Search Bonuses and Hiring Subsidies: A Theoretical Comparative Analysis

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Abstract

This paper investigates the relative performance of employment subsidy schemes in tackling long term unemployment. Should the government intervene on the supply side or the demand side of the labor market? Is targeted intervention desirable and, in this particular case, what should be the target group?

Two policy instruments - a hiring subsidy and a search bonus- and three unemployed workers target groups -all the unemployed, the short term and the long term unemployed- are considered. Equilibrium properties of the policy schemes are assessed in a search and matching model where both search effort and wages are endogenous. Analytical and computational investigations reveal that a search bonus paid to short term unemployed only produces the best outcome in terms of labor force composition and welfare.

JEL classification: J 64

Keywords: Long term unemployment; Search bonuses; Hiring subsidies

1 Introduction

In Europe, unemployment has become essentially a problem of long-term unemployment. In the European Union countries, almost half of the unemployed have been unemployed for a year or more. Long-term unemployment has negative effects on the individuals who suffer it (e.g. scarce labor market opportunities, physical and mental distress), and subsequently on the society as a whole. The fight against long term unemployment has been put at the top of the policy agenda. Although “re-integrative” strategies still prevail in most of the existing policy schemes, there is a growing interest in policy strategies oriented towards prevention of long-term unemployment essentially in order to prevent the stigmatic effects of long unemployment spells on job seekers from appearing.

Machin and Manning (1998) observe that the rise in incidence of long term unemployment has been the consequence of the collapse of outflow rates at all durations of unemployment. They stress the fact that preventing unemployed workers from entering long-term unemployment is likely to be more desirable than improving the job perspective of long-term unemployed. “Re-integrative” strategies may reduce the exogenous burden inherent in long unemployment spells only slightly, unless they have a strong positive impact on the outflow rates of shorter term unemployed. Rather, most of the supply-side-oriented policy approaches, like the so called unemployed “activation” approaches, are in line with the above argument. Measures aiming to activate the unemployed often operate by making unemployment benefit conditional on job-search and acceptance criteria, or by time-limiting the receipt of unemployment benefit. Such approaches usually aim at reducing the attractiveness of remaining on benefit. Other measures, which can be interpreted to be either an alternative or
complimentary, rather attempt to increase the attractiveness of taking up a job, like “making-work-pay”\(^1\) and “welfare-to-work”\(^2\) measures.

The scope of this paper is twofold. First it is to assess the performance of some demand-side-oriented relative to supply-side-oriented policy measures. In other words, it is to determine whether intervening on one side of the labor market rather than on the other side, could lead to different general equilibrium effects. Second, controlling for the side of the labor market chosen for policy implementation, it is to assess the importance of the timing of intervention. In particular, the paper attempts to assess the relative performance of some general (universal) “re-integrative” and, “preventive” measures.

More precisely, we consider two policy instruments and three unemployed workers target groups. The first policy instrument, the demand-side-oriented instrument, is a hiring subsidy paid to firms that recruit a targeted unemployed worker. The second policy instrument is a search bonus, the supply-side-oriented instrument, paid to unemployed workers of the targeted group who return to employment. The three target groups are respectively, all the unemployed, the short term unemployed and, the long term unemployed\(^3\). Among these targeted policy instruments two are of particular relevance: the hiring subsidy paid to firms for the recruitment of a long term unemployed and the search bonus paid to the newly unemployed who return to employment quickly. These two specific instruments have been extensively assessed whether theoretically or empirically.

Subsidizing the hiring of the long-term unemployed has been contemplated as a possibly ecient means to increase the outflow from long-term unemployment. However, such schemes have always attracted scepticism on the grounds that they could have high levels of deadweight and substitution: some hiring within the subsidy scheme would have occurred in any case or occurs at the expense of some uncovered unemployed workers. Scepticism about hiring subsidies, is strongly supported by many micro level empirical evaluation studies, like the well-documented studies by Fay (1996); and by Meager and Evans (1998). In that empirical context, most of the demand-side-oriented labor market policies launched in European countries, and particularly subsidies for employers to recruit the long-term unemployed, were found to have high deadweight and significant substitution effects. Nevertheless, at a macro-economic level, there is some evidence of a reduction in both the level and the share of long term unemployment\(^4\). From a theoretical point of view, substitution effects have been minimized by various economists, notably Layard (1997). However, hiring subsidy schemes are generally found to have ambiguous equilibrium properties, in particular in the context of job search and matching theory. In Richardson (1997), a lump-sum hiring subsidy paid to firms hiring a long term unemployed person affects both the long term, and the short term unemployed outflow rates positively and thus reduces equilibrium unemployment, as long as wage pressure remains low. In this circumstance, no substitution effect appears. In Millard and Mortensen (1997) and, Mortensen and Pissarides (1998), ambiguity is also embodied in the rise in the separation rate, which was assumed to be constant in Richardson’s work. Their numerical results indicate a negative performance for hiring subsidies. They found to increase unemployment incidence (the unemployment inflow rate) more than unemployment hazard (the unemployment outflow rate), leading to a rise in unemployment and suggesting the occurrence of strong substitution effects. Nevertheless, the latter result relies on the fact that “subsidy-farming” can take place. If some eligibility restrictions were introduced to prevent such a behavior, then unemployment incidence might be reduced and the overall performance of hiring subsidies may be improved.

Search bonuses, when they take the form of a cash payment to short term unemployed able to find a job rapidly, adhere to the spirit of unemployment “activation” policy approach in the sense that,\(1\)

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1 As presented in Bertola (1999), making-work-pay policies consist of wage subsidies and in-work benefits for low-pay employment relationships. They can be regarded as preventive policies against long term unemployment as they are directed mainly to the low-skilled workers, who are usually identified as the most “at-risk” group.

2 As described in Boeri and alii (2000), welfare-to-work measures include measures aiming to activate the unemployed, to tackle information deficits and labour market “friction”, to offer job-search training and to guarantee some vocational training and/or active implication in socially-useful or NGO projects.

3 Fugazza (2000) compares the search bonus scheme where only short term unemployed are eligible, with a sequence of unemployment benefits that declines over time. Both measures stimulate unemployed search effort and reduce total unemployment and the incidence of long term unemployment. However, because the insurance dimension of unemployment benefits matters, workers and specially, the long term unemployed are always better off in the search bonus scheme.

4 See Layard and alii (1991) and OECD (1993) for a comprehensive presentation of available results.
as they effectively represent an incentive payoff, they push unemployed workers back to employment more quickly. Such a result is obtained by Mortensen (1987) in a search-model of labor supply with an exogenously given distribution of wage offers. Kim (1992) comes to the same conclusions within the paradigm of labor contract theory. In that context, he concludes that an appropriate provision of reemployment bonus enhances the likelihood of self-compliance, and in turn reduces the costs of monitoring unemployed workers’ search effort. Moreover, Search bonuses are not “new” as a policy instrument. They have been “tested” extensively in various experiments in the United States between 1984 and 1989. These experiments have been implemented in order to define possible directions for unemployment insurance (UI, thereafter) system reforms. The proposed reforms, cash bonuses included, have sought to curb unemployment and reduce the budgetary costs of the UI system. Two major conclusions can be drawn from these experiments: search bonuses effectively reduce unemployment duration spells of the bonus claimants, but are not cost effective from the perspective of the UI system. This has been interpreted to call for more narrowly targeted reemployment bonuses by the way of a worker profiling mechanism based on objective characteristics. By doing so, when serving exclusively dislocated or low-skilled workers, search bonuses may also be included in the set of “making-work-pay” policy instruments. Nevertheless, this paper shows that other positive features of search bonuses, not captured by empirical estimates, could still make their extended use be socially desirable, even if cost-ineffectiveness may be observed.

As pointed out in Meyer (1995); like in case of hiring subsidies for the long-term unemployed, another important concern surrounding the reemployment bonus scheme is that it may have high deadweight and substitution effects. The deadweight effect refers to the fact that people knowing with certainty that they will be re-employed within a given period of time, may be tempted to declare themselves as being unemployed in order to cash in the bonus. The argument is quite convincing, however it is valid for any kind of financial support during unemployment. Although such deadweight effects are not precisely assessable, it is realistic to think that they keep reasonable dimensions. As a matter of fact, to be eligible for UI, that is for the search bonus program, people must not have been reduced by their employers for reasons related to their productive behavior. It is hence plausible to believe that, in most cases, people involuntarily entering unemployment are “true job seekers”. Those who know that they will be re-employed with certainty either switch from job to job or quit their present position on a voluntary basis, renouncing de facto UI. Indeed, the occurrence of an agreement between a firm and an employee to declare the latter laid off rather than having left willingly, in order to allow the worker to cash in unemployment benefits, may only happen in very particular and likely rare cases. Such an event is even more improbable in the presence of financial support during unemployment. Although such deadweight effects are not precisely assessable, it is realistic to think that they keep reasonable dimensions. As a matter of fact, to be eligible for UI, that is for the search bonus program, people must not have been reduced by their employers for reasons related to their productive behavior. It is hence plausible to believe that, in most cases, people involuntarily entering unemployment are “true job seekers”. 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The second side effect of cash bonuses, namely the substitution effect, refers to the fact that by taking jobs more quickly, those eligible for a bonus automatically reduce the number of vacant positions available to other unemployed workers. Again, this might be true. However, like in the case of hiring subsidies, it is hard, whether theoretically or empirically, to identify a clear-cut net effect. Davidson and Woodbury (1993) calibrate two partial equilibrium matching models, one with exogenous and the other with endogenous wages, in order to evaluate whether search bonuses displace workers not covered by the bonus program. Their computational exercises show that the substitution effect is countered by two offsetting effects, namely a gross job creation effect and a relatively small rivalry effect, suggesting that, on net, the substitution effect is small to nonexistent.

In order to assess the relative performance of the three profiles of search bonuses and hiring subsidies presented previously, this paper develops a discrete search and matching model where both wage and unemployed workers search effort are endogenous. Unemployed workers become long-term unemployed after one period of unemployment. Long-term unemployment is assumed to lead to attrition of the skills of the long-term unemployed person. As a consequence, when hiring a long-term unemployed person, firms must bear the cost of initial retraining. As in Richardson (1997), we assume that training costs are stochastic and match specific, that is, their level is revealed once the contact
between a long-term unemployed and a rm has occurred. Such a cost is considered to be either a proper re-training cost or a measure of employers’ discrimination against the long-term unemployed. As argued by Meager and Evans (1998), there is indeed persuasive evidence that employers often regard long-term unemployed as having less favorable characteristics than otherwise similar job applicants. Then, if the training cost proves to be too high, it becomes optimal for a rm to reject the worker and continue searching for a short-term unemployed worker, fully productive at no cost, or a long-term unemployed worker with lower training cost. Such an assumption introduces some state-dependence, the outflow rate from unemployment for long-term unemployed workers being lower than for short-term ones. Moreover, with endogenous search effort, a “discouraged worker effect” shows up, and amplifies the state-dependence phenomenon.

As for the policy instruments under study, we define hiring subsidies as a lump sum cash payment to rns hiring a targeted unemployed individual. Search bonuses take the form of a lump sum cash payment to unemployed workers who belong to the target group. When short term unemployed are the target group, the cash bonus is a cash transfer made to those unemployed who return to employment within their rst period of unemployment.

The main points of this works are the following. First, whether policy is applied to the demand side or to the supply side of the labor market matters as long as wage determination is not able to internalize fully the policy instrument. Second, whatever the wage bargaining rule is, prevention oriented measures better perform than universal and reintegration oriented schemes.

We consider three types of wage determination rules: exogenously determined wages, collective Nash bargaining and decentralized Nash bargaining. The latter is, somewhat by definition, the only determination rule among these three that does not introduce any “labor-market-side bias”. Analysis with exogenously determined wages is used essentially to enable us to identify the respective “transmission” mechanisms of the various policy strategies. Collective Nash bargaining is chosen because it is quite representative of European experiences.

In the context of exogenous wages, a hiring subsidy paid for any recruitment and a hiring subsidy paid only for the recruitment of a long term unemployed both generate search disincentives for the short term unemployed. In the rst place, hiring subsidies reduce long-term unemployed rejection by rns and stimulate overall job creation. They are thus accompanied by a positive “gross job creation effect”. As a matter of fact, as hiring a (long-term) unemployed person becomes on average less costly, extra resources are made available for new vacant positions to be opened. Moreover, the more the probability that a (long term) unemployed worker will be rejected decreases, the more intensively he is pushed to search. For the same reason however, short-term unemployed willingness to search for a job may be reduced. As a matter of fact, a decrease in the inherent disutility of being long-term unemployed may arise. When perceiving the higher probability of returning to employment when becoming long-term unemployed, short-term unemployed workers may tend to decrease their level of search effort. In sum, in such a context of even indirect substitution effects, the impact of hiring subsidies on aggregate unemployment can not be properly signed. The latter result holds also when the search bonus is paid only to long-term unemployed workers. Short-term unemployed would delay their search to become eligible for the bonus. As to search bonuses paid to the short term unemployed, they stimulate both short-term and long-term unemployed in their job search. For both types of unemployed workers, this stimulation in the rst place corresponds to an “entitlement effect”. Because long-term unemployed workers discount more heavily the payment of the bonus, such an entitlement effect is much stronger for short-term unemployed. In the case of a universal search bonus, this difference persists but it becomes less important. A hiring subsidy paid for the recruitment of short term unemployed does not induce any search disincentive and is expected to have an impact qualitatively similar to search bonuses paid to short term unemployed. The difference comes from the crossed reaction of job creation and search incentives.

When wages are made endogenous, analytical investigations do not lead to clear cut results, specially because in the rst place wages react to the instruments in opposite directions. In order to throw some light on the consequent ambiguities and eventually introduce welfare considerations, a calibration of the model is undertaken. The policy schemes are evaluated in a labor market expenditures activation perspective. Except for search bonuses paid to long term unemployed all policy...
schemes are found to curb unemployment, to reduce the proportion of long term unemployment in total unemployment and, to generate aggregate welfare improvements. Among these schemes, search bonuses paid to short term unemployed display the best performance in all respect.

This paper is organized as follows. The next section presents the base-line framework and describes search bonuses and hiring subsidies schemes. Their equilibrium impacts are analyzed for exogenous wage and taxation in section 3. Wage determination issues are considered in section 4. A broad discussion based on analytical insights into partial equilibrium comparative statics is presented in section 5. In section 6, both wages and taxation are considered to be endogenous in a calibrated version of the model. An aggregate welfare index is also presented and computational results are discussed. Concluding remarks are contained in section 7.

2 The Base-Line Framework

2.1 Basic Concepts and Labor Flows

We use a discrete time search and matching framework à la Pissarides (1990) with variable search intensity. There is a fixed labor force normalized to unity. Individuals have infinite horizons and perfect foresights. At any time, a worker is either employed, or short-term unemployed, or long-term unemployed. Long-term unemployed are unemployed workers whose unemployment spell is longer than one period.

A representative competitive firm is also in the economy. It produces with constant returns to scale and is composed of drifted and vacant jobs.

Job seekers and the representative firm are brought together by a matching function

$$M(v; \hat{A}(e_s) u_s + \hat{A}(e_l) u_l)$$

where $u_s$ and $u_l$ denote the proportion of short-term and long-term unemployed respectively, and $e_s$ and $e_l$ their respective search effort intensity. $\hat{A}(\cdot)$ is an increasing and concave function of search effort. The matching function has the standard properties: increasing in both its arguments and constant returns to scale. Contacts are random and any vacancy is contacted by no more than one worker per period.

Let $\mu = \frac{\hat{A}(e_s) u_s + \hat{A}(e_l) u_l}{v}$ denote the labor market tightness. Then we can define the probability that any vacancy is contacted by an unemployed worker as

$$q(\mu) = \frac{M(v; \hat{A}(e_s) u_s + \hat{A}(e_l) u_l)}{v}$$

Since matching is random and the firm is contacted by at most one worker per period, $\frac{\hat{A}(e_s) u_s}{\hat{A}(e_s) u_s + \hat{A}(e_l) u_l} q(\mu)$ and $\frac{\hat{A}(e_l) u_l}{\hat{A}(e_s) u_s + \hat{A}(e_l) u_l} q(\mu)$ are respectively the probabilities that any given vacancy is contacted by a short- and long-term unemployed worker per period.

For an unemployed worker of type $i$, $i = s; l$; the probability of contacting a vacancy is defined as

$$p_i(\mu e_i) = \hat{A}(e_i) \frac{M(v; \hat{A}(e_s) u_s + \hat{A}(e_l) u_l)}{\hat{A}(e_s) u_s + \hat{A}(e_l) u_l} = \hat{A}(e_i) \mu q(\mu)$$

From the properties of the matching function we have that $q(\mu)$ and $p(\mu)$ are respectively decreasing and increasing with respect to $\mu$

Note that $p_s(\mu e_s)$ is also the short-term unemployment outflow rate. However, a match between a firm and a long-term unemployed, once contact has occurred, may fail to become productive. I assume that some cost $\frac{1}{4}$ is attached to the hiring of a long-term unemployed worker. This cost can be interpreted as either explicit or implicit. It could be the consequence of some skills loss incurred by long-term unemployed worker and would thus correspond to some on-the-job re-training cost. Or, it could be seen as a measure of employers’ discrimination against the long-term unemployed8. $\frac{1}{4}$

8See Meager and Metcalf (1988) and Atkinson and ali (1996) findings from employer survey evidence in the UK.
Figure 1: Labor Market Flows

is randomly drawn from a known distribution \( G(\frac{1}{4}) \) and is match specific. As we will see in the following sections of the paper, production is undertaken only for \( \frac{1}{4} \) values falling below a threshold \( \frac{1}{4} \): As all long-term unemployed workers and firms are ex ante identical, the reservation cost, \( \frac{1}{4} \); is common to all pairs of jobs and workers coming from long-term unemployment. Hence, the long-term unemployment outflow probability is given by \( G(\frac{1}{4}) \) \( \tilde{A}(\epsilon) \mu q(\mu) \). Then, the existence of costs inherent to long-term unemployed hiring is likely to generate state-dependence, which translates, though roughly, into a negative duration dependence of unemployment outflow. \( G(\frac{1}{4}) \) is assumed to be identical for any match with a long-term unemployed worker. Independently of the length of his unemployment spell length the degree of duration dependence remains identical within long-term unemployment. Nevertheless, various empirical studies have underlined the difficulty in disentangling true duration dependence from unobserved individual heterogeneity, playing down the need for a more sophisticated cost specification.

A match is destroyed at an exogenous probability \( \lambda \) at the end of each period. Then the job would become unproductive and the worker short-term unemployed.

Figure 1 illustrates flows in the labor market.

We consider stationary equilibria in which unemployment is constant. Hence, at steady state, flows into and out of unemployment must be equal, that is

\[
\lambda (1 - u) = \tilde{A}(\epsilon) \mu q(\mu) u_s + G(\frac{1}{4}) \tilde{A}(\epsilon) \mu q(\mu) u_l
\]

and

\[
u_s = \lambda (1 - u)
\]

where \( (1 - u) \) stands for the aggregate level of employment and reads as

\[
(1 - u) = 1 - u_s - u_l
\]

Since short-term unemployment lasts only one period, the existing stock must always be “renewed” by the end of the period. The stock of short-term unemployment is thus given by the inflow into unemployment.

Combining equation (1) with equations (2) and (3), we obtain

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9 See Machin and Manning (1998) for a general discussion.
\[
\frac{u_s}{u} = \frac{\hat{A}(e) G(\frac{1}{4}) \mu q(\mu)}{1 - \frac{\hat{A}(e) \mu q(\mu)}{A(e) G(\frac{1}{4}) \mu q(\mu)}}
\]  

From appendix A, it can be easily shown, that the proportion of short-term unemployment varies positively with all endogenous variables. This result is mainly the consequence of the assumptions that short-term unemployment lasts one period only and job destruction is exogenous and constant.

The equations of the model are written in a static form as we limit ourselves to a comparison of steady states.

2.2 Firm Behavior

The representative competitive firm has many positions either vacant or occupied and productive. We assume that each position is treated individually. Hence, the job remains the basic unit of analysis. Vacant jobs incur a per period cost of \( v \), whilst filled positions produce \( y \) units of output. Production occurs at the end of each period.

Let \( J^v \) be expected present discounted value of a vacant position, \( J^f \) be the present discounted value of a job filled with a worker coming from short-term unemployment and \( J^{s,f} \) be the expected present discounted value of a job filled with a worker coming from long-term unemployment. The expression for \( J^v \) is given by

\[
J^v = e^{-r} \frac{\hat{A}(e)}{\bar{A}(e) \mu q(\mu)} + A \cdot 
\]

where \( r \) is the discount rate. \( A = \frac{\hat{A}(e) u_s}{\bar{A}(e) u_s + \hat{A}(e) u_l} q(\mu) G(\frac{1}{4}) \frac{\hat{A}(e) u_s}{\bar{A}(e) u_s + \hat{A}(e) u_l} q(\mu) \); the probability that a position remains vacant is composed by the probability that the position is not contacted by a short-term unemployed and, the probability that if contacted by a long-term unemployed, the match does not form. \( J^f \) represents the ex ante expected value of a match such that the cost associated with hiring a long-term unemployed worker is low enough that the worker is retained. In general, once contact has been made between a vacancy and a job seeker, both parties have to decide whether or not to accept the match. Workers accept the match as long as the expected discounted value they attach to being employed is at least as great as the expected discounted value of remaining unemployed. In a non-trivial equilibrium this is always the case for short-term unemployed persons. As for firms, if they refuse to match, they get \( J^v \), which is zero in equilibrium, because we assume free entry of vacancies. If a firm is contacted by a short-term unemployed worker then the return for forming the match is given by \( J^{s,f} \). Again, in a non-trivial equilibrium \( J^{s,f} \) is positive and firms always accept short-term unemployed workers. However, if a firm is contacted by a long-term unemployed worker, with a realized hiring cost of \( \frac{1}{4} \), then it will accept to match only if the subsequent return is at least equal to zero, that is if

\[
J^{s,f}(\frac{1}{4}) \leq 0
\]

As all firms are identical and all workers have the same intrinsic characteristics, all firms share a common reservation value for hiring costs which solves

\[
J^{s,f}(\frac{1}{4}) \leq \frac{1}{4} = 0
\]  

A combination of condition (6), the free entry condition \( J^v = 0 \) with equation (5) will provide us with a firm's vacancy opening condition.

It is assumed that hiring cost \( \frac{1}{4} \) is incurred during the first period only. It is further assumed that production occurs at the end of each period and that the productivity of a match is the same for all types of workers. In other words, long-term unemployed workers are made fully productive
since their rst period in employment by the payment of cost \( \frac{1}{2} \). Hence, any filled position generates \( y \) units of output per period. Firms must pay a wage \( w_s \) to their workers coming from short-term unemployment and a wage \( w_l \) to their workers coming from long-term unemployment. We leave the discussion on wage determination for the next section. However, the wage paid to workers who have entered employment from short-term unemployment differs from the wage paid to workers who have entered employment from long-term unemployment if cost \( \frac{1}{2} \) or any other element, like a policy instrument, specific to unemployment status, is taken explicitly into account in the labor contract and thus in the wage determination. Firms must also pay a proportional wage tax \( \frac{1}{2} \). We have seen that separation between a worker and a rm occurs with an exogenous probability \( \frac{1}{2} \) at the end of each period. In the light of existing works, e.g. Mortensen and Pissarides (1998), it might be argued that making job destruction endogenous is necessary for a proper assessment of hiring and search subsidies’ performance mainly because both could have a substantial impact on separations. As already mentioned, this eect would be largely contained as soon as “subsidy-farming” is prevented. In the Mortensen-Pissarides model, this would become possible if wage renegotiation, rather than being automatic with the occurrence of a match productivity shock, was made possible by mutual agreement only, like in Cahuc and Zylberberg (1999). Endogenous separation, though desirable, would not change drastically our results from a qualitative point of view.

Hence, \( J_s^E \) and \( J_l^E \) respectively read as

\[
J_s^E = y \left( 1 + \frac{1}{2} \right) w_s + -i \frac{1}{2} \cdot J_V + (1 - i) \cdot J_s^E \tag{7}
\]

\[
J_l^E = y \left( 1 + \frac{1}{2} \right) w_l + -i \frac{1}{2} \cdot J_V + (1 - i) \cdot J_l^E \tag{8}
\]

By substituting condition (6) into (8) we will obtain a reservation hiring cost condition.

### 2.3 Worker Behavior

Workers have no access to a capital market, so, as the good produced is assumed to be non-storable, individuals consume all their income in each period. They have identical preferences expressed by the utility function: \( u(x; e) = x - e \), where \( x \) stands for income and \( e \) for the search efort. As no search on the job takes place, the search efort of an employed worker is nil. We further assume that consumption occurs at the end of each period. Hence \( u(x; e) \) expresses expected utility at the beginning of the period.

Let \( J_s^E \) and \( J_l^E \) denote the respective present discounted value of being employed when entering employment from short-term unemployment and when entering employment from long-term unemployment. Let \( J_s^U \) and \( J_l^U \) be the present discounted value of being respectively short-term unemployed and long-term unemployed. In equilibrium, \( J_s^E \), \( J_l^E \), \( J_s^U \) and \( J_l^U \) satisfy

\[
J_s^E = w_s + -i \cdot J_s^U + (1 - i) \cdot J_s^E \tag{9}
\]

\[
J_l^E = w_l + -i \cdot J_l^U + (1 - i) \cdot J_l^E \tag{10}
\]

\[
J_s^U = z \cdot e_s + -\frac{1}{2} A(e_s) \cdot \mu(\mu) \cdot J_s^E + (1 - i) \cdot A(e_s) \cdot \mu(\mu) \cdot J_l^U \tag{11}
\]

\[
J_l^U = z \cdot e_l + -\frac{1}{2} A(e_l) \cdot G(\frac{1}{2}) \cdot \mu(\mu) \cdot J_s^E + (1 - i) \cdot A(e_l) \cdot G(\frac{1}{2}) \cdot \mu(\mu) \cdot J_l^U \tag{12}
\]

Employed workers are separated from their jobs and enter short-term unemployment at the exogenous rate \( \frac{1}{2} \). \( z \) is workers income from unemployment and is interpreted to be unemployment benefts.
paid by the government. It has been argued that short-term unemployed workers always accept a job offer in a non-trivial equilibrium. Indeed, we always assume that $w_s > z + e$. However, as part of hiring costs can be passed on to long-term unemployed workers, the latter might reject the job offer if $w_l$ falls below some reservation value $w_l^*$ defined as the wage value which solve $w_l^* = z + e$. As it is shown below, $w_l^*$ corresponds, in equilibrium, to the value of $\frac{1}{2}$ which satisfies condition (6). In other words, firms and workers share the same reservation hiring costs $\frac{1}{2}$ in equilibrium.

Moreover, each type of unemployed worker chooses a search intensity in order to maximize the value of unemployment.

Equations (11), (12) give the following first order conditions for optimal search efforts $e_s$, $e_l$ respectively:

$$i + A_s(e_s) \mu_l(\mu) - \int \frac{1}{j} j^U \mathbf{C} = 0$$

$$i + A_l(e_l) G(\frac{1}{2}) \mu_l(\mu) - \int \frac{1}{j} j^U \mathbf{C} = 0$$

Equations (13), (14) imply that, at steady state, $e < e_s$ as $G(\frac{1}{2}) < 1$. As the probability of contacting a firm willing to hire them decreases, unemployed workers when entering long term unemployment search less intensively. This is the “discouraged-worker effect”.

2.4 Policy Instruments

2.4.1 Definitions and Motivation

The two policy instruments under study are a search bonus and a hiring subsidy. Both of them are introduced in a targeted form. Three target groups are considered: all the unemployed, the short-term unemployed and the long-term unemployed workers.

The search bonus takes the form of a cash payment made to eligible unemployed workers who succeed in finding a job and accept it. For example, if short-term unemployed represent the scheme eligible, then they would be paid the bonus if they are able to find a job within their first period of unemployment. We assume that the bonus, thereafter denoted by $b$, is paid to and consumed by the beneficiary at the end of the last period of unemployment, after she met with the firm.

The hiring subsidy corresponds to a lump sum payment made to firms hiring a worker coming from the unemployed target group. For example if the target group includes only long-term unemployed workers, the payment is made by the government once the firm and a long-term unemployed has decided to match. Thereafter, the hiring subsidy is denoted by $\lambda$.

As mentioned in the introductory part of the paper the motivation of these exercises is twofold: it is first to determine to what extent the side of the labor market considered for policy intervention matters and, second it is to identify the unemployed cohort that should be targeted in order to generate the best results in terms of long-term unemployment containment and welfare. In other words we try to answer two fundamental questions. Would unemployed workers react more strongly to policy transfers than firms or vice versa? In order to curb substantially long-term unemployment which is assumed to generate a cost, and eventually to reduce welfare, should we prevent unemployed workers from entering long-term unemployment or should we try to increase the exit rate of long term unemployed instead? While the answer to the latter intuitively makes little doubt, the answer to the former is not a priori clear cut. Before turning to these various points in the next sections we briefly introduce budgetary constraints even though they will be fully taken into consideration only in the calibration exercises.

2.4.2 The Government Budget Constraint

We assumed that a payroll tax, proportional to current wages, is imposed on all productive firms. It is further assumed that the whole expenses of the government are devoted to unemployment benefits and policy instruments financing. Let $p_s = A_s(e_s) \mu_l(\mu)$ and $p_l = A_l(e_l) \mu_l(\mu)$. Accordingly, the government budget constraint reads as
\[ n_s w_s + n_l w_l = uz + (p_u u_s + G \left( \frac{1}{4} \right) p_u u_l) b \]

in the general search bonus scheme,

\[ n_s w_s + n_l w_l = uz + p_u u_b b \]

in the search bonus scheme with short term unemployed as the eligible group, and

\[ n_s w_s + n_l w_l = uz + G \left( \frac{1}{4} \right) p_u u_b b \]

in the search bonus scheme with long term unemployed as the eligible group.

\[ n_s \text{ and } n_l \text{ represent employed workers coming form short and long-term unemployment respectively. We verify that } n_s + n_l = 1. \]

\[ p_u u_s \text{ corresponds to the number of workers leaving short-term unemployment for employment, } G \left( \frac{1}{4} \right) p_u u_l \text{ is the number of unemployed workers returning to employment.} \]

The government budget constraint is given by

\[ n_s w_s + n_l w_l = uz + \left( G \left( \frac{1}{4} \right) p_u u_s + p_u u_b \right) \bar{A} \]

in the general hiring subsidy scheme

\[ n_s w_s + n_l w_l = uz + p_u u_b \bar{A} \]

in the hiring subsidy scheme with short term unemployed as the eligible group, and

\[ n_s w_s + n_l w_l = uz + G \left( \frac{1}{4} \right) p_u u_b \bar{A} \]

in the hiring subsidy scheme with long term unemployed as the eligible group.

### 3 Wage Determination

The wage determination rule is expected to play a central role in the relative performance of policy schemes. This section is dedicated to the corroboration of such an argument.

In the context of a decentralized bargain à la Nash where wealth is transferable between agents, whether it is the supply or demand side of the labor market that is subsidized, does not generate any difference in the general equilibrium properties of the schemes. Wages fully integrate the policy instrument and the respective responses of the agents. For example, a search bonus paid to short-term unemployed and a hiring subsidy paid to the firm that recruit a worker coming from short term unemployment, both generate identical general equilibrium effects. Indeed, if we take the surplus of a match denoted by \( S \), we have

\[ S = \int_{E}^{v_{i}} E_{i} + \int_{S}^{v_{i}} F_{i} + \int_{U}^{v_{i}} U_{i} + \text{subsidy} \]

Whether the subsidy is paid to the firm or to the worker does not affect the very value of the match surplus. Of course, equilibrium wages would react differently in the two schemes but the overall effects would be identical\(^{10}\). Different outcomes would be observed only if different target groups are considered. This point is discussed in the last section of the paper.

\(^{10}\) Appendix D presents a special case of individual labor contract which can be easily extended to a more general case.
However, when this “full-adjustment” mechanism is removed from the analysis, not only the target group but also the side of the labor market that receives the subsidy are likely to affect the general equilibrium properties of policy intervention.

The next sub-sections assess the performance of the policy schemes in the presence of exogenously given wages and of wages bargained collectively. As shown below, the latter situation leads to a unique wage at equilibrium despite heterogeneity of matching outcomes. The analysis remains a qualitative one and does not allow for a precise quantitative comparison between hiring subsidies and search bonuses. As a consequence we first proceed with an assessment of hiring subsidies and search bonuses along the three configurations presented previously. We present crossed comparisons in the following two sections.

3.1 Exogenous Wages

In order to bring to light policy schemes equilibrium effects we first consider the case of a unique and exogenously determined wage. Moreover, in order to identify clearly fundamental transmission channels we also assume that the tax rate $\tau$ is kept constant.

With exogenous wage, firms are unable to pass on hiring costs to workers. In other words they bear the full burden of hiring a long-term unemployed person. Then, the present discounted value of a filled position is unique, that is,

$$J^F_s = J^F_l = J^F.$$

Thus equation (5) together with the free entry condition now reads as

$$0 = \frac{\dot{A}(e_s) u_s}{A(e_s) u_s + A(e_l) u_l} q(\mu) J^F + \frac{\dot{A}(e_l) u_l}{A(e_s) u_s + A(e_l) u_l} q(\mu) Z \int_0^{\frac{\mu}{\tau}} j F i \frac{\partial \phi}{\partial G} (\frac{\mu}{\tau}) \, dG (\frac{\mu}{\tau}) (15)$$

where $J^F$ is given by

$$J^F = \frac{\mu_1 + (1 + \xi) W}{\mu_1 (1 + \xi)}$$

The reservation cost condition (6) becomes

$$J^F \frac{\gamma \phi}{\gamma_0} = 0 \quad (17)$$

By substituting for $J^F$ from condition (17) into condition (15), the vacancy opening condition becomes

$$\frac{\dot{A}(e_s) u_s}{A(e_s) u_s + A(e_l) u_l} q(\mu) Z \int_0^{\frac{\mu}{\tau}} j F i \frac{\partial \phi}{\partial G} (\frac{\mu}{\tau}) \, dG (\frac{\mu}{\tau}) = 0 \quad (18)$$

By combining equation (16) with equation (6), we obtain the reservation training cost condition

$$\frac{\mu_1 + (1 + \xi) W}{\mu_1 (1 + \xi)} \frac{\gamma \phi}{\gamma_0} = 0 \quad (19)$$

With exogenous wage $J^F = J^F$ and, as long as equilibrium is non-trivial, both types of unemployed workers always accept a job offer. Conditions for optimal search are re-written as

$$1 + \dot{A}(e_s) \mu q(\mu) \mu_1 \frac{\partial \phi}{\partial G} (\frac{\mu}{\tau}) = 0 \quad (20)$$

$$1 + \dot{A}(e_l) G (\frac{\mu}{\tau}) \mu q(\mu) \mu_1 \frac{\partial \phi}{\partial G} (\frac{\mu}{\tau}) = 0 \quad (21)$$
The difference between the present discounted value of being employed and the present discounted value of being unemployed is given by

$$J^E_i \cdot J^U_i = \frac{w_i \cdot z_i \cdot e_i \cdot (1 + \gamma) \cdot e_i}{1 + (\gamma)} + G(\gamma) \cdot p_i \cdot (1 + \gamma) + p_i$$

where $p_i = \tilde{A}(e_i) \cdot u_i(\mu)$ and $p = \tilde{A}(e_i) \cdot u_i(\mu)$.

3.1.1 Hiring Subsidy

2 General Scheme

The hiring subsidy, denoted by $\tilde{A}$, takes the form of a lump sum payment to firms hiring a worker coming from both short-term and long-term unemployment. The reservation training cost now satisfies

$$J^E_i \cdot \gamma^e + \tilde{A} = 0$$

Thus, the new training cost condition reads as

$$\gamma^e = \frac{(1 + \tilde{A}) \cdot w_i}{1 + (1 + \gamma)} \cdot \gamma^e + \tilde{A} = 0$$

As effective labor costs $(1 + \tilde{A}) \cdot w$ are exogenously given by assumption, equation (22) can be solved for $\gamma^e$ independently of any other endogenous variables. It is straightforward to check that $\partial \gamma^e = 1$.

The opening vacancy condition is still given by (18). Tightness is not affected by the subsidy itself but it would vary in reaction to the change in $\gamma^e$. As shown in appendix B.2 $\partial \gamma^e > 0$. By reducing rejection of long-term unemployed, it increases the overall probability of opening a vacant position.

As for optimal unemployed workers search intensities, a similar observation can be formulated.

As for search efforts, $\tilde{A}$ does not appear in the conditions for optimal $e_i$ and $e_i$ which are still given by (20) and (21), respectively. Then, search efforts are not affected by the hiring subsidy directly but by the effect of the latter on reservation training cost $\gamma^e$.

The rise in $\gamma^e$, implied by a rise in the subsidy $\tilde{A}$, increases the probability that a long term unemployed person enters employment and reduces the gap between the expected lifetime income for the employed and the long-term unemployed worker, that is $J^E_i \cdot J^U_i$ decreases. Appendix B.1 shows that $\partial \gamma^e < 0$ and $\partial \gamma^e > 0$. The first partial derivative result is explained by the fact that a rise in $\gamma^e$, by increasing the chance of finding a job while long-term unemployed, reduces $J^E_i \cdot J^U_i$; that is the relative cost of becoming long-term unemployed. $\partial \gamma^e$ is positive because the positive impact on the transition rate to employment prevails over the impact on $J^E_i \cdot J^U_i$.

The general equilibrium properties of a rise in the subsidy are ambiguous as the opposite reactions of $e_i$ and $e_i$ to $\tilde{A}$ translate into an ambiguous effect upon $\tilde{A}(e_i) \cdot u_i(\mu)$ and thus on $\mu$. Moreover, the net impact on $e_i$ remains ambiguous as $\mu$ shows a tendency to rise with $\tilde{A}$ implying that short term unemployed person would be encouraged to search more intensively.

2 Short-term Unemployed as the Target Group

$\tilde{A}$ now takes the form of a lump sum payment to firms hiring a worker coming from both short-term unemployment exclusively. In this scheme only the vacancy opening condition is modified. It reads as

$$\tilde{A}(e_i) \cdot u_i(\mu) \cdot q(e_i) \cdot (\gamma^e + \tilde{A}) = 0$$

Following appendix B-2 it is easy to verify that $\partial \gamma^e > 0$: Even though only the recruitment of short-term unemployed is subsidized, as contacts are random, firms perceive it as an overall increase in posting job vacancies. We also have that higher tightness is likely to induce both types of unemployed workers to search more intensively. As a consequence the proportion of short-term unemployed expressed in terms of search efficiency units and, aggregate employment can be expected to increase.
2 Long-term Unemployed as the Target Group

In this scheme only firms that hire a worker coming from long-term unemployment are paid the subsidy \( \bar{A} \). The reservation training cost condition is given by (22) and the job opening condition re-writes as

\[
\frac{\bar{A}(e_i) u_s}{\bar{A}(e_i) u_s + \bar{A}(e_i) u_l} q(\mu) (1/\bar{A}) + \frac{\bar{A}(e_i) u_l}{\bar{A}(e_i) u_s + \bar{A}(e_i) u_l} q(\mu) Z = 0 \quad (24)
\]

As shown in appendix B.2, \( \frac{\partial r}{\partial q} > 0 \) and \( \frac{\partial r}{\partial \mu < 0} \), the net partial effect being positive. As mentioned previously, by reducing rejection of long term unemployed, the subsidy increases the overall probability of opening a vacant position even though \( \frac{\partial \mu}{\partial q} \) is negative. The latter effect is due to the fact that the lower bound of \( J^F \) corresponding to \( 1/\bar{A} \) is reduced, by thus reducing the average return of filling a vacancy. However, this effect remains negligible and tightness is expected to increase with the subsidy level. Consequently, a training subsidy tends to generate a positive “job creation effect” which would benefit not only long-term unemployed but also the short-term unemployed workers.

As for search intensities, the disincentive effect on short-term unemployed search effort is observed as in the case of a general subsidy scheme. A gain the general equilibrium properties of the scheme can not be properly determined.

3.1.2 Search Bonus

2 General Scheme

In this scheme, unemployed workers, unconditionally on the length of their unemployment spell receive a lump sum transfer from the government once they know they would return to employment in the next period. Equations (20) and (21) are respectively re-written as

\[
i \frac{\partial r}{\partial q} + \frac{\partial \mu}{\partial q} = 0
\]

\[
i \frac{\partial r}{\partial q} + \frac{\partial \mu}{\partial q} = 0
\]

where \( \frac{\partial r}{\partial q} \) can be expressed as

\[
J^E_i \frac{\partial r}{\partial q} = w i \frac{[ - \bar{p}_i \frac{\partial r}{\partial q} G (1/\bar{A}) p (1 + - \bar{e}_i)] b_i - \bar{e}_i (1 + - \bar{e}_i) e_i}{1 \bar{p}_i \frac{\partial r}{\partial q} G (1/\bar{A}) p (1 + - \bar{e}_i) + - \bar{p}_i}
\]

It is straightforward to check that \( \frac{\partial r}{\partial q} \) is likely to vary negatively with \( q \). However, as \( \frac{\partial r}{\partial q} \) increases with \( q \) both types of unemployed workers search more intensively in response to the scheme. We have

\[
\frac{\partial r}{\partial q} = \frac{1 \frac{\partial r}{\partial q} (1 + - \bar{e}_i)}{1 \bar{p}_i \frac{\partial r}{\partial q} G (1/\bar{A}) p (1 + - \bar{e}_i) + - \bar{p}_i}
\]

Then \( \frac{\partial r}{\partial q} > 0 \) is both positive and because long-term unemployed face a positive probability of being rejected \( \frac{\partial \mu}{\partial q} > 0 \). In other words, short-term unemployed anticipate the fact that if they enter long-term unemployment, the probability to cash in the bonus would fall relative to the current one. It is also easy to check, not surprisingly, that the impact of a positive search bonus on search intensities is lower as \( - \bar{e} \) increases. If rejection would never occur, then search intensities reactions to \( q \) would be driven exclusively by the value of time preference rate.

2 Short-term Unemployed as the Target Group
In this scheme, a lump sum amount \( b \) is paid to unemployed individuals able to find a job by the end of their first period of unemployment. In other words, only short term unemployed workers succeeding in their job search are eligible for the search bonus. Equation (20) is re-written as:

\[
\begin{align*}
J^E_1 \cdot J^U_1 &= \frac{1}{1 + \frac{A^E_1(\varepsilon_1) \mu_1 \mu_1}{A^U_1(\varepsilon_1) \mu_1 \mu_1}} \\
J^E_1 \cdot J^U_1 &= \frac{1}{1 + \frac{A^E_1(\varepsilon_1) \mu_1 \mu_1}{A^U_1(\varepsilon_1) \mu_1 \mu_1}} \cdot b = 0
\end{align*}
\]

where the difference \( J^E_1 \cdot J^U_1 \) reads as

\[
J^E_1 \cdot J^U_1 = \frac{w_1 \cdot z + \mu \cdot b \cdot \varepsilon_1 + (1 + \mu) \cdot \varepsilon_1 + \mu \cdot \varepsilon_1 \cdot \mu_1 \cdot \mu_1}{1 + \frac{A^E_1(\varepsilon_1) \mu_1 \mu_1}{A^U_1(\varepsilon_1) \mu_1 \mu_1}}
\]

For \( w \) and \( z \) given, \( \frac{b}{\mu} \) is not affected by the introduction of a search bonus. In the first place, \( b \) affects conditions (25) and (14). As shown in appendix B.3, \( \frac{b}{\mu} \) and \( \frac{b}{\mu^2} \) are both positive. Short-term unemployed search intensity is increasing with \( \mu_1 \) on the one hand because the payment of the bonus is made contingent upon the event that the worker finds a job and, on the other hand, because the opportunity cost of remaining unemployed, measured by \( J^E_1 \cdot J^U_1 \) also increases with \( b \). These two effects stimulate short-term unemployed search intensity. \( \frac{b}{\mu} \) is also positive because of the increase in the opportunity cost of remaining unemployed just mentioned. This can be interpreted to be a “discounted entitlement effect”. Indeed, long-term unemployed workers anticipate the fact that they would also be covered by the search bonus if they were to become unemployed after some periods of employment. In this scheme, we obtain

\[
\frac{\partial J^E_1 \cdot J^U_1}{\partial b} = \frac{1}{1 + \frac{A^E_1(\varepsilon_1) \mu_1 \mu_1}{A^U_1(\varepsilon_1) \mu_1 \mu_1}} \cdot \mu \cdot \varepsilon_1 \cdot \mu_1 \cdot \mu_1
\]

and

\[
\frac{\partial J^E_1 \cdot J^U_1 \cdot b}{\partial b} = \frac{1}{1 + \frac{A^E_1(\varepsilon_1) \mu_1 \mu_1}{A^U_1(\varepsilon_1) \mu_1 \mu_1}} \cdot \mu \cdot \varepsilon_1 \cdot \mu_1 \cdot \mu_1 + b
\]

It is obvious that ceteris paribus, the effect on short-term unemployed search effort is strong in this scheme than in the general scheme. The impact on long term unemployed search intensity is, as we stated positive, but as concerns its amplitude, little can be said with certainty relative to the one observed in the general scheme. From appendix A, we have that the impact of \( \varepsilon_1 \) on the ratio of short-term unemployment over aggregate unemployment, again expressed in efficiency search units, is nil, which indicates that behavioral properties of \( \frac{A^E_1(\varepsilon_1) \mu_1 \mu_1}{A^U_1(\varepsilon_1) \mu_1 \mu_1} \) are exclusively driven by short-term unemployed search effort. Then the impact on this ratio is stronger in the current scheme than it was in the previous one. As a consequence the impact on job creation is also stronger in the current scheme. Moreover, higher \( \mu \) also implies higher \( \varepsilon_1 \) and higher \( \varepsilon_1 \). Thus, we can expect that partial equilibrium comparative statics translate into qualitatively similar general equilibrium comparative statics. All partial equilibrium effects go in the same direction and reinforce each other.

Note that Davidson and Woodbury (1993), interpret \( \frac{\partial}{\partial b} > 0 \) to be a “rivalry effect”. Namely, the increased search effort of short-term unemployed makes it more difficult for long-term unemployed to find employment as both types of unemployed are competing for the same jobs, thus, in order to recover previous jobs opportunities, long-term unemployed increase their search effort as well. Such an effect does not appear here, as \( \frac{\partial}{\partial b} \frac{\partial}{\partial b} \) and \( \frac{\partial}{\partial b} \frac{\partial}{\partial b} \) are both nil as shown in appendix B.2. Instead, long-term unemployed persons increase their search effort because they anticipate the fact that they might be covered by the scheme at some future date.

Long-term Unemployed as the Target Group

In this scheme only individuals recruited from long-term unemployment are eligible for the search bonus. That is only the condition for optimal search of the long-term unemployed workers is modified. Equation (21) is re-written as
\[ i + A^0(e) \mu_l(j) F_i \sum_{j} \mu^j + b = 0 \]

where the divergence \( J^E_i J^U_j \) reads as

\[
J^E_i J^U_j = \frac{w_i z_i (1 + \epsilon) G(\frac{r}{s}) p b_i - e_i (1 + \epsilon) e_i}{1 + \frac{G(\frac{r}{s}) p (1 + \epsilon) + p_s}{p_s}}
\]

Following previous derivations, it is easy to check that \( \frac{\partial J}{\partial s} \) is positive but that \( \frac{\partial J}{\partial \epsilon} \) turns to be negative as the introduction of a search bonus paid to long-term unemployed workers reduces \( J^E_i J^U_j \). Short-term unemployed workers become less eager to enter employment rapidly as they know to be covered by the search bonus scheme once treated as long-term unemployed workers. As a consequence it is expected that the proportion of short-term unemployed workers, expressed in efficiency units, falls as the latter does not respond to variations in \( q \). Then, the overall profitability of a hired position is reduced and equilibrium \( \mu \) falls. As long as the tax rate \( \bar{\epsilon} \) is not modified, reservation hiring cost \( \frac{\bar{r}}{s} \) is not affected by the search bonus. Thus aggregate unemployment is likely to rise unless the positive impact of search bonus on \( e_i \) is strong enough to counter the fall in \( e_i \) and \( \mu \). Unemployment composition response is not clear cut but might be biased towards a larger proportion of long-term unemployment.

### 3.2 Endogenous wages

We assume that equilibrium wages are the outcome of collective bargaining at the firm level and that the insiders, whose job tenure is at least one period, set the wage for all workers belonging to the firm. This corresponds to a special case of complete labor contracts in which individual renegotiation never occurs\(^1\). This assumption is fully plausible for European labor markets where more than 80% of the workers are covered by collective agreements. This approach generates wage uniqueness despite ex-post heterogenous match values.

For the clarity of the exposition, the tax rate is still considered to be exogenously given.

Because of wage uniqueness, equilibrium conditions are similar to the ones presented in the section dedicated to the case of exogenous wages. Thus, we have that \( J^E_i J^U_j = J_i^F J^V_i \) and \( J^E_j J^U_i = J^E_i J^E_j \).

Collective bargaining implies that neither the sunk cost of hiring a long-term unemployed person, nor the benefits of policy instruments form part of the wage bargain. Moreover, workers choose the expected utility of a short-term unemployed as their fallback pay-off. Hence, wages are set to maximize the Nash bargain

\[
\max J^E_i J^U_j \quad \text{subject to} \quad J^V_i J^V_j = 0
\]

By imposing the free entry condition \( J^V = 0 \), the first order condition of this maximization problem reads as

\[
J^E_i J^U_j = \frac{\bar{r}}{s} \frac{J^F}{1 + \bar{\epsilon}}
\]

where \( J^E_i J^U_j \) is given by

\[
J^E_i J^U_j = \frac{(w_i z_i) [1 + \epsilon (p_i G(\frac{r}{s}) p_s)] + e_i [1 + \epsilon (p_i G(\frac{r}{s}) p_s)] + e_i (1 + \epsilon) e_i}{1 + \frac{G(\frac{r}{s}) p (1 + \epsilon) + p_s}{p_s}}
\]

Table 1 contains the respective sign of the partial derivative of \( w \) with respect to the four endogenous variables of the model. Formal result derivations are shown in appendix C.

\(^1\) Nonetheless, this would also be the outcome in the case of incomplete wage contracts where renegotiation can occur at any time, meaning that the equilibrium wage is renegotiation proof.
becomes dampened in the search bonus scheme. As a consequence the job creation effect is reinforced in the hiring subsidy the latter scheme, as the search bonus increases the expected return of becoming unemployed, wage acceptable bound of hiring costs and by thus reduces the expected return of filling a vacancy. In with respect to

In what follows we make the two policy instruments appear simultaneously in the following ex-

2 General Schemes

If we consider a general scheme for both hiring subsidies and search bonuses, the wage equilibrium solves

$$\frac{\{w, z\}[1 + (1 - (1; G(\frac{\mu}{2})p) + e_t(1 - (1; G(\frac{\mu}{2})p))] + e_t(1; p) + b_t(1 - (1; p) + \frac{\mu}{2})}{1 \cdot [1 + (1; G(\frac{\mu}{2})p) + \frac{\mu}{2}]} \quad \text{if } \quad \frac{\mu}{2} = \frac{1}{\phi} \quad \text{and } \frac{\mu}{2} \neq 0$$

It is easy to state that the partial derivative of \( w \) with respect to \( \bar{A} \) is negative and it is positive with respect to \( b \). In the former scheme it reflects the fact that the hiring subsidy reduce the lower acceptable bound of hiring costs and by thus reduces the expected return of filling a vacancy. In the latter scheme, as the search bonus increases the expected return of becoming unemployed, wage revendication are higher. As a consequence the job creation effect is reinforced in the hiring subsidy scheme, while it is likely to be dampened in the search bonus scheme.

2 Short-term Unemployed as the Target Group

When recruitment or employment of the short-term unemployed is subsidized, the wage equation becomes

$$\frac{\{w, z\}[1 + (1 - (1; G(\frac{\mu}{2})p) + e_t(1 - (1; G(\frac{\mu}{2})p))] + e_t(1; p) + b_t(1 - (1; p) + \frac{\mu}{2})}{1 \cdot [1 + (1; G(\frac{\mu}{2})p) + \frac{\mu}{2}]} \quad \text{if } \quad \frac{\mu}{2} = \frac{1}{\phi} \quad \text{and } \frac{\mu}{2} \neq 0$$

In that case the hiring subsidy paid to firms hiring short-term unemployed has no effect on the equilibrium wage as it does not enter the decision rule for the reservation hiring cost. Like in the previous scheme the impact of the search bonus remains positive. However, the impact of the search bonus is of smaller amplitude. We can easily verify that

$$(1; \bar{s}) \bar{p}_t + \frac{\mu}{2} \bar{P} > \bar{p}_t (1; \bar{s}) (1; G(\frac{\mu}{2})p)$$

that is \( \frac{\mu}{2} > \frac{\mu}{2} \bar{s} \)

2 Long-term Unemployed as the Target Group

In these schemes the equilibrium wage solves

$$\frac{\{w, z\}[1 + (1 - (1; G(\frac{\mu}{2})p) + e_t(1 - (1; G(\frac{\mu}{2})p))] + e_t(1; p) + b_t(1 - (1; p) + \frac{\mu}{2})}{1 \cdot [1 + (1; G(\frac{\mu}{2})p) + \frac{\mu}{2}]} \quad \text{if } \quad \frac{\mu}{2} = \frac{1}{\phi} \quad \text{and } \frac{\mu}{2} \neq 0$$

Table 1: Wage Properties

<table>
<thead>
<tr>
<th>µ</th>
<th>( \frac{\mu}{2} )</th>
<th>( e_t )</th>
<th>( \bar{q} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>+</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>

The fact that \( w \) increases with \( \mu \) and \( \frac{\mu}{2} \) is standard. Higher \( \mu \) implies a higher probability for unemployed workers to leave search unemployment, meaning that the cost attached to being unemployed is reduced. Then, unemployed workers bargain over a higher outside option leading to higher wage. Higher \( \frac{\mu}{2} \) affects principally the overall profitability of a productive match as it corresponds, on average, to lower vacancy duration. As part of this gain goes to the worker due to bargaining, the equilibrium wage rises. The last two results of table 1 indicate that wage does not respond to variations in search intensities. This is due to the assumed additive separability between consumption and searching.

The fact that \( w \) increases with \( \mu \) and \( \frac{\mu}{2} \) is standard. Higher \( \mu \) implies a higher probability for unemployed workers to leave search unemployment, meaning that the cost attached to being unemployed is reduced. Then, unemployed workers bargain over a higher outside option leading to higher wage. Higher \( \frac{\mu}{2} \) affects principally the overall profitability of a productive match as it corresponds, on average, to lower vacancy duration. As part of this gain goes to the worker due to bargaining, the equilibrium wage rises. The last two results of table 1 indicate that wage does not respond to variations in search intensities. This is due to the assumed additive separability between consumption and searching.
Partial equilibrium properties are similar to the ones found in the general scheme. Namely, we have that $\frac{\partial w}{\partial \mu} < 0$ and $\frac{\partial w}{\partial b} > 0$. We also observe that $\frac{\partial w}{\partial \mu} > \frac{\partial w}{\partial j_G} > \frac{\partial w}{\partial j_{1,T}} > \frac{\partial w}{\partial j_{2,T}}$. The impact on the equilibrium wage is the lowest of the three search bonus schemes. However, as $\mu$ was found to respond negatively to the introduction of bin that particular scheme, we expect an even worse response of job creation in the presence of endogenous wages.

4 Hiring Subsidies versus Search Bonuses

These short characterizations of these policy instruments allow us to throw some light on their intrinsic properties. On the one hand, the search bonus scheme attempts principally to introduce search incentives for short-term unemployed persons in order to prevent them from entering “costly” long-term unemployment. On the other hand, the aim of the hiring subsidy scheme is, in the first place, to reduce the cost inherent to the hiring of long-term unemployed workers and thus, to increase outflows from long-term unemployment. A comparative analysis of search bonuses and hiring subsidies will reveal to what extent preventing long-term unemployment is more desirable than curing it.

Partial equilibrium analysis results of the previous section are summarized in Table 2.

![Table 2: Partial Equilibrium Properties](image)

The left side of Table 2 indicates that for exogenous wages and taxation, three policy schemes appear to perform better, in qualitative terms, than any other. These three schemes are the general search bonus, the search bonus paid to short-term unemployed and the hiring subsidy paid to firms that hire a short-term unemployed. All the other schemes may generate some negative effect on at least one of the four endogenous variables considered in this exercise. These findings underline the important role played by search incentives. The best target group for policy intervention proves to be the short-term unemployed simply because any subsidy attached to long-term unemployed leads to search disincentives for short-term unemployed workers. Preventing people from entering long-term unemployment seems to be more productive in terms of impact on unemployment composition than acting on the exit rate from long-term unemployment. However, quantitative analysis produces less clear-cut outcome and partial-equilibrium results are likely to be modified and possibly reverted once wage effects are assessed. For endogenously determined wages, the equilibrium conditions system becomes too complex to allow for a deep comparative statics analysis. However, as summarized in the last column of Table 2, $w$ tends to increase with $b$ and decrease with $\mu$ in all schemes. The equilibrium wage is an additional channel through which the policy instruments affect equilibrium values.

As for the hiring subsidy, the fall in $w$ would amplify the effect of $\bar{A}$ on $\frac{\mu}{\gamma}$ and thus on $\mu$. However, lower $w$ would also reduce the opportunity cost of remaining unemployed, so that $J_{E_l} J_{J^U}$ decreases. The latter effect would push long-term unemployed workers to contain their propensity to intensify their search and short-term ones to refrain from increasing theirs. Hence, an ambiguous impact of $\bar{A}$ on search efforts is expected. As a consequence, this works against the positive impact a rise in $\bar{A}$ has on labor tightness through the dampening effect lower search efforts would have on the $A(e_s j_{1,T} + A(e_l j_{2,T})$ ratio: As concerns the search bonus, as in the first place it tends to increase the wage level, $\frac{\mu}{\gamma}$ falls. Consequently, the probability of opening up a vacant position decreases and $\mu$ will tend to fall. However, this downward effect is dampened by the positive response of search intensities, in particular $e_s$, to the introduction of the search bonus. Moreover, the rise in the equilibrium wage
further stimulate unemployed workers search, implying that the downward tendency of $\mu$ may be completely reversed by higher values of $\frac{A(\varepsilon)_{us}}{A(\varepsilon)_{ul} + A(\varepsilon)_{ul}}$. Nevertheless, the impact of a positive search bonus remains uncertain because of the ambiguous overall response of labor market tightness.

In order to shed some light on these ambiguities and to introduce welfare considerations, the next section presents quantitative results obtained thanks to calibration exercises.

5 Some Quantitative Exercises

5.1 Calibration

The scope of the section is to evaluate the impact of search bonus and hiring subsidy schemes when wages are endogenous. A particular attention is paid to the case of collective bargaining, which, as mentioned previously, remains the closest to European experiences. To do so, we use a calibrated version of the base-line model that gives a 11% unemployment rate, a proportion of long-term unemployed workers equal to 60% and a 0.4 elasticity of unemployment durations with respect to unemployment benefits. A proportion of long-term unemployed workers of 60% is reasonably close to the OECD countries average in 1995 for a time period of unit length interpreted to be a semester as estimated by Machin and Manning (1998). A partial equilibrium elasticity of unemployment durations with respect to benefits of 0.4 is fairly in the middle range of the estimates of Layard and alii (1991). The rate of job destruction, interpreted to be half-yearly, is set equal to 0.05: This corresponds to an average match duration of ten years. This could be seen as a relatively high figure, but it becomes more realistic once we consider the fact that quits and job-to-job movements are not taken into account. The discount factor, also interpreted to be half-yearly, is given a value of 0.975.

The matching function is assumed to be Cobb-Douglas, such that $q(\mu) = \cdot \mu^\cdot$. $\cdot$ is set equal to 0.5 which corresponds to the upper range of the estimates in Blanchard and Diamond (1989). $\cdot$ is set equal to 0.5. The search efficiency function is homogenous of degree a: It is given by, $A(\varepsilon) = \varepsilon^a$. Elasticity a is set equal to 0.6. The distribution of the idiosyncratic match specific training cost $\frac{1}{4}$ is uniform over the interval $[k_l; k_u]$, i.e.,

$$G(\frac{1}{4}) = \frac{\frac{1}{4} \cdot k_l}{k_u - k_l} \cdot \frac{1}{4} \cdot k_u$$

$k_l$ is set equal to 0 and $k_u$ to 3 in both calibration exercises. This is a fairly wide interval and it might be argued that the very efficiency of hiring subsidies relative to search bonuses, hinges on the choice of $k_u$. However, this is true only to some extent and the important element in determining potential efficiency is the difference between the benchmark value of $\frac{1}{4}$ and $k_u$ compared to initial short-term unemployed search efforts. We will see that parametrization goes more in favor of hiring than search subsidies.

Some other distributions could be retained, but our results would not be dramatically qualitatively modified.

We set the workers' bargaining power equal to the elasticity of the matching function with respect to vacancies, i.e. $\bar{\varepsilon} = 0.5$. This choice is not empirically motivated but is known to provide efficient in the decentralized equilibrium in this type of theoretical environment when there is no policy intervention. By setting $\bar{\varepsilon} = \bar{\varepsilon}$ we avoid any bias that could play in favor of one or the other policy scheme as both of them introduce some inequity under that condition.

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12 See Hosios (1990) and Pissarides (1990), Chap. 7.
We set the level of unemployment benefits such that it corresponds to a replacement ratio $\frac{1}{2}$ equal to 0.4 with respect to initial wages\textsuperscript{13}. This is a fair average value for OECD countries in 1995 according to the estimates in Martin (1996). The match productivity is set equal to unity and the vacant jobs cost $c$ is 0.6 in both wage setting environments.

5.2 The Experiments

Since one of the main interest of the paper is to assess the relative performance of search bonus and hiring subsidy schemes, we set up a simple experiment based on an element common to the two schemes. The common element, that allows for a direct comparison, is the level of public expenditures. We assume the latter to be constant. Then, the experiment consists of turning progressively unemployment benefits into either search bonuses or hiring subsidies. In other words, the scope of the experiment is to identify the best expenditures "activation" strategy. The impact on unemployment and on its composition is considered. We also introduce a welfare indicator. The latter corresponds to a social welfare functional $W$, taken to be utilitarian. With collective labor contracts, as hiring costs are sunk and fully paid by firms hiring long term unemployed workers, $W$ is adjusted for the expected training costs incurred by the latter. Then, $W$ is equal to

$$W = (1 - \bar{u}) \cdot (1 - u) J_E + u J_U + (1 - u) J_F + u J_V,$$

Because of free entry of vacancies the term $u J_V$ is zero and thus disappears from the above expressions. A gain, the presence of the expected hiring cost in the definition of $W$ is motivated by the fact that firms bear it fully without being able to pass it on to other agents in the economy. Without its explicit introduction into the above welfare index definition the expected hiring cost would not be properly taken into account. This could bias quantitative results and distort eventually normative assessments.

5.3 Results

Figures are displayed in appendix E. The x-axis reads as the negative variation, expressed in percentage, of the level of unemployment benefits relative to its value in the baseline economy. Regular curves refer to search bonus schemes outcomes while dashed curves refer to hiring subsidy schemes outcomes. This presentation allows for a direct comparison of the two instruments directed towards a common unemployed workers group.

5.3.1 General Schemes

Results are displayed in figures 2 and 3. As indicated by the upper left graph of figure 2, short-term unemployed increase their search intensity in both schemes, even though, this increase is stronger in the search subsidy scheme. The reverse is true for long-term unemployed (not shown in the graphs). In previous analytical investigation short-term unemployed search intensity was found to fall with the hiring subsidy. However, the rise in the short-term unemployed search effort could be, at least in part, due to the fall in unemployment benefits which translates into a rise in the opportunity cost of remaining unemployed. The upper right graph reveals that the reservation hiring cost increases, as expected, in the hiring subsidy scheme and falls in the search bonus scheme. This is explained by the fact that labor costs increase in both schemes by similar proportions, and firms are compensated for the loss in their returns only in the hiring subsidy scheme. We also observe that tightness increases in both situations even though this trend is more pronounced in the hiring subsidy scheme. As a result, as shown in the middle and lower graphs of figure 2, unemployment hazards increase in both schemes and so does the share of short-term unemployment in total unemployment. Again, the respective impacts of the two policy instruments are quantitatively different. Short-term unemployment hazard

\textsuperscript{13}In the complete labor contracts set up we refer to wages as wages gross of hiring costs, meaning that wages eventually are identical for all types of workers, although earnings are not.
rate takes larger values in the search bonus scheme as it is driven essentially by short-term unemployed search effort. As to long-term unemployment hazard rate behavior, the reverse is observed even though the gap is wider than in the previous case. This comes from the fact that the later is driven not only by long-term unemployed search intensity and tightness, but also by the hiring reservation cost. Not surprisingly, the impact of hiring subsidies on $\frac{u}{\mu}$ is more important in the hiring subsidy scheme than in the search bonus scheme.

Looking at Figure 3, we observe that the equilibrium wage slightly increases and in a similar fashion in both schemes. We also have that realized hiring costs fall in both policy frameworks but the trend is sharper in the search bonus scheme. This is due to the fact that the short-term unemployment hazard rate increases by more in the latter than in the hiring subsidy scheme. As to welfare, it increases for all types of workers by comparable amplitudes in both schemes. Firms are worse off in both schemes and their loss is slightly larger in the hiring subsidy scheme. This is due on the one hand to a bigger fall in $q(\mu)$ and on the other hand to the hiring subsidy itself, as it reduces the acceptance boundary of $J_F$.

5.3.2 Short-Term Unemployed as the Target Group

Results are reported in Figures 4 and 5. They prove to be much in line with previous analytical outcomes. Namely, both short-term and long-term unemployed increase their search effort, $\mu$ takes larger values and the reservation cost moves in the same direction in both schemes. Long-term unemployed respond in a similar way to both instruments. However, as could have been expected, short-term unemployed search more intensively in the search bonus scheme relative to the hiring subsidy scheme while, tightness responds more strongly to the latter than to the former. We also have that labor costs increase in both schemes, which implies lower reservation values for the hiring costs. The increase is stronger in the search bonus scheme mainly because of relatively higher wages. Nevertheless, these results translate into higher unemployment hazards in both schemes. On the one hand short-term unemployment hazard takes increasingly larger values in the search bonus relative to the hiring subsidy scheme. On the other hand long-term unemployment hazard upward trend is sharper in the hiring subsidy scheme. Nonetheless, the share of short-term unemployment in total unemployment is always larger in the search bonus scheme compared to the hiring subsidy scheme and total unemployment falls by more in the former than in the latter. As in the previous exercise, effective hiring costs falls by more with search bonuses than with hiring subsidies. As to aggregate welfare it is always higher in the search bonus scheme. The same is true for all types of workers. Again, firms loose less in the search bonus scheme than in the hiring subsidy one which is due to higher employment levels.

5.3.3 Long-Term Unemployed as the Target Group

Results are depicted by Figures 6 and 7. Again, they prove to correspond to analytical findings presented previously. Short-term unemployed reduce their search effort in both schemes by similar amplitudes. Long-term unemployed on the contrary increase theirs, specially in the presence of a hiring subsidy. This is explained essentially by the observed rise in acceptance probability which is a consequence of the upward trend of both $\frac{1}{\mu}$ and $\mu$. This two variables are found to respond only slightly to the introduction of the search bonus. Consequently, the hazard rate of short-term unemployment slightly increases in the hiring subsidy scheme and falls in the search bonus scheme. As to the long-term unemployment hazard rate, it displays an upward trend in both schemes but the latter is sharper in the hiring subsidy scheme. Then, $\frac{u}{\mu}$ increases in the latter while it remains somewhat constant in the search bonus scheme. As a consequence the divergence observed in unemployment behavior. It falls with hiring subsidies and increases with search bonuses. Another consequence is that effective hiring costs increase with both instruments and specially with the search bonus. As to welfare, all workers are better off in the hiring subsidy scheme while they are made worse off by search bonuses. This is also verified at the aggregate level as can be seen from the lower left graph of Figure 7. Part of these welfare gaps is due to the wage differential which stands in favor of the hiring subsidy scheme.
5.4 An Overall Assessment

The previous subsection offers clear indications to answer the questions asked in this paper. Should the government intervene on the supply or the demand side of the labor market? Is targeted intervention desirable and what should be the target group?

As mentioned previously, the first question is not really relevant as long as wage determination is able to fully internalize the impact of the implemented instrument, like in the case of a fully decentralized wage bargain à la Nash. However, we have seen that when this is not the case any more, then the answer to that question is not trivial as wages reaction may not go in the right direction.

The previous subsection reveals that the two subsidy schemes generate different equilibrium outcomes when the unemployed target group corresponds to either the short-term or the long-term unemployed. Search bonuses are found to generate more desirable results than hiring subsidies, both in terms of long-term unemployment reduction and welfare, when the short-term unemployed are the target group. These results are driven essentially by the strong effect, the entitlement effect, of bonuses on short-term unemployed effort. Welfare improvements are stronger in the search bonus scheme for two reasons. First the equilibrium wage shows a similar reaction path in both schemes. Second as the exit rate from short term unemployment is always larger in the search bonus scheme relative to the hiring subsidy scheme, long-term unemployment is strongly short-circuited in the former and consequently, effective hiring costs are much lower. When the long-term unemployed are taken as the target group, the results are reversed. In that case, disincentive effects on short-term unemployed search effort are observed in both schemes. However, because hiring subsidy push the hiring reservation cost upward and raise the overall returns of job creation, long term unemployed search effort increases substantially. This is also observed in the search bonus scheme because of the entitlement effect. However, the latter is dampened by the downward reaction of \( \mu \) which implies that the observed increase in \( \bar{e} \) is much lower than it is in the hiring subsidy scheme. Then, \( \frac{\bar{e}}{e} \) increases with hiring subsidies and remains constant with the search bonus. In addition, employment is increased by hiring subsidies while it falls when search bonuses are implemented. Not surprisingly search bonuses lead to lower aggregate welfare and, more specifically, to lower welfare for all types of workers. However, and essentially because labor costs are reduced, \( \text{rms} \) are slightly better off. As to hiring subsidies, symmetric outcomes are obtained. When general schemes are implemented, supply or demand side interventions lead to similar welfare outcomes.

Then, policy intervention whether directed towards the supply or the demand side of the labor market has equivalent welfare effects only in the case of general subsidy schemes.

In answer to the second question, it becomes clear that taking the short-term unemployed as the target group is the best policy strategy. Figure 8 presents the performance of a subsidy, either a search bonus or a hiring subsidy, devoted respectively to short-term and long-term unemployed, in the case of individual wage bargaining. As one can see, taking the short-term unemployed as the target group as the more desirable policy strategy: lower unemployment and long term unemployment incidence and, higher total welfare. However, in the light of the previous results, with collective bargaining, a preference should be given to search bonuses as they always generate higher \( \frac{\bar{e}}{e} \) ratios, lower unemployment rates and higher aggregate welfare levels, than hiring subsidies. Moreover, even though hiring subsidies paid to \( \text{rms} \) that recruit a long-term unemployed are more welfare improving than any general scheme, it seems difficult to justify, form a policy intervention point of view, focussing on long term unemployed. However, it should be remembered that our results are related to steady-state policies. Obviously, starting from a position with an existing stock of long term unemployed, then a policy of tackling the stock could still be justified, at least on equity terms.

6 Conclusion

The scope of this paper has been to answer the following interrelated questions. To what extent, in the labor market, should supply-side-oriented prevail over demand-side-oriented strategies? To what extent are strategies directed towards long-term unemployment “prevention”, more desirable than strategies directed towards long term unemployed “re-integration”? In order to do so, we determine the relative performance of search bonus (the supply-side-oriented instrument) and hiring subsidy (the
demand-side-oriented instrument) schemes. Three unemployed workers target groups are considered: all the unemployed, the short term unemployed and, the long term unemployed respectively.

The answer to the first question is found to rely essentially on the wage determination rule assumed to prevail in the labor market. We argue that when wages are bargained, à la Nash, at the job level then, the side chosen for policy intervention plays no fundamental role. However, even in that context of “fully internalizing” wages, the impact of policy instruments varies across the different target groups. Indeed, preventive strategies, that is instruments biased towards short-term unemployed, prove to generate better outcomes than re-integrative strategies. A major analytical attention is devoted to collective bargaining. In that context, as policy instruments are never part of the bargain surplus, their respective impact on equilibrium proves to differ even though the target group is the same. Differently speaking, not only the timing of intervention matters but also the side of the labor market policy intervention is directed towards. We find that all policy schemes, but search bonuses paid to long term unemployed workers, lower total unemployment and the proportion of long term unemployment and, increase aggregate welfare. However, the best performer proves to be the search bonus scheme in which only short term unemployed are eligible.

References


A \( \frac{\hat{A}(e_s) u_s}{\hat{A}(e_s) u_s + \hat{A}(e_l) u_l} \) Behavioral Properties

We first re-write \( \frac{\hat{A}(e_s) u_s}{\hat{A}(e_s) u_s + \hat{A}(e_l) u_l} \) as \( 1 + \frac{\hat{A}(e_s) (u_l u_s)}{\hat{A}(e_s) u_s} \) and focus on the behavior of the expression into brackets in order to retrieve the partial derivative of the former with respect to \( \frac{1}{u_s}, \mu, e_s \) and \( e_l \) respectively. From expression (??), \( 1 + \frac{\hat{A}(e_s) (u_l u_s)}{\hat{A}(e_s) u_s} \) reads as

\[ \Delta = 1 + \frac{1}{u_s} \frac{\hat{A}(e_s) \mu q(\mu)}{\hat{A}(e_s) G(\frac{1}{u_s}) \mu q(\mu)} \]

Then,
$$\frac{\partial A}{\partial \gamma_i} = i \frac{\partial \gamma_i}{\partial \beta} \frac{A(e_i) \mu(q) (1 + \gamma_i A(e_i) \mu(q))}{(A(e_i) G(\gamma_i) \mu(q))^{2}} < 0$$

$$\frac{\partial A}{\partial q} = i \frac{\partial q}{\partial \beta} \frac{A(e_i) G(\gamma_i)}{(A(e_i) \mu(q) G(\gamma_i))^{2}} < 0$$

$$\frac{\partial A}{\partial e} = i \frac{\partial e}{\partial \beta} \frac{A^0(e_i) G(\gamma_i) \mu(q)}{(A(e_i) G(\gamma_i) \mu(q))^{2}} < 0$$

$$\frac{\partial A}{\partial \gamma_i} = 0$$

Thus, \( \frac{A(e_i) \mu(q)}{\partial (e_i) \mu(q)} \) responds positively to a rise in \( \gamma_i \), \( \mu \), \( e \) and is not affected by changes in \( e \):

The last result is due to the fact that the pool of short-term unemployed is "renewed" each period.

**B Selected Partial Derivatives (Exogenous \( w \) and \( \dot{q} \))**

Note that, \( \frac{A(e_i) \mu(q)}{\partial (e_i) \mu(q)} \) is denominated by II, henceforth

**B.1 Search Intensities**

As concerns optimal \( e \), let \( \Omega^2 \) be defined as

$$\Omega^2 = \frac{\partial \gamma_i}{\partial (e_i) \mu(q)} \frac{A(e_i) \mu(q) (1 + \gamma_i A(e_i) \mu(q))}{(A(e_i) G(\gamma_i) \mu(q))^{2}} C_0 = 0$$

where \( p = A(e) \mu(q) \) and \( p_e = A(e_i) \mu(q) \) and \( C_0 = w \), \( z_i \), \( - e_i + (1 + \gamma_i) e \).

$$\Omega^2_{e_i} = A^0(e_i) \mu(q) C_0 < 0$$

$$\Omega^2_{\gamma_i} = i \frac{\partial A^0(e_i) G(\gamma_i) \mu(q)}{(A(e_i) G(\gamma_i) \mu(q))^{2}} C_0 = 0$$

Thus

$$\frac{\partial e}{\partial \gamma_i} = i \frac{\partial \gamma_i}{\partial (e_i) \mu(q)} < 0$$

Consider optimal \( e \) and, let \( \Omega^3 \) satisfies

$$\Omega^3 = \frac{\partial \gamma_i}{\partial (e_i) \mu(q)} \frac{A(e_i) \mu(q) (1 + \gamma_i A(e_i) \mu(q))}{(A(e_i) G(\gamma_i) \mu(q))^{2}} C_0 = 0$$

$$\Omega^3_{e_i} = A^0(e) G(\gamma_i) \mu(q) C_0 < 0$$

$$\Omega^3_{\gamma_i} = i \frac{\partial A^0(e) G(\gamma_i) \mu(q)}{(A(e_i) G(\gamma_i) \mu(q))^{2}} C_0$$
Using $\Omega^3$, $\Omega^3_{\mu}$ reduces to

$$\Omega^3_{\mu} = \frac{\Theta_3}{\Theta_4} (\frac{\partial}{\partial \phi}) [1_{i, \mu} - (1_{i, \mu} + \rho_{es})] > 0$$

Hence

$$\frac{\Theta_3}{\Theta_4} = i \frac{\Omega^3_{\mu}}{\Omega^3_{\theta}} > 0$$

We can also check that in all schemes $i^j_{E_i} j^y_{\Phi}$ does not depend upon neither $e_s$ nor $e_l$. For example, in the general subsidy scheme, we verify that

$$\frac{\Theta_3}{\Theta_4} = \frac{(1 + \rho_{es})(1 + \mu_{q})}{[1_{i, \mu} - (1_{i, \mu} + \rho_{es})]} = 0$$

B.2 Hiring Subsidy Schemes

For $\phi$ and $w$ exogenously given, equation (22) can be solved for $\phi$ independently of any other endogenous variable. Thus, by taking $e_s$ and $e_l$ as given, equation (25) gives equilibrium $\mu$ for any particular value of $\phi$ obtained from (22). As to the effect of $\Phi$ on $\mu$, it reads as

$$\frac{d\mu}{d\phi} = 1 + \frac{d\Phi}{d\phi}$$

where according to equation (22) $\frac{d\Phi}{d\phi} = 1$ and according to $\frac{d\Phi}{d\phi} = 0$ as hiring is subsidized for both types of unemployed workers.

Let $\Omega^1$ be the function characterized by equation (25) and, $\Omega^2_{\mu}$ its derivative with respect to argument $h$. We have

$$\Omega^1_{\mu} = \frac{\Theta_1}{\Theta_3} q(\mu) A + \frac{\Theta_2}{\Theta_3} (1 - \frac{\mu}{\mu}) II \phi + (1_{i, \mu}) Z_{\phi} \phi + \frac{\Theta_4}{\Theta_3} \frac{d\Phi}{d\phi} (\phi \phi)$$

where

$$A = \frac{\mu}{\mu} (1_{i, \mu} \phi \phi + \frac{d\Phi}{d\phi} = \frac{d\Phi}{d\phi} (1_{i, \mu} \phi \phi + \frac{d\Phi}{d\phi} (\phi \phi))$$

$A$ represents the difference between the ex ante expected value of contacting a short-term unemployed worker and the one of contacting a long term unemployed worker. It is easy to check that this difference is always positive.

As the proportion of short term unemployed workers in the unemployed workers pools increases with $\mu$ the rst term in the LHS of $\Omega^1_{\mu}$ is positive and, as $\frac{d\Phi}{d\phi} < 0$ the second term is negative. This re ects two opposite effects of a rise in $\mu$ on the expected proability of a led job. The rst term expresses the fact that higher $\mu$ increases the chances of forming a match with a short term unemployed worker which is more proable that forming a match with a long term unemployed worker. The second term corresponds to the loss in the overall expected proability of a match due to a lower contact probability brought about by higher $\mu$. If the rst effect had to prevail then a
pernicious mechanism would arise. Namely, firms would be tempted to open an infinite number of vacancies. Thus we restrict the analysis to the case where $\Omega^1_{\mu} < 0$.

As concerns $\Omega^1_{\lambda}$ and $\Omega^1_{\nu}$ we have

$$\Omega^1_{\lambda} = 0$$

and

$$\Omega^1_{\nu} = \frac{\partial I}{\partial \lambda} q(\mu) A + \Pi q(\mu) + (1 \Pi) q(\mu) \frac{\partial \Phi}{\partial \Phi}$$

where $B = \int_0^{\frac{\nu}{\lambda}} (\frac{\nu}{\lambda} - \nu) \, dG(\nu)$ and $\frac{\partial \Phi}{\partial \Phi}$ is assumed to be positive in order to avoid any "perverse" effect. This is an assumption usually adopted in the literature. Thus, as $\frac{\partial \Phi}{\partial \Phi} > 0$ and $\Lambda > 0$, $\Omega^1_{\nu} > 0$.

By using the implicit-function theorem and by using $\frac{\partial \Phi}{\partial \Phi} = 1$, we find

$$\frac{d\mu}{d\lambda} j_{w;e;e;e} = i \frac{\Omega^1_{\nu}}{\Omega^1_{\mu}} > 0$$

2 Hiring subsidy for the short term unemployed

Using the same assumptions than in the previous scheme, the effect of $\lambda$ on $\mu$ is again written as

$$\frac{d\mu}{d\lambda} j_{w;e;e;e} = \frac{\partial \Phi}{\partial \Phi} + \frac{\partial \mu}{\partial \lambda} \frac{d\Phi}{d\lambda}$$

where according to equation $\frac{\partial \Phi}{\partial \Phi} = 0$ but $\frac{\partial \Phi}{\partial \Phi} \neq 0$.

Let $\Omega^{10}$ be the function characterized by equation (23) and, $\Omega^{10}_{\lambda}$ its derivative with respect to argument $h$. We have

$$\Omega^{10}_{\mu} = \frac{\partial I}{\partial \lambda} q(\mu) A^0 + \frac{\partial I}{\partial \lambda} \Pi (\frac{\nu}{\lambda} + \hat{A}) + (1 \Pi) q(\mu) \frac{\partial \Phi}{\partial \Phi}$$

where

$$A^0 = (\frac{\nu}{\lambda} + \hat{A}) \int_0^{\frac{\nu}{\lambda}} (\frac{\nu}{\lambda} - \nu) \, dG(\nu) = i \frac{\nu}{\lambda} + \hat{A} \frac{\nu}{\lambda} \int_0^{\frac{\nu}{\lambda}} (\frac{\nu}{\lambda} - \nu) \, dG(\nu)$$

As before, $A^0$ represents the difference between the ex ante expected value of contacting a short-term unemployed worker and the one of contacting a long term unemployed worker. $A^0$ is strictly positive for positive values of $\hat{A}$. Again, we restrict the analysis to the case where $\Omega^{10}_{\mu} < 0$.

$\Omega^1_{\lambda}$ is given by

$$\Omega^1_{\lambda} = q(\mu) \Pi > 0$$

By using the implicit-function theorem, we find

$$\frac{d\mu}{d\lambda} j_{w;e;e;e} = i \frac{\Omega^1_{\lambda}}{\Omega^1_{\mu}} > 0$$

2 Hiring subsidy for the long term unemployed

Again the effect of $\lambda$ on $\mu$ reads as
where as in the case of the general scheme \( \frac{d\mu}{d\theta} = 1 \). However we also have that \( \frac{d\mu}{d\theta} = 0 \):

Let \( \Omega^{\text{G}} \) be the function characterized by equation (24) and, \( \Omega^{\text{H}}_h \) its derivative with respect to argument \( h \). We have

\[
\Omega^{\text{G}}_h = \frac{\partial}{\partial \theta} q(\mu) A^{\text{G}} + \Pi (\frac{\partial}{\partial \mu} q(\mu) \frac{\partial}{\partial \theta} A^{\text{G}})
\]

where

\[
A^{\text{G}} = \left( \frac{\partial}{\partial \mu} \right) (1 + \Pi) \frac{dG}{d\theta} = J^{\text{G}} + A \frac{dG}{d\theta}
\]

Again, \( A^{\text{G}} \) represents the difference between the ex ante expected value of contacting a short-term unemployed worker and the one of contacting a long term unemployed worker. We restrict our analysis to values of \( \theta \) such that the latter difference is kept positive. As before, we restrict the analysis to the case where \( \Omega^{\text{H}}_h < 0 \).

\( \Omega^{\text{H}}_h \) and \( \Omega^{\text{G}}_h \) are given by

\[
\Omega^{\text{H}}_h = \frac{\partial}{\partial \mu} q(\mu) \Pi < 0
\]

and

\[
\Omega^{\text{G}}_h = \frac{\partial}{\partial \theta} q(\mu) A^{\text{G}} + \Pi q(\mu) \frac{\partial}{\partial \theta} A^{\text{G}}
\]

where as before \( B = \int_{\frac{dG}{d\theta}} (1 + \Pi) \frac{dG}{d\theta} > 0 \). Thus, as \( \frac{d\mu}{d\theta} > 0 \) and \( A^{\text{G}} > 0 \), \( \Omega^{\text{G}}_h > 0 \):

By using the implicit-function theorem and by using \( \frac{d\mu}{d\theta} = 1 \), we find

\[
\frac{\partial}{\partial \mu} j_{w;e;\theta} \frac{\partial}{\partial \theta} \mu > 0
\]

as

\[
\Omega^{\text{H}}_h + \Omega^{\text{G}}_h = \frac{\partial}{\partial \mu} q(\mu) A^{\text{G}} + (1 + \Pi) q(\mu) \frac{\partial}{\partial \theta} A^{\text{G}} > 0
\]

B.3 Search Bonus Schemes

\( \Omega^4 \) characterizes optimal \( e_b \). We have

\[
\Omega^4 = \frac{F}{\theta} e_b \frac{dG(\mu)}{d\theta} + \frac{D}{\theta} + \frac{\partial}{\partial \mu} q(\mu) \frac{dG(\mu)}{d\theta} \frac{d\mu}{d\theta} C_1 = 0
\]

where \( p_i = A(e_i) \frac{dG(\mu)}{d\theta} \) and \( p_i = \frac{A(e_i)}{\theta} \frac{dG(\mu)}{d\theta} \),

\[
C_1 = 1 \quad D = \frac{dG(\mu)}{d\theta} p_i (1 + \cdot) e_b + (1 + \cdot) e_b \]

and

\[
D = 1 \quad (1 + \cdot) \frac{dG(\mu)}{d\theta} p_i (1 + \cdot) e_b + (1 + \cdot) e_b
\]
\[\Omega^4_{e_s} = \dot{A}^0(e_s) \mu q(\mu) [\pi C_1 + b D] < 0\]

\[\Omega^4_{b} = \dot{A}^0(e_b) \mu q(\mu) [1 i \sim (1 i \sim)] > 0\]

Thus

\[\frac{\partial e_s}{\partial b} = i \frac{\Omega^4_{e_s}}{\Omega^4_{b}} > 0\]

As to optimal \(e_s\), we define \(\Omega^5\) as

\[\Omega^5 = \dot{A}^0(e_s) G(\nicefrac{1}{4}) \mu q(\mu) b_i \pi D + \dot{A}^0(e_s) G(\nicefrac{1}{4}) \mu q(\mu) C_1 = 0\]

\[\Omega^5_{e_s} = \dot{A}^0(e_s) G(\nicefrac{1}{4}) \mu q(\mu) [\pi C_1 + b D] < 0\]

\[\Omega^5_{b} = \dot{A}^0(e_b) G(\nicefrac{1}{4}) \mu q(\mu) [1 i \sim (1 i \sim)] > 0\]

We obtain

\[\frac{\partial e_s}{\partial b} = i \frac{\Omega^5_{e_s}}{\Omega^5_{b}} > 0\]

2 Bonus to the short-term

\(\Omega^{4_0}\) characterizes optimal \(e_s\). We have

\[\Omega^{4_0} = \dot{A}^{0_0}(e_s) \mu q(\mu) b_i \pi D + \dot{A}^{0_0}(e_s) \mu q(\mu) C_1^0 = 0\]

where \(p = \dot{A}(e_s) \mu q(\mu)\) and \(p_b = \dot{A}(e_b) \mu q(\mu)\).

\(C_1^0 = w j z + \sim A(e_s) \mu q(\mu) b_i \sim e_b + (1 + \sim) e_s\). We have

\[\Omega^{4_0}_{e_s} = \dot{A}^{0_0}(e_s) \mu q(\mu) [\pi C_1^0 + b D] < 0\]

\[\Omega^{4_0}_{b} = \dot{A}^{0_0}(e_b) \mu q(\mu) \frac{\mathbf{f}}{\pi} D + \sim^2 \dot{A}(e_s) \mu q(\mu) > 0\]

Thus

\[\frac{\partial e_s}{\partial b} = i \frac{\Omega^{4_0}_{e_s}}{\Omega^{4_0}_{b}} > 0\]

As to optimal \(e_b\), we define \(\Omega^{5_0}\) as

\[\Omega^{5_0} = i D + \dot{A}^0(e_s) G(\nicefrac{1}{4}) \mu q(\mu) C_1^0 = 0\]
\[\Omega_{\theta}^{50} = \hat{A}^0(\theta) G (\frac{1}{4}) \mu \theta (\mu)^{2} - (C_1^0 = D) < 0\]

\[\Omega_{b}^{50} = \hat{A}^0(\theta) G (\frac{1}{4}) \hat{A}(\theta) (\mu \theta (\mu)^{2} - 2 > 0\]

We obtain

\[\frac{\partial \theta}{\partial b} = j \frac{\Omega_{\theta}^{50}}{\Omega_{b}^{50}} > 0\]

\(^2\) bonus to the long term

\(\Omega^{50}\) characterizes optimal \(\theta\). We have

\[\Omega^{50} = i D + \hat{A}^0(\theta) \mu \theta (\mu)^{2} C_1^0 = 0\]

where \(p = \hat{A}(\theta) \mu \theta (\mu)^{2}\) and \(p_0 = \hat{A}(\theta) \mu \theta (\mu)^{2}\).

\[C_1^0 = w_i z (1 + . ) p \bar{b}_i . \bar{e}_b + (1 + . \bar{e}_b. \bar{e}_b\).

We have,

\[\Omega_{\theta}^{50} = \hat{A}^0(\theta) \mu \theta (\mu)^{2} - (C_1 = D) < 0\]

\[\Omega_{b}^{50} = i \hat{A}^0(\theta) \mu \theta (\mu)^{2} [1 + . \bar{p}] < 0\]

Thus

\[\frac{\partial \theta}{\partial b} = j \frac{\Omega_{\theta}^{50}}{\Omega_{b}^{50}} < 0\]

As to optimal \(\theta\), we define \(\Omega^{50}\) as

\[\Omega^{50} = \hat{F}(\theta) G (\frac{1}{4}) \mu \theta (\mu)^{2} b_1 \bar{b}_1 [1 + . \bar{b}_1 \bar{b}_1 + \hat{A}^0(\theta) G (\frac{1}{4}) \mu \theta (\mu)^{2} C_1^0 = 0\]

\[\Omega_{\theta}^{50} = \hat{A}^0(\theta) G (\frac{1}{4}) \mu \theta (\mu)^{2} \mu \theta (\mu)^{2} [1 + \bar{b}_1 + \bar{b}_1] < 0\]

\[\Omega_{b}^{50} = \hat{A}^0(\theta) G (\frac{1}{4}) \mu \theta (\mu)^{2} [1 i \bar{1} i \bar{1} + \bar{p}_0] > 0\]

We verify

\[\frac{\partial \theta}{\partial b} = j \frac{\Omega_{\theta}^{50}}{\Omega_{b}^{50}} > 0\]

\(^2\) Tightness
From previous results it is possible to assess the partial equilibrium impact of the search bonus schemes on equilibrium tightness. Let $\Omega^0$ characterize the job opening condition in the search bonus schemes.

We have

$$\Omega^0 = q(\mu) \int_0^{\frac{1}{2} \mu} Z \frac{\partial \Omega}{\partial s} \text{d}G.$$ \(\frac{\partial \Omega}{\partial s} > 0\)

We have shown that $\Pi$ is increasing with $e_s$ and $\frac{\partial \Pi}{\partial s} = 0$. Then, in generic terms, the partial equilibrium effect of bonus labor tightness reads as

$$\frac{\partial \mu}{\partial b} = \frac{\partial \mu}{\partial \Pi} \frac{\partial \Pi}{\partial s} \frac{\partial e_s}{\partial s} \frac{\partial b}{\partial b}.$$ \(\frac{\partial \mu}{\partial b} > 0\)

In the general search bonus scheme, $\frac{\partial \mu}{\partial b} > 0$ as $\frac{\partial e_s}{\partial b} > 0$.

A positive impact is also obtained in the case of a search bonus paid to short-term unemployed exclusively. Moreover one can easily verify that this effect is stronger in the latter scheme than in the former.

C. Wage Behavior

The following results are derived in a context where $\zeta$ is not allowed to adjust.

We know that equilibrium wage satisfies

$$J^E_j^s_j^U = \frac{\partial \Omega}{\partial s} \frac{\partial s}{\partial e_s} \frac{\partial e_s}{\partial s} \frac{\partial b}{\partial b}.$$ \(\frac{\partial \mu}{\partial b} > 0\)

By re-writing $J^E_j^s_j^U$ as

$$J^E_j^s_j^U = \frac{w_i Z + e_s + (1_i \cdot p_s) i^j E_j^U^U}{1 + \zeta},$$

where $i^j E_j^U^U$ is given by (26). We also have that, in equilibrium, the expected discounted value of a filled position must be equal to the reservation hiring cost. That is

$$J^F = \frac{\partial \mu}{\partial b}.$$ \(\frac{\partial \mu}{\partial b} > 0\)

It is also straightforward to check that,

$$\frac{\partial \mu}{\partial b} = \frac{1}{1 + \zeta} \frac{\partial \mu}{\partial b} \frac{\partial e_s}{\partial s} \frac{\partial b}{\partial b} = 0.$$ \(\frac{\partial \mu}{\partial b} < 0\)

that,

$$\frac{\partial \mu}{\partial b} = \frac{\partial \mu}{\partial b} \frac{\partial e_s}{\partial s} \frac{\partial b}{\partial b} = 0.$$
and that,

$$\Omega^6 = - \frac{(1 \ i \ p_\mu) \partial^E \ i \ J \ u^\mu}{1 + \omega} \ i \ p_\mu - 1 \ i \ \partial_1 + \zeta < 0$$

We also have that,

$$\Omega^6 = - \frac{(1 \ i \ p_\mu) \partial^E \ i \ J \ u^\mu}{1 + \omega} \ i \ p_\mu - 1 \ i \ \partial_1 + \zeta < 0$$

Thus,

$$\frac{\partial V}{\partial \omega} > 0; \frac{\partial V}{\partial \gamma} > 0; \frac{\partial V}{\partial \pi} = 0$$

D Decentralized Labor Contracts

We consider here the case wage bargaining at the job level. Rather than the “standard” individual wage bargaining à la Nash we introduce a more specific labor contract, both ...rms and workers initially commit to. Renegotiation never occurs. However, equivalent results would be obtained in the more standard framework but analytical outcomes would be less transparent.

We restrict our analysis to the case in which long term unemployed are the target group. When a ...rm and a worker coming from long-term unemployment meet, the contract wage is the solution of a Nash bargain where the fall back pay-off of the worker is, like in the previous case, expected utility of the long-term unemployment.

We further assume that the realized hiring cost \( \frac{\bar{\gamma}}{e_{i}} \) is shared between the two parties according to their bargaining power. In other words, \( \frac{\bar{\gamma}}{e_{i}} \) is not included in the wage bargain but the two parties take into account the fact that they both would bear part of it. The wage outcome holds until the job is destroyed. Then, ...rst period workers earnings are equal to \( w_{i} \ i \ (1 \ i \ \bar{\gamma}) \) and ...rst period ...rm’s effective labor costs are given by \( (1 + t) w_{i} + \bar{\gamma} \).

Consequently equation (5) together with the free entry, \( J^V = 0 \), condition now reads as

$$0 = i \ \bar{\omega} + \frac{\partial^E (e_{i}) u_{i}}{\partial (e_{i}) u_{i} + A(e_{i}) u_{i}} q(\mu) J^F + \frac{\partial^E (e_{i}) u_{i}}{\partial (e_{i}) u_{i} + A(e_{i}) u_{i}} q(\mu) \cdot Z \ \bar{\gamma} \ i \ J^F \ i \ \bar{\gamma} \frac{\partial G}{\partial \gamma} \frac{\bar{\gamma}}{e_{i}}$$

where \( J^F \) and \( J^F \) are still expressed by equations (7) and (8) respectively.

We assume that any subsidy follows the same sharing rule than hiring costs.

Thus, in the hiring subsidy scheme, the reservation cost condition (6) becomes

$$J^F \ i \ [\bar{\gamma} \ i \ (1 \ i \ \bar{\gamma})] = 0$$

and the vacancy opening condition is written as

$$\frac{\partial^E (e_{i}) u_{i}}{\partial (e_{i}) u_{i} + A(e_{i}) u_{i}} q(\mu) \ [\bar{\gamma} \ i \ (1 \ i \ \bar{\gamma})] + \frac{\partial^E (e_{i}) u_{i}}{\partial (e_{i}) u_{i} + A(e_{i}) u_{i}} q(\mu) \ [\bar{\gamma} \ i \ (1 \ i \ \bar{\gamma})] = 0$$

And the reservation hiring cost condition is given by

$$\frac{\bar{\gamma} \ i \ (1 + \bar{\omega}) w_{i}}{1 \ i + (1 \ i \ \bar{\gamma})} \ [\bar{\gamma} \ i \ (1 \ i \ \bar{\gamma})] = 0$$

Value functions for workers are given by (9); (10); (11) and

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Conditions for optimal search they are given by (13) and

\[ i \left( 1 + \hat{A}(\varepsilon) b \frac{q(\mu)}{q(\mu)} \right) \frac{Z}{v_{4}} + \left( 1 + G(\frac{1}{2}) p \right) J_{i}^{U} = 0 \]

Turning to wages outcomes, wage \( w_{s} \) paid to workers leaving short-term unemployment solves

\[
\max_{w_{s}} \left( \int_{s} \left( J_{E} - J_{U} \right) \right) \left( (1) \left( 1 + \frac{q(\mu)}{q(\mu)} \right) \frac{Z}{v_{4}} + \left( 1 + G(\frac{1}{2}) p \right) J_{i}^{U} \right) = 0
\]

while wage \( w_{l} \) paid to workers leaving long-term unemployment supports the outcome

\[
\max_{w_{l}} \left( \int_{l} \left( J_{E} - J_{U} \right) \right) \left( (1) \left( 1 + \frac{q(\mu)}{q(\mu)} \right) \frac{Z}{v_{4}} + \left( 1 + G(\frac{1}{2}) p \right) J_{i}^{U} \right) = 0
\]

As workers accept all jobs that satisfy \( J_{E} \), and all matches that satisfy \( J_{V} \), reservation hiring cost \( \frac{1}{2} \) that solves the above reservation cost condition also satisfies condition (13).

Differently put, \( \frac{1}{2} \) is a common reservation cost and both parties agree to reject all job matches generating hiring costs \( \frac{1}{2} > \frac{1}{2} \).

We verify that in equilibrium

\[ w_{l} = w_{s} \]

that is, \( w_{l} \) and \( w_{s} \) are equal for any value of \( \hat{A} \).

Let \( w_{0}^{l} \) be the expected effective wage received by workers coming from long-term unemployment in their first period of employment. We have seen that, by assumption,

\[ w_{0}^{l} = w_{s} \left( (1 + \frac{q(\mu)}{q(\mu)} \right) \frac{Z}{v_{4}} + \left( 1 + G(\frac{1}{2}) p \right) J_{i}^{U} \]

In the search bonus scheme considered here we have that the reservation cost condition (6) becomes

\[ J_{i}^{F} \left( \frac{1}{2} \left( 1 + \frac{q(\mu)}{q(\mu)} \right) \right) \frac{Z}{v_{4}} + \left( 1 + G(\frac{1}{2}) p \right) J_{i}^{U} = 0 \]

and the vacancy opening condition is now written as

\[
\frac{\hat{A}(\varepsilon) u_{s}}{\hat{A}(\varepsilon) u_{s} + \hat{A}(\varepsilon) u_{l}} q(\mu) \left( (1 + \frac{q(\mu)}{q(\mu)} \right) \frac{Z}{v_{4}} + \left( 1 + G(\frac{1}{2}) p \right) J_{i}^{U} = 0
\]

The reservation hiring cost condition is given by

\[ \frac{\hat{Y}}{\hat{A}(\varepsilon) u_{s} + \hat{A}(\varepsilon) u_{l}} q(\mu) \left( (1 + \frac{q(\mu)}{q(\mu)} \right) \frac{Z}{v_{4}} + \left( 1 + G(\frac{1}{2}) p \right) J_{i}^{U} = 0 \]

The value function for the long term unemployed is given by

\[ J_{i}^{U} = z_{i} \left( 1 + \frac{q(\mu)}{q(\mu)} \right) G(\frac{1}{2}) \left( (1 + \frac{q(\mu)}{q(\mu)} \right) \frac{Z}{v_{4}} + \left( 1 + G(\frac{1}{2}) p \right) J_{i}^{U} \]

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Conditions for optimal search of the long term unemployed reads as

\[ 1 + \hat{A}_i^0(G) G^{(1/4)} \mu Q(\mu) - \frac{\mu}{J} \int_0^{\mu} Z \frac{dG}{d\mu} = 0 \]

It is straightforward to see that for identical values, \( b \) and \( \bar{A} \) equilibrium effects are also identical. The side of the labor market chosen for policy intervention is of no relevance in the context of labor contracts introduced at the job level.
E  Graphical Outcomes

Figure 2: Unemployment (General)

Figure 3: Welfare (General)
Figure 4: Unemployment (Short Term)

Figure 5: Welfare (Short Term)
Figure 6: Unemployment (Long Term)

Figure 7: Welfare (Long Term)
Figure 8: Individual Labor Contracts