax schedule $T_d(\theta_d, q_d)$ must be such that

$$q_d^{*a}(\theta) \in \arg \max_{q_d} \{\pi(\theta_d, q_d, q_f = 0)\},$$

or,

$$\begin{cases} \alpha_d(\theta_d) = -\frac{1}{3}(1 + \beta_d)(a_d - \theta_d), \\ 4 + \beta_d \geq 0, \end{cases} \quad (48)$$

the fixed part being fixed so as to leave the firm with no rent in equilibrium. The key point is that, since government G_d is under complete information with respect to the firm, the quadratic part β_d is a free parameter for government G_d that can be freely chosen in $(-4, +\infty)$ to modify the firm's behavior with respect to government G_f .

Note that under (47) and (48), the term

$$\frac{1 + \frac{\partial^2 T_d}{\partial \theta_d \partial q_d}(\theta_d, q_d)}{4 + \frac{\partial^2 T_d}{\partial q_d^2}(\theta_d, q_d)} = \frac{1}{3} \quad (49)$$

and is constant with respect to θ_d .

Conditions for government G_f to be unable to offer an informative contract Let us come back on government G_f 's problem. For any tax schedule offered in the domestic country, one can try to characterize government G_f 's optimal quantity profile as done in Section 5. The optimal quantity profile would therefore be given by (45).

Using (49), we can differentiate (45) to obtain

$$\frac{\partial}{\partial \theta_d} [3q_f(\theta) + 2q_d(\theta)] = -\frac{2}{3} \frac{d}{d\theta_d} \frac{1 - H_d(\theta_d)}{h_d(\theta_d)}.$$

The left-hand side can be rewritten as follows (by differentiating the equivalent of Equation (17) for the foreign country)

$$\begin{aligned} \frac{\partial}{\partial \theta_d} [3q_f(\theta) + 2q_d(\theta)] &= 3 \frac{\partial q_f}{\partial \theta_d}(\theta) + 2 \frac{\partial q_d}{\partial \theta_d}(\theta) \\ &= \frac{\partial q_f}{\partial \theta_d}(\theta) \frac{3(4 + \beta_d) - 4}{4 + \beta_d} - \frac{2}{3}. \end{aligned}$$

Hence under the monotone hazard rate assumption, $\frac{d}{d\theta_d} \frac{1 - H_d(\theta_d)}{h_d(\theta_d)} \leq 0$, we have

$$\text{Sign} \left[\frac{\partial q_f}{\partial \theta_d}(\theta) \right] = \text{Sign} [3(4 + \beta_d) - 4]. \quad (50)$$

This enables us to state the following lemma.

Lemma 5 *Assume that government G_d uses a quadratic tax schedule given by (47). Assume that (48) holds. If government G_d chooses the quadratic part β_d of his tax schedule in $(-4, -\frac{8}{3})$, then government G_f is forced to offer a non informative contract.*

Indeed, under these conditions the equivalent of the Spence-Mirrlees condition is satisfied

$$\begin{aligned} \frac{\partial^2 \hat{\pi}_f}{\partial q_f \partial \theta_d}(\theta_d, q_f(\theta)) &= -\frac{\partial \hat{q}_d}{\partial q_f}(\theta_d, q_f(\theta)) \left[1 + \frac{\partial^2 T_d}{\partial \theta_d \partial q_d}(\theta_d, \hat{q}_d(\theta_d, q_f(\theta))) \right] \\ &= \frac{2}{3} > 0. \end{aligned}$$

However, when $\beta_d \in (-4, -\frac{8}{3})$, from (50) we have $\frac{\partial q_f}{\partial \theta_d}(\theta) < 0$ for all θ , implying that the monotonicity condition on the quantity profile is not satisfied in the foreign country.

Government G_f 's optimal non informative contract We denote by $\{q_f^{ni}(\theta_f), T_f^{ni}(\theta_f)\}$ the non informative contract offered in the foreign country. We assume without loss of generality that government G_f wants to make all types of firm produce. This implies that the non informative contract must satisfy the following participation constraint

$$[p_f(q_f^{ni}(\theta_f)) - \theta_f] q_f^{ni}(\theta_f) + \hat{\pi}_f(\underline{\theta}_d, q_f^{ni}(\theta_f)) - T_f^{ni}(\theta_f) \geq \pi_f^{nr}(\underline{\theta}_d).$$

This constraint will be binding in equilibrium and substituting the value of the tax $T_f^{ni}(\theta_f)$, we can rewrite the objective of the government G_f as follows

$$SW_f^{ni} = \int_0^{q_f^{ni}(\theta_f)} p_f(x) dx - \theta_f q_f^{ni}(\theta_f) + \hat{\pi}_f(\underline{\theta}_d, q_f^{ni}(\theta_f)) - \pi_f^{nr}(\underline{\theta}_d). \quad (51)$$

Notice that given the particular specification of the transfer in the domestic country, we have

$$\hat{q}_d(\underline{\theta}_d, q_f^{ni}(\theta_f)) = \frac{a_d - \underline{\theta}_d - \alpha_d(\underline{\theta}_d) - 2q_f^{ni}(\theta_f)}{4 + \beta_d}. \quad (52)$$

Replacing (52) into (51) we obtain the following lemma.

Lemma 6 *Assume that government G_d uses a quadratic tax schedule given by (47). Assume that (48) holds and that government G_d chooses the quadratic part β_d of his tax schedule in $(-4, -\frac{8}{3})$. When G_f decides to implement a non informative contract then his objective is a convex function of the quantity.*

Immediate computations show that

$$\frac{\partial SW_f^{ni}}{\partial q_f^{ni}} = a_f - \theta_f - 3q_f^{ni}(\theta_f) - 2\hat{q}_d(\underline{\theta}_d, q_f^{ni}(\theta_f)),$$

and

$$\frac{\partial^2 SW_f^{ni}}{\partial q_f^{ni2}} = -3 - 2 \frac{\partial \hat{q}_d}{\partial q_f^{ni}}(\underline{\theta}_d, q_f^{ni}(\theta_f)) = -3 + \frac{4}{4 + \beta_d} > 0.$$

This implies that the non informative quantity implemented in the domestic country is equal to either a_f or 0. When government G_f decides to implement a quantity equal to a_f the welfare in the foreign country will be negative if and only if

$$3a_f\beta_d + 6(4 + \beta_d)\underline{\theta}_f + 4(4 + \beta_d)(a_d - \underline{\theta}_d) \geq 0.$$

This condition is the least demanding when $\beta_d = -\frac{8}{3}$, or

$$\frac{a_d - \underline{\theta}_d}{a_f - \underline{\theta}_f} \geq \frac{3}{2}.$$

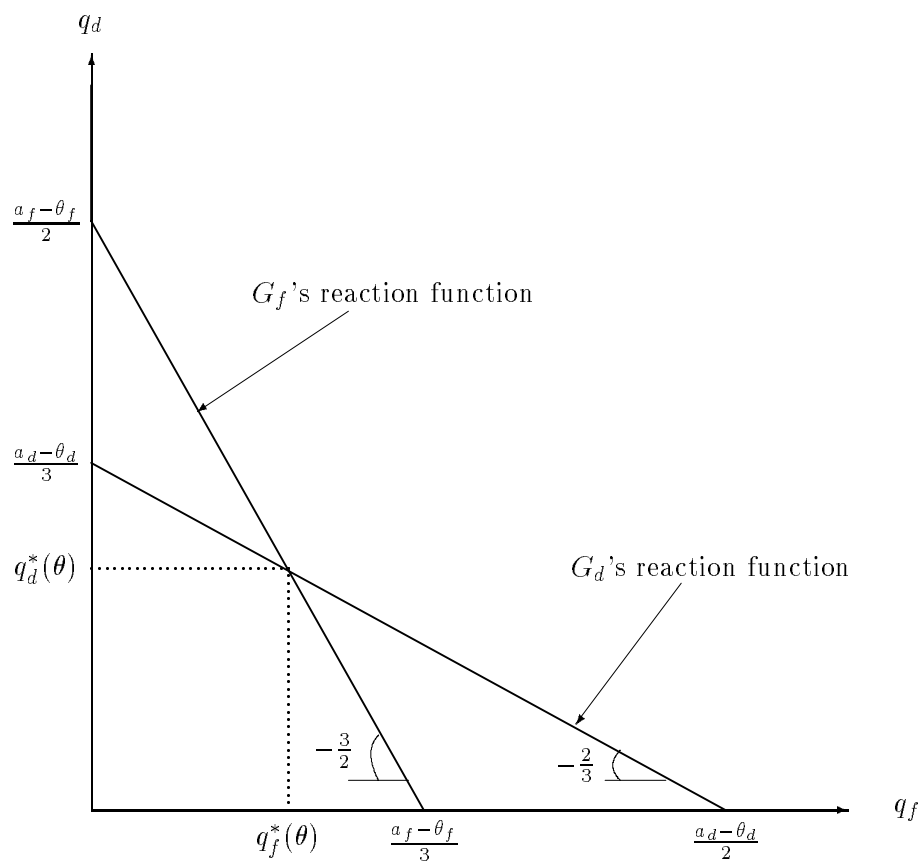


Figure 1: Governments' best-response functions under complete information.

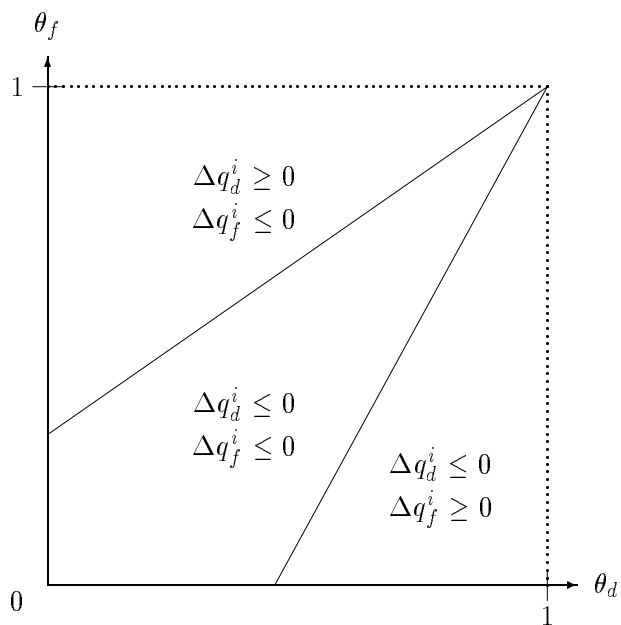


Figure 2: Difference between complete information and informative equilibrium quantities.

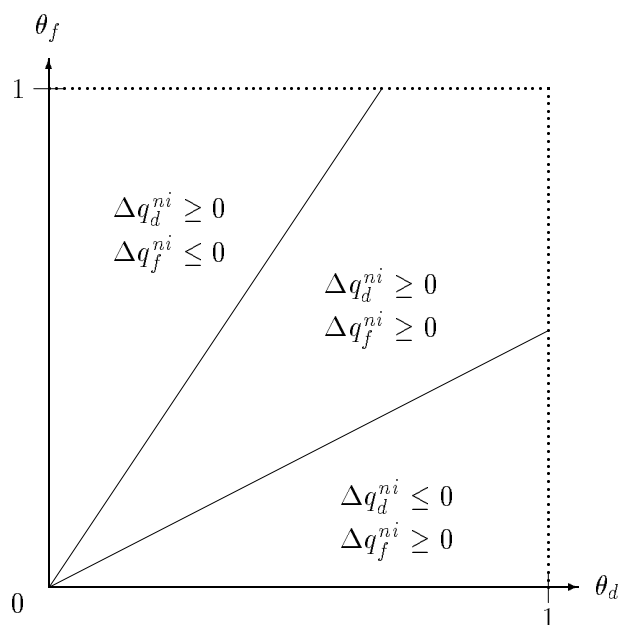


Figure 3: Difference between complete information and non-informative equilibrium quantities.