Search Bonuses: An Alternative to Declining Unemployment Benefits ?

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Abstract

This paper presents search bonuses as a possible alternative to unemployment benefits decreasing with the increase in the length of unemployment. Search bonuses are paid to unemployed workers who find a job within their first period of unemployment. Both job search intensity and wages are determined endogenously. Both policy schemes restore search incentives, but while decreasing unemployment benefits penalize low levels of search effort, search bonuses reward high levels of search effort. In the presence of risk adverse workers, the insurance dimension of unemployment benefits matters. Quantitative exercises indicate that, for a similar impact on the unemployment rate, search bonuses may generate higher welfare levels both at the individual and aggregate level.

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1. Introduction

Unemployment benefits have become a prominent institution in European countries. The main rationale for public unemployment insurance is to provide income insurance for risk adverse workers when financial markets are imperfect. However, unemployment benefits are also accused of creating disincentives for looking for work and therefore of increasing the length of the period of unemployment. This is confirmed by empirical studies[1], which show a positive relation between the degree of persistence of unemployment and the duration of unemployment benefits. These empirical findings have reinforced beliefs regarding the existence of a "benefit trap", and, together with increasing budgetary difficulties, have led policy makers to reform UI systems. Reforms have been directed towards unemployed ``activation" measures. The reform of the unemployment insurance system recently proposed by the French employer's union (MEDEF) for instance has been substantially inspired by such a policy strategic line. Activation measures take two broad forms. The first is based principally on sanctions and compulsion, and usually involve strict benefit eligibility criteria relying on the individual's job search activity and availability for work. This is the case of declining unemployment insurance schemes. The second type of measures are also incentive-based but aims at pushing unemployed workers back to work by making employment more attractive relative to unemployment. Approaches based on in-work benefits and the like, as well as search bonuses WE discuss below, belong to this second set of activation measures.

Turning to existing theoretical contributions, a series of papers, initiated by the seminal work of Shavell and Weiss (1979), contribute to defining what an optimal UI scheme would look like in the presence of moral hazard associated with search effort. For that purpose, Shavell and Weiss (1979) as well as Hopenayn and Nicolini (1997) use a dynamic principal-agent model, where the principal, i.e. the government, maximizes the welfare of the agent, i.e. an unemployed worker, subject to a budget constraint. The probability for an unemployed worker to find a job depends on his search effort, the latter being unobservable by the government. Both papers conclude that the level of unemployment benefits should take the form of a decreasing time sequence. Hopenayn and Nicolini (1997) also find that the tax rate after re-employment should increase with the length of the unemployment spell.

Justifications for declining unemployment benefits other than moral hazard can be found in Wright (1986) and Cremer, Marchand and Pestieau (1995). In a model with neither moral hazard nor monitoring problems and with exogenous transition rates, Wright (1986) finds that a declining unemployment benefits scheme maximizes the welfare of an employed worker, the latter being the median voter. This comes from the fact that any employed worker benefits only in the future from income insurance, whereas its cost, that is the taxes imposed on her wage, must be borne immediately. Cremer, Marchand and Pestieau (1995) consider the occurrence of mismatching between job offers

[1] See, for example, Layard and *alii* (1991) or Bean (1994) for comprehensive surveys.

and job demands. In such a framework an adverse selection problem occurs in addition to the moral hazard one. Again, the optimal UI should pay benefits decreasing over time.

If the works mentioned above were focused on the supply side perverse effects of unemployment benefits, their influence on wages and labor costs, that is, on labor demand is also well known. As a matter of fact, some contributions have addressed the issue of defining the optimal insurance schedule in a general equilibrium with both endogenous wage and search effort. In such a context, Fredriksson and Holmund (1998) find that a decreasing profile of unemployment benefits over the unemployment spell is socially optimal when aggregate welfare is measured by an utilitarian welfare function.

In a similar theoretical environment, Cahuc and Lehmann (1999) show that these findings are mitigated when the Ralws' justice criterion is adopted in order to assess the welfare consequences of the introduction of a decreasing rather than a flat compensation scheme.

Restoring search incentives is a strong argument in favor of a declining scheme. However, as the insurance dimension of unemployment benefits is somewhat compromised, some welfare losses, particularly at the individual level, may be generated by such a scheme.

This chapter proposes search bonuses as an alternative to decreasing unemployment benefits. As previously mentioned, search bonuses, like declining unemployment benefits, indeed represent an unemployed activation measure and could be interpreted to be a candidate for ``welfare-to-work"[2] oriented policy reforms. Indeed, search bonuses reward high search effort and as such, stimulate unemployment outflow. As already mentioned, decreasing unemployment benefits generate similar search incentives. However, they penalize low search effort rather than reward high search effort. In terms of equity, such a difference may matter. Moreover and most of all, the insurance dimension of unemployment benefits does not have to be reduced within a search bonus program.

In this chapter, search bonuses and unemployment benefits decreasing with the unemployment spell are analyzed in a discrete search and matching model where both wage and search effort are endogenous. As in the previous chapter, search bonuses are paid to job seekers who find a position within their first period of unemployment. As to unemployment benefits, they are kept constant in relative terms when search bonuses are introduced, and decrease *una tantum* after the first period of unemployment. Wages are determined by bargaining *à la* Rubinstein-Shaked-Sutton. In that context, workers are paid a fixed share of the match outcome unless the latter falls below their outside option. In such a case workers get exactly their outside option. Analytical investigations reveal that both policy schemes stimulate unemployed workers search. As search bonuses may introduce wage pressure in the case workers outside option is binding, firms might become more reluctant in opening new vacancies. Hence, the overall impact on unemployment is *a priori* ambiguous. Contrary to a search bonus, more declining unemployment compensations may contribute to lower wages. In that event, jobs profitability may increase and firms may become more eager to open up new positions. As a consequence unemployment may sensitively fall.

In order to take into consideration the welfare properties of the two schemes a calibrated version of the model is proposed. Computational results confirm previous analytical findings and conjectures. They also allow to identify a positive trade-off between welfare and lower unemployment in the search bonus scheme even when wages strongly respond. This is never the case when UI is made more declining. Welfare falls unambiguously, despite the fall in the unemployment rate. In addition, in the search bonus scheme, workers, and in particular unemployed workers, are better off. On the contrary, their welfare situation worsens when unemployment compensations are made more declining. This suggests that search bonuses are likely to satisfy the Rawls' criterion.

The rest of the chapter is organized as follows. The next section presents the model in general terms. Equilibrium is defined and some general equilibrium properties of the two schemes are described analytically in section 3. Section 4 presents some indicators our welfare is built on. Computational strategies and numerical results are presented in section 5. Concluding remarks are contained in section 6.

2. The Model

2.1 The Labor Market

We use a discrete time search and matching framework *à la* Pissarides (2000) with variable search intensity which follows Cahuc and Lehman (1999). There is a fixed labor force normalized to unity. Individuals have infinite horizons and perfect foresights. At any time, a worker is either employed, or short-term unemployed, or long-term unemployed. Long-term unemployed are unemployed workers whose unemployment spell is longer than one period. Employed workers are paid a wage w and are separated from their jobs and enter unemployment at the exogenous rate λ. In our benchmark economy, nothing distinguishes long-term from short-term unemployed, both receive an unemployment benefit denoted by z. z is proportional to w, namely $z = \rho w$ where ρ represents the replacement ratio.

Then, we consider two alternative policy schemes: a search bonus scheme and a declining unemployment benefits scheme. In the search bonus scheme, both types of unemployed receive ρw as unemployment benefits. However, an amount $B = bw$ is paid to unemployed searchers able to find a job by the end of their first period of unemployment. That is, only short term unemployed succeeding in their job search are eligible for the search bonus. In the declining unemployment benefits scheme, short term unemployed are given $z_s = \rho_s w$ as unemployment benefits whereas long-term unemployed receive $z_l = \rho_l w$ with $\rho_l < \rho_s$.

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[2] See Boeri and *alii* (2000) for a general presentation and definition of the concept.

A representative competitive firm is also in the economy. It produces with constant returns to scale and is composed by filled and vacant jobs.

Job seekers and the representative firm are brought together by a matching function

$$M\left(v, \phi(e_s)u_s + \phi(e_l)u_l\right)$$

where u_s and u_l denote the proportion of short-term and long-term unemployed respectively, and e_s and e_l their respective search effort intensity. $\phi(.)$ is an increasing and concave function of search effort. The matching function has the standard properties: increasing in both its arguments and constant returns to scale. Contacts are random and any vacancy is contacted by no more than one worker per period.

Let $\theta = \dfrac{v}{\phi(e_s)u_s + \phi(e_l)u_l}$ denote the labor market tightness. Then, we can define the probability that any vacancy is contacted by an unemployed worker as

$\phi(e_s)u_s + \phi(e_l)u_l$

$$q(\theta) = \frac{M\left(v, \phi(e_s)u_s + \phi(e_l)u_l\right)}{v} \qquad (1)$$

By the same reasoning, we can define the exit rate of an unemployed worker of type *i*, *i* = *s*,*l* as

$$p_i(\theta) = \phi(e_i)\frac{M\left(v, \phi(e_s)u_s + \phi(e_l)u_l\right)}{\phi(e_s)u_s + \phi(e_l)u_l} = \phi(e_i)\theta q(\theta) \qquad (2)$$

$\phi(e_i)$

From the properties of the matching function we have that $q(\theta)$ and $p_i(\theta)$ are respectively decreasing and increasing with respect to θ.

Figure 1 illustrates flows in the labor market.

In equilibrium, flows into and out of unemployment must be equal, that is

$$\lambda(1-u) = \theta q(\theta)\left[\phi(e_s)u_s + \phi(e_l)u_l\right] \qquad (3)$$

and

$$u_s = \lambda(1-u) \qquad (4)$$

where $(1-u)$ stands for the aggregate level of employment reads as

$$(1-u)=1-u_s-u_l \quad (5)$$

Since short-term unemployment lasts only one period, the existing stock must always be ``renewed'' by the end of the period. The stock of short-term unemployment is thus given by the inflow into unemployment.

We assume that the government imposes a payroll tax to employers, proportional to the wage paid to workers. The tax rate is denoted by τ. Tax proceeds are used to finance UI and search bonuses.

In the decreasing UI scheme, the government budget constraint can be written

$\phi(e_s)u_s+\phi(e_l)u_l$

$$\tau(1-u_s-u_l)=\rho_s u_s+\rho_l u_l \quad (6)$$

In the search bonus scheme the government budget constraint reads as

$$\tau(1-u_s-u_l)=\rho(u_s+u_l)+b\phi(e_s)\theta q(\theta)u_s \quad (7)$$

2.2 Worker Behavior

Workers have no access to a capital market, so, as the good produced is assumed to be non-storable, individuals consume all their income in each period. They have identical preferences expressed by the utility function: $\upsilon(x,e)=\ln(x)-e$, where *x* stands for the income and *e* for the search effort. As no search on the job takes place, the search effort of an employed worker is nil. We further assume that consumption occurs at the end of each period. Hence $\upsilon(x,e)$ expresses expected utility at the beginning of the period.

Let J^E, J_s^U and J_l^U denote the respective present discounted value of being employed, short-term unemployed and long-term unemployed. In equilibrium, J^E, J_s^U and J_l^U can be written as asset equations which satisfy

$$J^E=\ln(w)+\beta\left[\lambda J_s^U+(1-\lambda)J^E\right] \quad (8)$$

$$J_s^U = Z_j^s + \beta\left[\phi(e_s)\theta q(\theta) J^E + (1-\phi(e_s)\theta q(\theta)) J_l^U\right] \quad (9)$$

$$J_l^U = Z_j^l + \beta\left[\phi(e_l)\theta q(\theta) J^E + (1-\phi(e_l)\theta q(\theta)) J_l^U\right] \quad (10)$$

where Z_j^s and Z_j^s stand for the respective instantaneous utility of a short-term unemployed and a long-term unemployed in the policy scheme *j*, *j* = *d*,*b*, and β is the discount rate. Let Z_d^s and Z_d^l be the instantaneous utility of a short-term unemployed and a long-term unemployed in the declining UI policy scheme. They read as

$$Z_d^s = \ln(z_s) - e_s \quad (11)$$

$$Z_d^l = \ln(z_l) - e_l \quad (12)$$

Let Z_b^s and Z_b^l be the instantaneous utility of a short-term unemployed and a long-term unemployed in the search bonus policy scheme. They read as

$$Z_b^s = \phi(e_s)\theta q(\theta)\ln(z+B) + (1-\phi(e_s)\theta q(\theta))\ln(z) - e_s \quad (13)$$

$\phi(e_s)\theta q(\theta)$

$$Z_b^l = \ln(z) - e_l \quad (14)$$

Each type of unemployed persons chooses search intensity in order to maximize the value of unemployment.

In the declining UI scheme equations (9), (10), (11) and (12) imply the following first order condition for optimal search effort

$$-1 + \phi'(e_i)\theta q(\theta)\beta\left(J^E - J_l^U\right) = 0 \qquad i = s,l \quad (15)$$

Separability in the instantaneous utility function and the particular time structure of the model imply that both the short-term and the long-term unemployed choose the same search intensity denoted by e. The difference $\left(J^E - J_l^U\right)$ reads as

$$\left(J^E - J_l^U\right) = \frac{e + \lambda\beta \ln(\rho_s) - (1+\lambda\beta)\ln(\rho_l)}{1-\beta\left[1-\lambda-\phi(e)\theta q(\theta)\right]} \tag{16}$$

As shown in Appendix B.1, partial equilibrium analysis reveals that search intensity of both sort-term and long-term unemployed increases with ρ_s. The result that the long-term unemployed search intensity may rise when ρ_s is increased corresponds to the so-called entitlement effect from Mortensen (1977). The fact that search intensity among the short-term unemployed may rise as well was first noted by Shavell and Weiss (1979). With a more decreasing UI scheme, short-term unemployed workers expect a sharper drop in their future income in unemployment and thus have an incentive to increase their search intensity.

In the search bonus scheme equations (9), (10), (13) and (14) imply the following first order condition for an optimal search effort

$$-1 + \phi'(e_i)\theta q(\theta)\left[\beta\left(J^E - J_l^U\right) + \ln\left(\frac{\rho + b}{\rho}\right)\right] = 0 \tag{17}$$

$$-1 + \phi'(e_l)\theta q(\theta)\beta\left(J^E - J_l^U\right) = 0 \tag{18}$$

It can be deduced from (17) and (18) that in equilibrium, $\phi''(.) < 0$, $e_s > e_l$.

The difference $\left(J^E - J_l^U\right)$ now reads:

$$\left(J^E - J_l^U\right) = \frac{e_l + \lambda\beta(e_l - e_s) + \lambda\beta\phi(e_s)\theta q(\theta)\ln\left(\frac{\rho + b}{\rho}\right) - \ln(\rho)}{1-\beta\left[1-\lambda-\phi(e_l)\theta q(\theta) + \lambda\beta\theta q(\theta)(\phi(e_s) - \phi(e_l))\right]} \tag{19}$$

As shown in Appendix A, both $\frac{\partial e_s}{\partial b}$ and $\frac{\partial e_l}{\partial b}$ are positive. For the short-term unemployed, $\frac{\partial e_s}{\partial b} > 0$, on one hand, this is due to the Shavell and Weiss effect presented previously: as the search bonus increases the expected income from unemployment, the short-term unemployed anticipate a drop in their future income, hence they search more intensively. On the other hand, there is also an entitlement effect: by increasing their search effort, the short-term unemployed increase their chances of cashing in the search bonus. As concerns the long-term unemployed, the same kind of entitlement effect identified in the decreasing unemployment compensation scheme is observed. Then, a more declining sequence of unemployment insurance and a positive search bonus produce similar quantitative effects on both types of unemployed workers search intensities.

Now, if the difference $\left(J^E - J_l^U\right)$ takes the same value in the two alternative schemes, which is a non zero mass event, then the existence of an entitlement for short-term unemployed leads to $e_s > e_l = e$, implying that the effects on aggregate unemployment are more pronounced in the search bonus scheme that in the decreasing unemployment compensation scheme.

Because of this direct effect on the search efforts of short-term unemployed, expressed by $\phi'(e_s)\theta q(\theta)\ln\left(\frac{\rho + b}{\rho}\right)$, in condition (17), search bonuses may have stronger positive effects on aggregate unemployment that a declining UI scheme.

2.3 Firm Behavior and Wage Determination

The representative competitive firm has only one position that can be either occupied or vacant. When filled with a worker, the position produces *y* units of good. When vacant, the position incurs a cost denoted by *γ*. J^V and J^F stand for the present discounted value of a vacant and an occupied job respectively. At steady state they verify

$$J^F = y - (1+\tau)w + \beta\left[\lambda J^V + (1-\lambda)J^F\right] \tag{20}$$

$$J^V = -\gamma + \beta\left[q(\theta)J^F + (1-q(\theta))J^V\right] \tag{21}$$

where as already mentioned, τ is the payroll tax rate.

Free entry of vacancies and decreasing returns in the matching process ensures that in equilibrium the number of positions available in the economy is finite and corresponds to condition *V = 0*. Then, from equations (20) and (21) we obtain

$$J^F = \frac{y - (1+\tau)w}{1-\beta(1-\lambda)} = \frac{\gamma}{\beta q(\theta)} \tag{22}$$

$(1+\tau)w$

According to condition (22) a rise in the real labor cost, namely a rise in $(1+\tau)w$, induces firms to post fewer vacancies in order to compensate for this discounted profitability loss. Indeed, by doing so, they increase the probability of filling a single vacancy and consequently also increase the expected returns attached to that vacancy.

Wages are assumed to be determined by bargaining à la Rubinstein-Shaked-Sutton[3] whereby

$(1+\tau)w$

$$w = \max\left\{g(\alpha, y, z_s), (1-\beta)J_l^U\right\} \tag{23}$$

where, α is the bargaining power of workers and z_s is the worker's disagreement outcome received during bargaining, namely the strike outcome. z_s is assumed to be non zero because of log utility functional and different from the unemployment benefits in order to obtain a well defined solution. As α, y and, z_s can be treated as exogenous parameters. Then without any loss of theoretical coherence and for the sake of clarity, $g(\alpha, y, z_s)$ can be re-written as $\alpha * y$. In other words, $\alpha *$ can be defined as the effective bargaining power of workers which depends on their utility functional or more precisely their risk-aversion, y and, their disagreement outcome within the bargain. Thus, wages are equal to some constant level, expressed as a proportion $\alpha *$ of output unless the outside option of the worker binds. The outside option is identical for both types of unemployed workers as short-term unemployment is defined as lasting only one period. This bargaining rule implies that wages are the same for both types of unemployed workers. Search bonuses do not appear in the bargaining because only the outside option of the parties involved matters. If workers coming from short-term unemployment were to reject the match, they would lose bonus coverage as it is contingent upon acceptance.

Combining (22) and (23) leads to a single equation determining equilibrium θ, which reads as

$$\frac{y-(1+\tau)\max\left\{\alpha * y, (1-\beta)J_l^U\right\}}{1-\beta(1-\lambda)} - \frac{\gamma}{\beta q(\theta)} = 0 \tag{24}$$

In the declining unemployment benefits scheme J_l^U is given by

$$\ln(w) + \frac{\lambda\beta^2\phi(e)\theta q(\theta)\ln(\rho_s) + (1-\beta(1+\lambda\beta\phi(e)\theta q(\theta)-\lambda))\ln(\rho_l) - (1-\beta(1-\lambda))e}{1-\beta[1-\lambda-\phi(e)\theta q(\theta)]} \tag{25}$$

In the search bonus scheme J_l^U reads as

[3] It can be argued that this wage bargaining rule has better microfoundations than Nash bargaining. See for instance Wolinsky (1987) or Costain (1996).

$$\ln(w)+\frac{[1-\beta[1-\lambda-\lambda\beta(p_l-p_s)]]\ln(\rho)+\lambda\beta^2 p_l\left(p_s\ln\left(\frac{\rho+b}{\rho}\right)-e_s\right)-[1-\beta(1-\beta\lambda p_s-\lambda)]e_l}{1-\beta[1-\lambda-(1+\lambda\beta)p_l+\lambda\beta p_s]} \qquad (26)$$

where *w* is defined by (23).

3. Equilibrium

3.1 Characterization

Uniqueness of equilibrium is guaranteed by the assumptions that the matching function has constant returns to scale and that search intensity enters the matching technology multiplicatively[4]. A proof sketch is presented in appendix B.2.

In the declining unemployment benefits scheme, conditions needed for fully characterized equilibrium are the condition for jobs (24) , where J_l^U is defined by (25), τ by (6), and the condition for optimal search effort (15) filled by (16). The subsequent two equation system can be solved for equilibrium θ and *e*. Then equilibrium τ, wages and unemployment rates can be recovered recursively.

In the search bonus scheme, conditions needed for fully characterized equilibrium are the condition for jobs (24), where J_l^U is defined by (26) and τ by (7), and conditions for optimal e_s and e_l, (17) and (18) respectively, both completed by (19). We obtain a three equation system that be solved for equilibrium θ, e_s and e_l. Again, equilibrium τ, wages and unemployment rates can be recovered recursively.

3.2 The Properties of the Instruments

Consider the case in which the unemployed worker's outside option never binds in the wage determination rule (23). Thus wages are simply a proportion α^* of the actual match productivity *y*, as defined previously, and equilibrium θ is affected by policy instruments only through subsequent variations in τ.

In the declining unemployment benefits scheme, by substituting τ from (6) into the condition for jobs (24) we obtain an equation in e and θ

$$\frac{y\left[1-\alpha*\left(1+\rho_s\lambda+\rho_l\lambda\frac{(1-\phi(e)\theta q(\theta))}{\phi(e)\theta q(\theta)}\right)\right]}{1-\beta(1-\lambda)}-\frac{\gamma}{\beta q(\theta)}=0 \qquad (27)$$

Equation (27) together with condition for optimal search (15), fully characterize equilibrium *e* and *θ*. It has been shown that with lower ρ_l, search efforts increase. As *e* increases, it is straightforward to check that $\frac{(1-\phi(e)\theta q(\theta))}{\phi(e)\theta q(\theta)}$ becomes smaller, meaning that the tax rate is also lower. It becomes even lower as ρ_l takes lower values.

Thus, job productivity unambiguously rises and firms are willing to open more vacancies. As a consequence, equilibrium *θ* is expected to rise, reinforcing the effect on *e*.

In the search bonus scheme, the impact on the tax rate remains ambiguous and so too is the effect on equilibrium tightness. *τ* can be expressed as

$$\tau=\rho\frac{\lambda[1-\phi(e_s)\theta q(\theta)+\phi(e_l)\theta q(\theta)]}{\phi(e_l)\theta q(\theta)}+b\lambda\phi(e_s)\theta q(\theta) \qquad (28)$$

We have shown that both e_s and e_l increase with *b*. It is easy to check from (28) that $\frac{\partial\tau}{\partial e_l}$ is negative, and that $\frac{\partial\tau}{\partial e_s}$ is negative for $\rho>b\phi(e_l)\theta q(\theta)$.

The latter condition is likely to be satisfied over a large range of *b* values. Nevertheless, even if both derivatives are negative, *τ* is pushed upward by the direct cost of implementing the search bonus scheme. When cost effectiveness is observed, the reinforcing effects defined in the declining unemployment benefits scheme would also occur with search bonuses. Otherwise, *θ* would tend to decrease, and thus offset the impact of higher search efforts on equilibrium values.

If we now turn to the case where the outside option of the unemployed worker is binding in the wage determination, results might be slightly modified. On one hand lower ρ_l would push wages downward while, on the other hand, search bonuses would push them upward. In the former case, profitability would be further increased as long as $J_l^U>\alpha*y$, and *θ* rises unambiguously. In the search bonus scheme, even assuming that *τ* falls in the first place, labor costs might rise if wage pressure is high enough and, as a consequence, *θ* might fall. If this occurs, then search efforts could be dampened and the overall equilibrium impact of bonuses becomes unclear.

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[4] See Pissarides (2000), chap. 5.

There is a third scenario in which the unemployed workers' outside options are not binding in the baseline situation, but become binding once *b* has reached some threshold value. In that case, wages would remain constant in the declining unemployment benefits scheme and the effects of a sharper unemployment compensations profile are the ones described earlier. In the search bonus scheme, wages would rise only for *b* values above a threshold level *b** corresponding to $J_l^U = \alpha * y$. For *b*>*b** the positive impact of the search bonus on search efforts and possibly on θ is played down as in the case where J_l^U is binding at $b = 0$.

4. Welfare

It has been shown that search bonuses could replicate declining insurance effects on unemployment. As mentioned previously, a declining sequence of unemployment benefits penalizes the long-term unemployed with respect to short-term unemployed workers. This feature explains why the short-term unemployed increase their search effort as the profile of unemployment compensations becomes more decreasing. As to search bonuses, they effectively represent a tangible asset for the short-term unemployed which incite the latter to increase their search efforts. Nevertheless, in contrast to increasing unemployment benefits they are not a ``malus'' for the long-term unemployed. They reward the search efforts of short-term unemployed workers without penalizing possible failure and preserve the same insurance properties as a flat profile of unemployment compensation. Thus, from a social welfare point of view, if the respective effects of the two policies on equilibrium values are similar, society should be better off in the search bonus scheme than in the declining UI scheme, always taking as a benchmark an economy paying a flat profile of unemployment benefits. Moreover, search bonuses are likely to satisfy Rawls (1971) justice criterion while this would not be the case for a declining sequence of unemployment compensations[5].

In our economy, workers are risk-adverse while entrepreneurs are risk-neutral. This heterogeneity raises the problem of choosing an appropriate welfare functional. Fredriksson and Holmlund (1998) use a standard utilitarian welfare functional, thus ignoring implicitly the heterogeneity among individuals. In order to assess welfare inclusive of uncertainty concerns, Cahuc and Lehmann (1999) use the discounted certainty equivalent of expected utilities of risk adverse individuals.

Following Cahuc and Lehmann (1999) we use as a measure of individual welfare, the discounted certainty equivalent of the expected utility for each type of worker and take the expected returns from filled and vacant positions for firms. Employed, short-term unemployed and long-term unemployed

[5] This is a central result of Cahuc and Lehmann (1999).

workers' welfare are given by the value of χ that solves respectively $\ln(\chi) = (1-\beta)J^E$, $\ln(\chi) = (1-\beta)J_s^U$ and $\ln(\chi) = (1-\beta)J_l^U$.

For instance, the value of χ that solves $\ln(\chi) = (1-\beta)J_s^U$, represents the level of income that leaves a short-term unemployed worker indifferent between consuming this income and pursuing a search.

These measures will permit us to consider the Rawls' justice criterion as a normative indicator. We also construct an aggregate social welfare index inclusive of all types of risk-adverse workers welfare and risk neutral firms' welfare. Then, aggregate social welfare reads as

$$SW = \chi_{wec} + (1-u)(1-\beta)J^F + u\theta(1-\beta)J^V$$

where χ_{wec} corresponds to the value of χ that solves

$$\ln(\chi) = (1-\beta)\left[(1-u)J_s^U + u_s J_s^U + u_l J_l^U\right]$$

and, $(1-\beta)J^F$ and $(1-\beta)J^V$ are the asset values of occupied and vacant jobs defined previously. Because of free entry $J^V = 0$.

As the previous analysis did not produce any clear analytical result, it becomes difficult to obtain a proper analytical welfare assessment. Hence, a calibrated version of the model is used in order to clarify the respective impact of the two policy schemes under study.

5. Some Quantitative Exercises

5.1 The Experiment

Previous analytical analysis revealed that a declining sequence of unemployment benefits has a positive impact on aggregate employment and that search bonuses could also be expected to lower unemployment as long as the inherent wage pressure was not too high. This depends on how far the unemployed workers' outside option in the wage determination is binding. To make the computational analysis of the model consistent with this feature, we consider three different situations. One is where the outside option of the unemployed worker is never binding whatever the policy scheme. Another is where the baseline economy is characterized by $J_l^U = \alpha * y$, in other words where wages would tend to rise in the search bonus scheme and remain constant in the declining UI scheme. The last is where the workers' outside option becomes binding only for values of *b* above some threshold *b** and remains unbinding in the declining UI scheme. These three situations can be analyzed in a unified calibration as the equilibrium wage in the baseline economy is in all cases equal to $\alpha * y$. A distinct calibration would be needed if we had to consider the case where the outside option of the unemployed worker remains binding for sharper UI profiles as well. Nevertheless, this would not add anything qualitatively significant to the overall conclusions. The objective of my computational exercises is twofold. First, we want to assess the equilibrium and welfare properties of both schemes in the three cases presented previously. Second, WE pay particular attention to the ``intermediate case'', where the outside option of the worker becomes binding only above a certain value *b**, and compare the respective effect on welfare indexes of both policy schemes for a given variation of the aggregate unemployment rate.

The benchmark is an economy where the government pays a flat UI profile, that is, $\rho_s = \rho_l = \rho$. We first introduce the search subsidy *b* with a flat UI scheme and look at the effect on aggregate unemployment obtained for increasing values of *b*. We then consider a declining UI scheme where $\rho_s = \rho > \rho_l$ for various values of ρ_l. In all cases, the government budget constraint remains balanced in equilibrium.

5.2 Calibration

Parameters are chosen such that we obtain a 11% unemployment rate and a 0.5 elasticity of unemployment duration with respect to unemployment benefits, which is in the middle range of the estimates found in Layard and *alii* (1991).

U	u_s/u_l	ρ	η	β	λ	α
12 %	40 %	0.4	0.5	0.05	0.975	0.5

Table 1: Baseline Equilibrium Values

A time period of unit length is interpreted to be one semester. Following Machin and Manning's (1998) average estimate for European countries, calibration generates a proportion of short term unemployment equal to 60%. The time discount factor β is set equal to 0.975. The separation rate λ is fixed at 0.05, corresponding to an average duration of a job of ten years. This could be seen as a relatively high figure, but it becomes more realistic once we consider the fact that quits and job-to-job movements are not taken into account.

The matching function is assumed to be Cobb-Douglas, such that $q(\theta) = k\theta^{-\eta}$. η is set equal to 0.5 which corresponds to the upper range of the estimates in Blanchard and Diamond (1989). *k* is equal to

0.2. The search efficiency function is homogenous of degree a. It is given by, $\phi(e_i) = e_i^a$, where a=0.4.

In the base-line calibration, the replacement ratio remains constant over time and is set equal to 0.4.This value falls in the middle range of Martin (1996) estimates for European countries.

In the three situations, the productivity of a firm-worker pair, y, is set to be 2.6 times larger than the cost of keeping a vacancy open, γ. The latter is set equal to 0.95 in the first situation *(A)*, to 0.9878 in the second situation *(B)* and 0.965 in the third situation *(C)*.

The model is solved recursively. We first solve for equilibrium search intensities and labor market tightness. These values are used to compute equilibrium (un)employment rates. Then, we retrieve equilibrium τ and the effective equilibrium wage.

5.3 Results

Results obtained in the three situations described previously are reported graphically in the next subsection. For each situation, we first report equilibrium properties, that is the effect of both policy schemes on unemployment (upper graphs), search efforts (middle graphs) and labor costs (lower graphs). Then, we report the effects on welfare, more precisely on aggregate welfare (upper graphs), on the welfare of firms (middle graphs) and on the welfare of workers (lower graphs).Finally, we present welfare effects for each category of worker. The upper graphs display welfare reactions expressed in absolute terms while the lower graphs present welfare reactions expressed in relative terms. Unless otherwise indicated, in the search bonus program the x-axis indicates absolute values taken by b. In the declining UI program the x-axis indicates ρ_s / ρ_l levels.

- **Situation A: Never Binding J_l^U**

In this situation, the outside option of workers in wage determination is never binding, in any of the policy schemes. As a consequence wages are constantly equal to $\alpha * y$.

The upper graph in figure 1 show that both policy schemes could lead to lower unemployment. However, as the search bonus takes larger and larger values, its downward effect is dampened and even reversed for b larger than 2. This never occurs when the UI profile becomes sharper and sharper. For $\rho_s / \rho_l = 4$, that is $\rho_s = 0.4$ and $\rho_l = 0.1$, unemployment falls to 7.36 %. Middle graphs confirm the analytical result stating that both schemes stimulate unemployed workers search effort. However, in the search bonus scheme, the long-term unemployed (dashed line) respond only weakly to bonus search incentives. This is explained by the fall in equilibrium θ, which is due to the rise in τ and thus in labor costs as displayed by the lower left graph in figure 1 (solid curve). As for the declining insurance scheme(lower right graph), as labor costs (solid line) are reduced because of lower τ, equilibrium tightness increases reinforcing the upward effect of more declining unemployment compensations on search efforts (identical for both workers).

Figure 2 displays the welfare properties of the two schemes. As a general remark, aggregate welfare never falls below its initial value in the search bonus schemes, at least for over the range of values given to *b*. On the contrary, in the declining UI scheme, a sharper profile is accompanied by a worsening of aggregate welfare. This is essentially explained by the fact that the search bonus scheme induces a rise in workers overall expectations income while in the declining unemployment benefits scheme the reverse occurs despite higher exit rates at all unemployment duration. Differently put, a negative trade-off between lower unemployment and aggregate welfare shows in the latter scheme. This trade-off is positive in the search bonus scheme, for *b*s generates lower unemployment. For *b*>2, the trade-off is somewhat reversed, in the sense that, unemployment starts increasing and aggregate welfare improvement is reduced.

The middle and lower graphs show that, in the search bonus scheme firms are worse off as *b* increases while workers enjoy increasing welfare. The former fact is explained by the rise in labor costs observed earlier. The latter is due principally to the rise in workers overall expected income induced by the search bonus. In the declining unemployment compensations scheme, converse results are obtained. The welfare of firms increases because of lower labor costs, and workers welfare falls because of lower overall expected income. Figure 3 reports welfare reactions by worker category. Upper graphs display welfares variations in absolute terms. In the search bonus scheme, the welfare of employed (dashed line), short-term unemployed (solid line) and long-term unemployed workers (dotted line) increases decreasingly with *b*. The converse is observable in the declining UI scheme. Lower graphs suggest that in the search bonus scheme, long-term unemployed workers are the ones who gain the least from the scheme. The employed to long-term unemployed welfare ratio (dashed line) increases in both schemes but in a stronger manner in the declining UI scheme. The short-term to long-term unemployed welfare ratio also increases in the two schemes. Again, welfare discrepancy widening is more pronounced in the declining UI scheme. As concerns the employed to short-term unemployed welfare ratio (solid line), it falls in the search bonus scheme and rises in the declining UI scheme. This reflects the fact that search bonuses effectively represent extra positive search return while lower replacement ratios represent search penalties.

- **Situation B: Initially Binding J_l^U**

In this situation, the outside option of workers in wage determination is binding in the baseline economy, that is, $\alpha * y = J_l^U$. As a consequence wages remain constantly equal to $\alpha * y$ in the declining UI scheme and are expected rise in the search subsidy scheme . As optimal search efforts do not depend on the wage level, identical reactions will be observed.

The upper graphs in figure 4 reveal that search bonuses still reduce unemployment over some range of *b* values. However, both the impact on unemployment and the range of values of *b* are smaller. This is explained by the rise in labor costs as shown in the lower left graph. Both wages (dashed line) and taxation (solid-dashed line) increase with the search bonus. Consequently, job profitability falls and θ is lower. Declining UI scheme performance is not affected as wages and search efforts remain as in situation *A*.

Figure 5 displays the welfare properties of both schemes. In the search bonus scheme, aggregate welfare responds in a way similar to the one observed in situation *A*. However, the relative impact is significantly stronger and the inverted U-shaped reaction profile is sharper for low values of *b* and smoother for high values. Firms welfare falls by less than workers welfare increases for values of *b* below 1.5 leading to a rise in the aggregate welfare index.

Figure 6 shows that all workers categories are better off in the search bonus scheme (upper left graph), although welfare gains are tiny for long-term unemployed workers. This translates in a deterioration in long-term unemployed welfare with respect to others workers (lower left graph, dashed and dotted lines). However, short-term unemployed welfare increases compared to employed workers. This is explained by the fact that the exit rate from long term unemployment deteriorates because of lower θ and the subsequent welfare loss is hardly offset by higher wages and discounted bonus coverage. In the declining UI scheme, all types of workers are made worse off as the UI profile gets sharper. As suggested by the lower right graph, the welfare of employed workers falls by less than any other welfare category. The long-term unemployed still lose the most from more declining unemployment compensations.

- **Situation C: ``Partially'' Binding** J_l^U

In this situation, the outside option of workers in wage determination becomes binding only for *b* values above some threshold *b**. Wages remain constantly equal to $\alpha * y$ in the declining UI scheme. Again conditions for optimal search intensity are unaffected by wage levels and reproduce situation *A* efforts reactions.

Situations *C* can be seen as intermediary between situation *A* and situation *B* as the following computational results assert. The left upper graph in figure 7 indicates than the downward effect of search bonuses on unemployment is reversed for *b* values larger than 1.65. J_l^U becomes binding for *b*>1.45, meaning that there is still a positive impact on employment despite the rise in wages as was found in situation *B*.

On the curve corresponding to labor costs (lower left solid) and wages (lower left dashed) reaction, a kink can be noted at *b* equal to 1.65. The latter indicates that the outside option is binding for any higher value of *b*. This also translates into a sharper decrease of θ, explaining the reversed tendency in unemployment.

As for welfare, tendencies revealed in situations *A* and *B* are repeated although with some nuances as shown in figure 8. The aggregate welfare reaction curve displays an inverted-U shape similar to the one observed in situation *A*. The welfare of firms falls and is characterized by a kink in its reaction curve as the workers outside option starts binding. Workers welfare rises with *b* and a kink also appears at b=1.65.

In the declining UI scheme firms welfare increases slightly with decreasing ρ_l. Workers welfare falls sensitively and by more than the welfare of firms welfare increases. The upper graphs in figure 9 suggests than long-term unemployed workers still benefit the least, in terms of welfare, from the introduction of search bonuses and lose the most from more declining unemployment compensations. Turning to relative analysis (lower graphs), the welfare gap between employed and long-term unemployed workers (dashed line) increases with *b*. The same is true as concerns short-term and long-term unemployed welfare discrepancy. A kink is also observed at b=1.65. Again employed and short-term unemployed workers welfare indexes converge as *b* increases.

Figure 10 presents the performance of search bonuses compared to that of declining unemployment compensations. The x-axis displays variations (negative) in the unemployment rate common to both schemes. Values corresponding to search bonuses make the solid curves and values corresponding to more declining UI the dashed curves. As regards aggregate welfare (the upper graph), search bonuses increasingly better perform than more declining unemployment compensations. The lower graph suggests that long-term unemployed workers are better off in the search bonus scheme while they are worse off with more declining unemployment compensations. Moreover, the difference between the two schemes increases as the impact on unemployment becomes stronger.

5.4 Graphical Outcomes

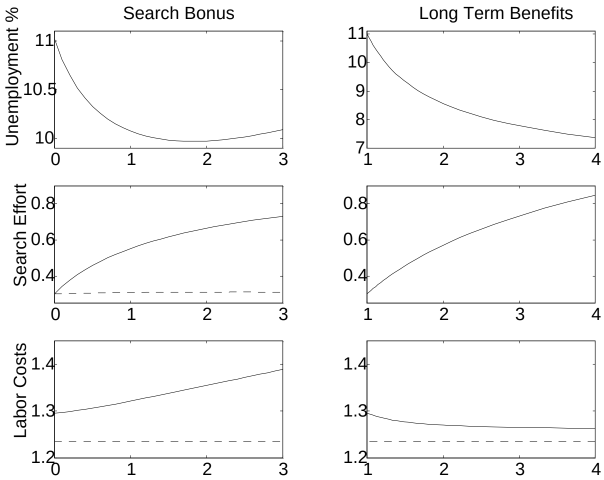

Figure 1: Equilibrium Properties (A)

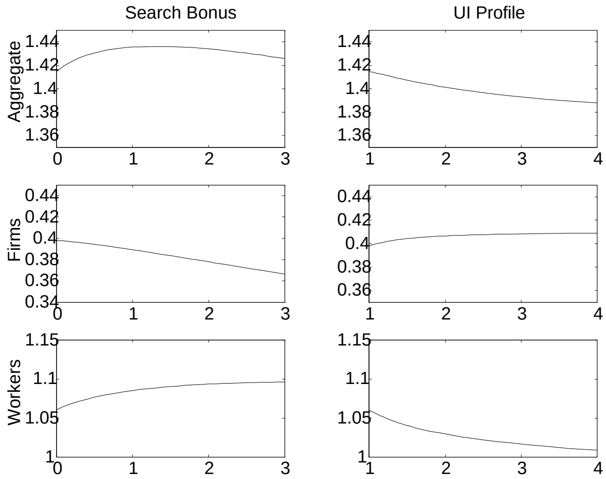

Figure 2: Welfare Properties (A)

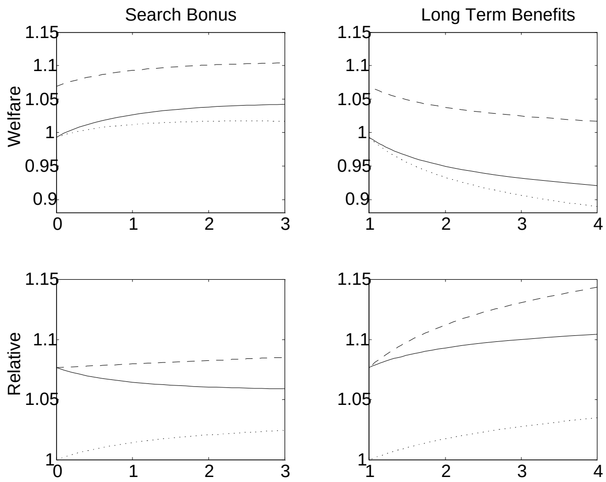

Figure 3: Workers Welfare (A)

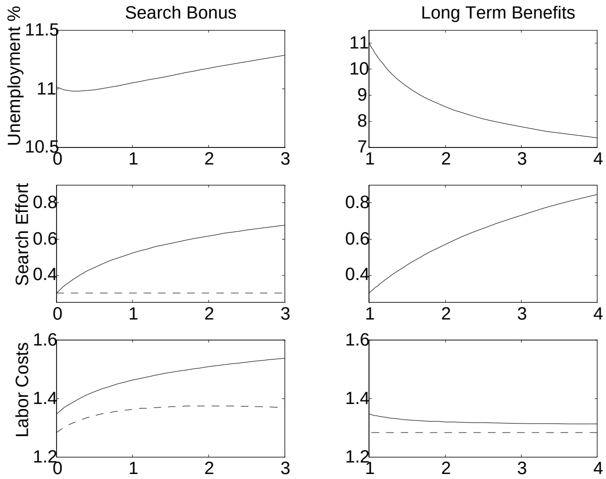

Figure 4: Equilibrium Properties (B)

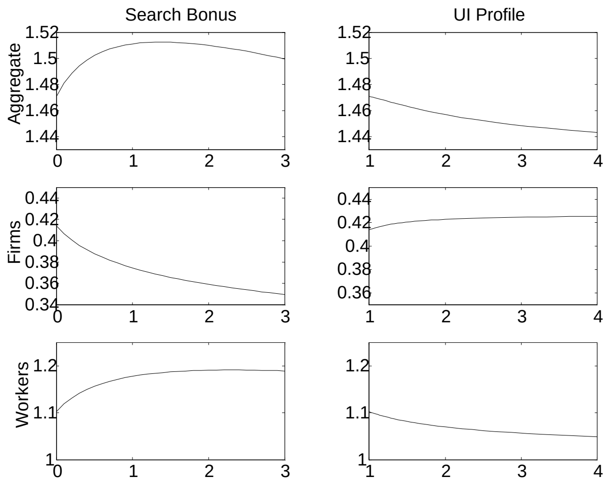

Figure 5: Welfare Properties (B)

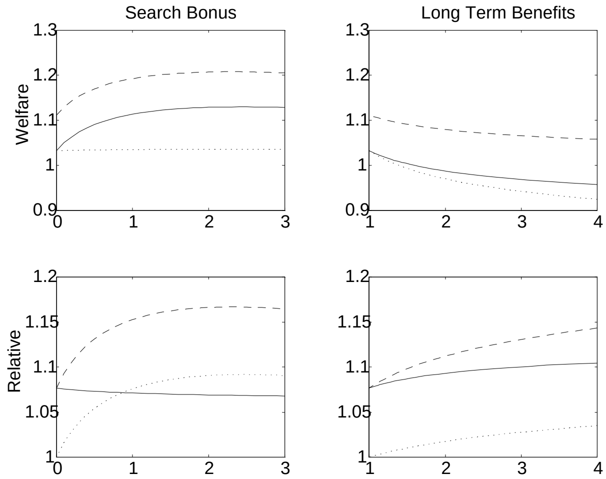

Figure 6: Workers Welfare (B)

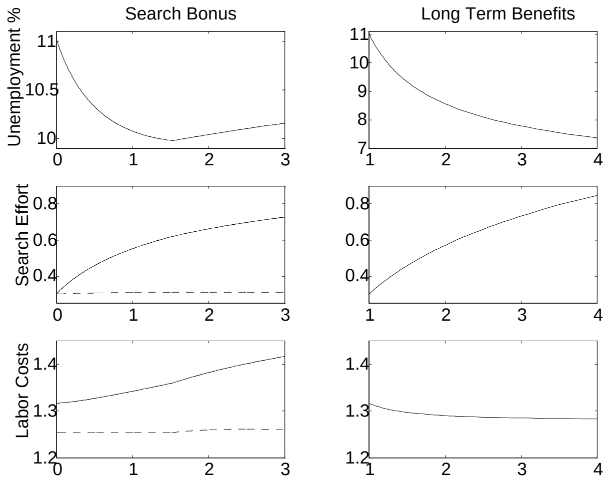

Figure 7: Equilibrium Properties (C)

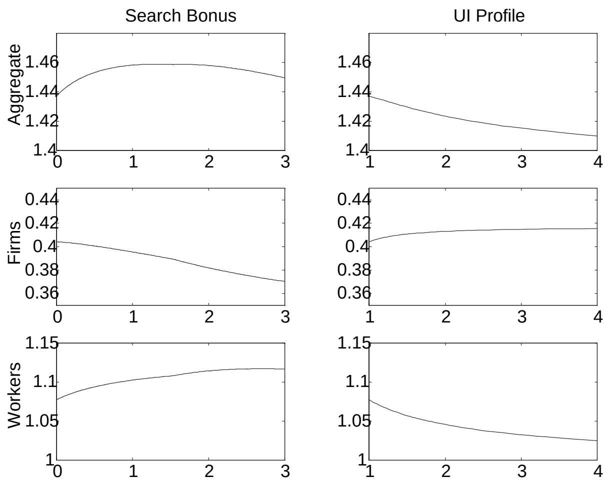

Figure 8: Welfare Properties (C)

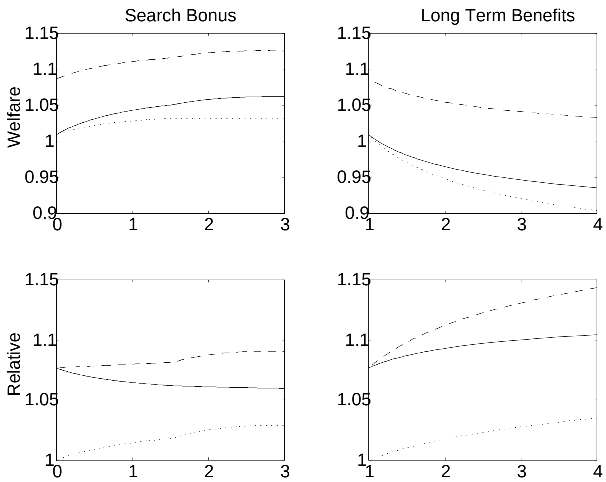

Figure 9: Workers Welfare (C)

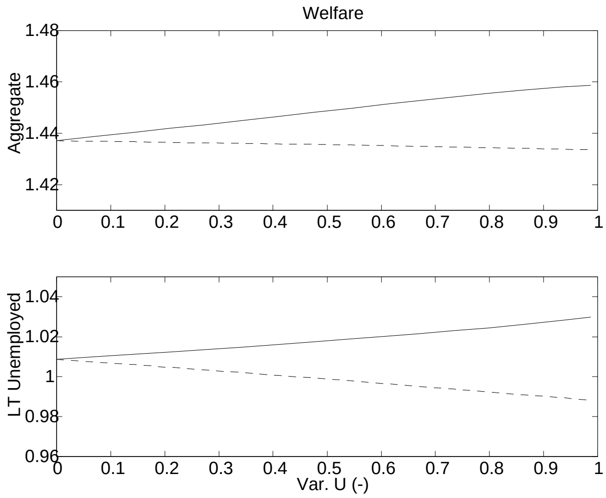

Figure 10: Relative Performance (C)

5.5 An Overall Assessment

Calibration exercises confirm previous analytical conjectures. Both schemes restore search incentives and can generate lower unemployment rates. However, downward effects on unemployment of search bonuses are dampened by wages and tax burden reactions. As wages become more and more responsive to search bonus levels (the extreme case was situation B), the impact of the latter on employment is found to decrease. In the declining UI scheme, wages remain constant in the three situations and a sharper profile always induces lower unemployment rates. Welfare analysis suggests that in all situations, search bonuses perform better than more declining unemployment compensations. While the former are found to be welfare improving if compared to the benchmark economy without policy, the latter proves to be welfare worsening. In other words, a negative trade-off between lower unemployment and welfare is identified in the declining UI scheme. As for search bonuses, this trade-off remains positive as long as unemployment is curbed and then turns to negative because both welfare and unemployment tendencies are reversed. However, welfare remains above its initial level.

Further analysis reveals that firms welfare falls in the search bonus scheme and rises in the declining UI one. As concerns workers, the converse occurs. All types of workers whether employed or unemployed, are better off in the search bonus scheme and worse off in the declining UI scheme. Long-term unemployed are also found to benefit less than short-term and employed worker from the implementation of search bonuses and suffer the most from sharper declining profile of UI. With search bonuses, this is a consequence of the fact that the fall in the outflow rate from long-term unemployment, because of lower equilibrium tightness, offset the rise in expected overall income induced by search bonus coverage and by wage pressure if such is the case. As for short-term unemployed workers, both their outflow rate from unemployment and overall expected earnings increase. Consequently short-term welfare index rises. Employed workers welfare horizon improves because welfare attached to unemployment has rises and because wages may rise. As UI is made more declining, the loss in the overall expected income of the long-term unemployed, because of lower long-term replacement ratio, exceeds the gains attached to higher outflow rates. This is also the case for the short-term unemployed and as a consequence for employed workers.

If we exclude firms from the analysis, unemployed workers are the worst off category in the baseline economy. Computational results suggest that unemployed workers, both short-term and long-term are made better off by the implementation of search bonuses while they are made worse-off by unemployment compensations increasingly declining over time. If we had to use Rawls' criterion for evaluating the desirability of the two schemes, only search bonuses would be approved despite their lower employment performance.

6. Conclusion

The scope of the paper was to define if search bonuses could be considered as an alternative to unemployment benefits which decline over time. The motivation for such a theoretical exercise was based principally on welfare considerations. Both policy scheme could potentially restore search incentives, and then stimulate unemployment outflow. However, while decreasing unemployment benefits penalize low search effort search bonuses reward high search effort. As individuals are risk adverse the insurance dimension of unemployment benefits matters. In that respect, the different functioning of the two schemes was thought to generate opposite effects on welfare and more precisely on long-term unemployed workers' welfare.

The computational findings based on the model confirmed theoretical expectations in large part. Long-term unemployed workers' welfare deteriorates as unemployment compensations become more declining. Search bonuses tend to generate more welfare for all types of workers and for the economy as a whole. Their performance is mitigated by their negative effect on labor market tightness expressed through continuously increasing labor costs and contained unemployment reduction. However, Rawls criterion considerations would suggest than a search bonus scheme should be preferred to declining unemployment compensations.

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Appendix

A. Optimal Search Intensities

In the declining UI scheme, by substituting expression (16) for $\left(J^E - J_l^U\right)$ in condition (15) we obtain

$$\Omega^1 = -[1-\beta[1-\lambda-\phi(e)\theta q(\theta)]] + \phi'(e)\theta q(\theta)\beta[e+\lambda\beta\ln(\rho_s)-(1+\lambda\beta)n(\rho_l)] = 0 \quad \text{(A.1)}$$

$\mathrm{n}(\rho_s)$

Differentiation with respect e and ρ_s gives respectively

$$\Omega_e^1 = \phi''(e)\theta q(\theta)\beta[e+\lambda\beta\ln(\rho_s)-(1+\lambda\beta)n(\rho_l)] < 0$$

$$\Omega_{\rho_s}^1 = \phi'(e)\theta q(\theta)\beta\left(\frac{\lambda\beta}{\rho_s}\right) > 0$$

$$\Omega_{\rho_l}^1 = -\phi'(e)\theta q(\theta)\beta\left(\frac{1+\lambda\beta}{\rho_l}\right) < 0$$

$$\Omega_\theta^1 = -\phi(e)(\theta q(\theta))'\beta + \phi'(e)(\theta q(\theta))'\beta[e+\lambda\beta\ln(\rho_s)-(1+\lambda\beta)n(\rho_l)]$$

From equation (A.1) $\phi'(e)\beta\,\dfrac{[e+\lambda\beta\ln(\rho_s)-(1+\lambda\beta)n(\rho_l)]}{\theta q(\theta)[1-\beta(1-\lambda-\phi(e)\theta q(\theta))]}$. Hence Ω_θ^1 can be re-written as

$$\Omega_\theta^1 = \frac{(\theta q(\theta))'}{(\theta q(\theta))}[1-\phi(e)(\theta q(\theta))\beta + \lambda\beta - \beta + \phi(e)(\theta q(\theta))\beta] > 0$$

Thus, using the implicit function theorem we have

$$\frac{\partial e}{\partial \rho_s} = -\frac{\Omega_{\rho_s}^1}{\Omega_e^1} > 0, \frac{\partial e}{\partial \rho_l} = -\frac{\Omega_{\rho_l}^1}{\Omega_e^1} < 0 \text{ and } \frac{\partial e}{\partial \theta} = -\frac{\Omega_\theta^1}{\Omega_e^1} > 0$$

In the search bonus scheme, by substituting expression (19) for $\left(J^E - J_l^U\right)$ in condition (17) we obtain for optimal e_s

$$\Omega^2 = B_2\left[\phi'(e_s)\theta q(\theta)\ln\left(\frac{\rho+b}{\rho}\right)-1\right]+\phi'(e_s)\theta q(\theta)\beta C_2 = 0 \qquad \text{(A.2)}$$

where $B_2 = 1-\beta\left[1-\lambda-\phi(e_l)\theta q(\theta)+\lambda\beta\theta q(\theta)(\phi(e_s)-\phi(e_l))\right]$

and $C_2 = e_l + \lambda\beta(e_l - e_s)+\lambda\beta\phi(e_s)\theta q(\theta)\ln\left(\frac{\rho+b}{\rho}\right)-\ln(\rho)$

We have

$$\Omega_{e_s}^2 = \phi''(e_s)\theta q(\theta)\left[B_2\ln\left(\frac{\rho+b}{\rho}\right)+\beta C_2\right]+\phi'(e_s)\theta q(\theta)\beta^2(1+\lambda)E_2 < 0$$

as $E_2 = \phi'(e_s)\theta q(\theta)\ln\left(\frac{\rho+b}{\rho}\right)-1<0$ from condition (A.2). We verify that

$$\Omega_b^2 = \phi'(e_s)\theta q(\theta)\left(\frac{\rho}{\rho+b}\right)\left[B_2+\beta^2\lambda\phi(e_s)\theta q(\theta)\right]>0$$

Moreover

$$\Omega_\theta^2 = \beta(\theta q(\theta))' D_2\left[\phi'(e_s)\theta q(\theta)\ln\left(\frac{\rho+b}{\rho}\right)-1\right]+\phi'(e_s)\theta q(\theta)\left[B_2\ln\left(\frac{\rho+b}{\rho}\right)+\beta C_2\right]$$

where $D_2 = \left[\beta\phi(e_s)-(1+\lambda\beta)\phi(e_l)\theta q(\theta)\right]$

A sufficient condition for Ω_θ^2 to be positive is

$$\frac{\beta}{(1+\beta\lambda)}\phi(e_s)<\phi(e_l) \qquad \text{(A.3)}$$

By using the implicit function theorem and assuming that condition (A.3) is verified we have

$$\frac{\partial e_s}{\partial b} = -\frac{\Omega_b^2}{\Omega_{e_s}^2}>0 \text{ and } \frac{\partial e_s}{\partial \theta} = -\frac{\Omega_\theta^2}{\Omega_{e_s}^2}>0$$

By substituting expression (19) for $\left(J^E - J_l^U\right)$ in condition (18) we obtain for optimal e_l

$$\Omega^{2'} = -B_2 + \phi'(e_1)\theta q(\theta)\beta C_2 = 0$$

We have

$$\Omega_{e_l}^{2'} = \phi''(e_l)\theta q(\theta)\beta C_2 < 0$$

$$\Omega_b^{2'} = \phi'(e_l)\theta q(\theta)\left(\frac{\rho}{\rho+b}\right)\beta^2\lambda\phi(e_s)\theta q(\theta) > 0$$

$$\Omega_\theta^{2'} = \beta(\theta q(\theta))' D_2 + \phi'(e_l)\theta q(\theta)\left[B_2 \ln\left(\frac{\rho+b}{\rho}\right) + \beta C_2\right]$$

(A.3) is a sufficient condition for $\Omega_\theta^{2'}$ to be positive is

By using the implicit function theorem and assuming that condition (A.3) is satisfied we have

$$\frac{\partial e_l}{\partial b} = -\frac{\Omega_b^{2'}}{\Omega_{el}^{2'}} > 0 \text{ and } \frac{\partial e_l}{\partial \theta} = -\frac{\Omega_\theta^{2'}}{\Omega_{e_l}^{2'}} > 0$$

B. Uniqueness of equilibrium

Consider the simplest case in which UI is constant over time, that is, in our theoretical context, $\rho_s = \rho_l = \rho$ and $b = 0$. Thus equilibrium condition for tightness and search effort can be expressed respectively as

$$\Psi^1 = \frac{y - \left(1 + \rho\lambda \frac{1}{\phi(e)\theta q(\theta)}\right)\max\left\{\alpha * y, (1-\beta)J_l^U\right\}}{1 - \beta(1-\lambda)} - \frac{\gamma}{\beta q(\theta)} = 0 \qquad \text{(B.1)}$$

$$\Psi^2 = -1 + \phi'(e)\theta q(\theta)\beta\left(J^E - J_l^U\right) = 0 \qquad \text{(B.2)}$$

where

$$J_l^U = \ln(w) + \frac{(1-\beta(1-\lambda))(\ln(\rho)-e)}{1-\beta[1-\lambda-\phi(e)\theta q(\theta)]}$$

and

$$\left(J^E - J_l^U\right) = \frac{e-\ln(\rho)}{1-\beta[1-\lambda-\phi(e)\theta q(\theta)]}$$

and *w* is given by equation (23). We can then show that $\frac{\partial J_l^U}{\partial e} = 0$. Indeed, we have

$$\frac{\partial J_l^U}{\partial e} = \frac{-(1-\beta(1-\lambda))}{1-\beta[1-\lambda-\phi(e)\theta q(\theta)]}\left[1+\frac{\phi'(e)\theta q(\theta)\beta(\ln(\rho)-e)}{1-\beta[1-\lambda-\phi(e)\theta q(\theta)]}\right] = 0$$

As from equation (B.2) we have

$$\frac{\phi'(e)\theta q(\theta)\beta(\ln(\rho)-e)}{1-\beta[1-\lambda-\phi(e)\theta q(\theta)]} = -1$$

Thus, condition (B.1) depends on *e* only through $\left(1+\rho\lambda\frac{1}{\phi(e)\theta q(\theta)}\right)$ which is the definition of the equilibrium tax rate. If the latter were kept constant then $\frac{\partial\theta}{\partial e}$ would be nil.

θ

Following appendix A it can be shown that $\frac{\partial e}{\partial\theta} > 0$. As to $\frac{\partial\theta}{\partial e}$ obtained from Ψ^1, it is nil for constant τ as argued earlier. Thus in the *(e,θ)* space, the *e(θ)* curve would be everywhere steeper than the *θ(e)* curve, as the latter displays a zero slope.

For endogenous τ, we can show that $\frac{\partial\theta}{\partial e}$ becomes positive. However, for reasonable parameters values, it is only slightly different from zero. A rise in e leads to lower τ and as a consequence, to a higher match probability. Then tightness is expected to increase. Despite the fact that higher θ would also contribute to lower τ and thus to higher profitability, the expected cost of a vacant position increases in an increasing manner, meaning that only a small variation in θ would offset the rise in match profitability. Again the $e(\theta)$ curve would be everywhere steeper than the $\theta(e)$ curve in the (e,θ) space.

The same argument hold when UI decreases over time. Search effort is identical for both types of searchers as shown in appendix A and, the expression τ is a monotonic transformation of the above expression.

In the search bonus scheme, by using similar arguments, it is possible to find a unique equilibrium combination of θ and e_s, and θ and e_l.. However, θ needs to be the same for both combinations in order to assert uniqueness of equilibrium. Equations (17) and (18) give an expression for $\frac{\phi'(e_s)}{\phi'(e_l)}$.

Then, by using the envelop theorem it can be shown that $\frac{\partial\frac{\phi'(e_s)}{\phi'(e_l)}}{\partial\theta}=0$ and thus that $\frac{\partial\frac{e_s}{e_l}}{\partial\theta}=0$. The condition for jobs opening can be used to determine the sign of $\frac{\partial\theta}{\partial\frac{e_s}{e_l}}$ in order to establish a second curve in the $\left(\frac{e_s}{e_l},\theta\right)$ space. As long as the slope of $\theta\left(\frac{e_s}{e_l}\right)$ remains finite in the positive quadrant, we obtain a unique θ for any given $\frac{e_s}{e_l}$.