

Corruption and Competition in Public Market Auctions

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Abstract

This paper investigates the effect of corruption on competition in government procurement auctions. Our assumption is that the bureaucrat (i.e. the agent that administers the market), if corrupt, may provide a favor in exchange for a bribe. The favor we consider in most of our analysis is the opportunity to readjust a bid. We show that a key effect of corruption is to facilitate collusion in price between firms. This can result in high public spending and inefficient allocation. We discuss the effect of other forms of bureaucratic discretion in the procurement process and analyze conditions under which unilateral anticorruption controls may restore price competition.

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1 Introduction

Is corruption only a transfer between bribers and bribees or does it inhibit competitive pressures and allocative efficiency? What are the links between corruption and competition? What are the impacts of controls and public market procedures on corruption, firms' profits and government expenditures? This paper addresses these important issues in a simple model of public market auctions. It shows that a key effect of corruption in public market is to undermine competition and to facilitate *implicit collusion in price* between competing firms. This may result in high public spending and inefficient allocation.

There is presently no need to motivate the importance of economic analyses of corruption. The experiences of developing and transition economies suggest that corruption is a crucial obstacle to economic development and growth. The fight against corruption has become a top priority for international institutions, like the World Bank and the IMF. The concern about corruption in international public markets has also been forcefully stressed by the WTO, where the issue is anticipated to be one of the new topics on the negotiation table in the next 2000 trade round.

Though there seems to be a general consensus on the undesirability of corruption in procurement auctions, little formal work has been done in this field.¹ Of particular interest are questions related to the link between competition and corruption. The purpose of this paper is to examine, in a corrupt environment, the performance of various procurement procedures with respect to the promotion of competition. We also investigate the effects of controls over firms or bureaucrats in this context.

As a benchmark, we consider the case where a public contract is allocated through a first price auction.² The candidate firms are differentiated in their cost structure. Without corruption, this type of auction mechanism induces competitive pressures on firms so that the price at which the contract is

¹Most theoretical studies of corruption have focused on the issue of collusion and optimal delegation in principal-supervisor-agent models of regulation (cf Tirole 1986, Laffont Tirole 1993). See Auriol (1998) for a recent analysis following this approach. For a model of government procurement and international trade (with no corruption) using an auction theory framework, see also Mc Afee and Mc Millan (1989).

²Our aim is to investigate the effect of corruption on *existing* public auction procedures and their impact on the outcomes for government procurement. In particular, we will not attempt to derive optimal "collusion proof" or "corruption proof" incentive mechanisms.

allocated tends to reflect the cost structure of the “most” efficient firms (in a sense that we make precise in section 2).

We then introduce corruption in the following simple way. We assume that the public official (the *bureaucrat*) is corrupt, which means that i) he may provide a favor to a firm in exchange for a bribe (the favor we consider will be defined shortly), and ii) he is only interested in the level of bribes he can extract. We will also assume that he is limited (by controls) in his capacity to receive illegal transfers.

The favor we consider throughout most of our analysis is the following one: after *initial bids* are made, the bureaucrat may give to a firm the opportunity to readjust its initial bid e.g., through manipulations of envelopes. Such readjustments may also arise in complex contracts where ambiguities in the *initial bids* open the way to modifications of the offers. The *official bids* in the auction with corruption are thus identical to the initial bids made by the firms, except possibly for one firm if it was given an opportunity to readjust its initial bid, or *resubmit*. The contract is then allocated to the firm with the *lowest official bid*.

Our first contribution is to show that in the corrupt environment described above, competition loses its force. In particular, the contract may be allocated at the government’s reservation price with probability 1. In the equilibrium we consider, all firms bid the government reservation price and then offer the highest bribe the bureaucrat would accept. The bureaucrat then randomly picks one of the firms. Intuitively, these strategies are in equilibrium because when a firm deviates in the initial price auction, other firms may still participate in the bribing game, and the winner of this bribing game then has the opportunity of slightly undercutting the deviators’ initial price bid, thereby making the initial price deviation unprofitable. The corrupt bureaucrat’s discretion thus provides firms with a mechanism *to sustain collusion* in the initial price auction.

We next examine the effect of controls and examine whether they can be used to reduce corruption and restore competition in prices. Our analysis should thus be contrasted with that of the literature on collusion in auctions which has emphasized the role of the reserve price as a way to fight collusion (see Graham and Marshall (1987), Mc Afee and Mc Millan (1992)). We will rather focus on the role of controls and contrast the impact of controls on bureaucrats as opposed to controls on firms.

A first result suggests that increasing control on the bureaucrat i.e., reducing the amount of illegal transfer he can accept, may not reduce the

ability of firms to collude. The intuition is that corrupt bureaucrats are in a sense in competition with firms for collusive rents. Increasing controls on civil servants reduces the “price” firms have to pay to sustain collusion, which in turn makes collusion even more profitable to the firms.

In contrast, corruption controls on efficient firms may restore competition in prices (to some extent). An efficient firm subject to controls is ready to harden price competition in the price auction stage: it needs to compensate for its comparative disadvantage in bribe competition. By proposing a low enough price, it can make sure that other less efficient firms cannot afford to compete in bribes. This behavior kills implicit collusion in the price auction game and, at the same time, mitigates corruption with the bureaucrat. In such a context, unilateral controls on a firm that is a potential winner dramatically reduces public spending. So does also the entry of a sufficiently efficient outsider (that lacks connections i.e., has no bribe capacity).

Our next result however mitigates the positive effects of unilateral controls on firms. We consider a situation where the bureaucrat has some limited discretion in the allocation of the contract (in addition to being able to offer an opportunity to resubmit). *Limited discretion* arises when the bureaucrat may choose the winner according to criteria that have not been well-defined ex-ante or to criteria that are not all objectively measurable. It also arises when the bureaucrat may readjust the contract in the course of completion, which may amount to a cost reduction to the winner. We model this additional limited discretion by departing from our initial assumption that the bureaucrat is constrained to choosing the *lowest official bid*. Instead, we assume that the bureaucrat may choose any firm whose official price does not exceed a pre-defined mark up above the lowest official bid. We call this procedure the “*best offer*” (rather than the lowest price offer) procedure.³ We show that under the best offer procedure, unilateral controls on firms may further *deteriorate efficiency*. Their sole effect may be to exclude the controlled firm(s) from the market. The intuition is that even if they are more efficient, the controlled firm (possible) cost advantage may not be sufficient to compensate for the favor received by the winner of the bribing game. Corruption then induces a bias in favor of uncontrolled firms.

Our analysis of the determinants of competition and corruption in public

³Such procedures are often used in practice when offers are not solely financial: the quality of the offers may differ and leaving a (benevolent) bureaucrat choose the project that suits best the interests of the government seems attractive.

auctions has interesting policy implications for the fight against corruption in international public markets. Typically, one of the main issues in that context is the effectiveness and desirability of unilateral controls i.e., anti-corruption regulations enacted by individual countries⁴. The point is that countries as a whole would benefit from fighting corruption and reducing extortions in international markets. At the same time, they fear that imposing severe legislations on their national firms while other countries still adopt a more permissive attitude, would prevent their firms from competing effectively in corrupt markets. Our framework sheds some light on this issue. As long as the foreign civil servants administering the auction do not have too much discretion (in the sense of being able to provide additional favors to the auction's winner for instance), we show that this fear may not be justified. On the contrary, unilateral controls, by stimulating price competition, will make domestic efficient firms more likely to win the market (though possibly with a lower expected profit than in the case of price collusion).

When foreign civil servants have a high degree of discretion (in evaluation or in implementation), our analysis explains why individual countries may be even more reluctant to impose unilateral controls, since the only effect of such controls would be to exclude their firms from these markets. This discussion suggests that i) anti-corruption controls on firms should be coordinated across countries, and ii) calls for voluntary unilateral controls on firms will be more likely to be followed once bureaucratic discretion in the allocation process and in the course of implementation has been reduced.

The paper is organized as follows. Section 2 describes a benchmark model of first price auction with no discretion. Section 3 introduces corruption and shows how corruption may allow implicit collusion to be sustained in the price auction game. Section 4 considers the situation with unilateral controls on the most efficient firm's bribe capacity. Section 5 investigates other forms of discretion. Section 6 discusses policy implications and section 7 concludes.

2 A simple model with no discretion

There is one contract to be allocated at a price not exceeding \bar{p} . This reservation price has been chosen by the government. We consider n firms $i \in \{1, \dots, n\}$ competing for the contract, and one bureaucrat in charge of allocating the contract. In the first benchmark model we consider, the allocation

⁴One example is the Corrupt Act of 1977 passed by the US Congress.

procedure imposed by the government on the bureaucrat is a first price auction, so that in effect, the bureaucrat has no discretion over the allocation process.

We assume that firm i has a cost c_i of completing the contract. The cost c_i is assumed to be drawn from a distribution with positive and continuous density $f_i(\cdot)$ on $[\underline{c}_i, \bar{c}_i]$. These distributions are assumed to be known to the firms only. For convenience, we will order firms so that

$$\bar{c}_1 \leq \bar{c}_2 \leq \dots \leq \bar{c}_n.$$

We will also let $\underline{c} = \min_i \underline{c}_i$. We wish to emphasize at this stage that, even if the distributions f_i are important in deriving equilibrium behavior, our results will only depend on the bounds \underline{c} and \bar{c}_i and not on the fine details of these densities.

In a first price auction, firms simultaneously submit a bid for the contract. Firm i submits a bid p_i and we denote by p^* the lowest bid:

$$p^* = \min_{i \in \{1, \dots, n\}} p_i.$$

If this price p^* is no larger than the reservation price \bar{p} , the firm i who submits the lowest bid obtains the contract at price p^* and makes a profit equal to

$$p^* - c_i.$$

In case several firms make the same lowest bid, the bureaucrat chooses one of these firms. In accordance to the fact that the bureaucrat may be ignorant about the costs of the firms (and about the distribution from which they are drawn), we shall assume that the bureaucrat chooses each lowest bidding firm with equal probability.

The following proposition illustrates the beneficial effect of competition on efficiency and public spending:

Proposition 1 *Assume that $\bar{c}_1 < \bar{p}$. Then in any Bayesian equilibrium, the first price auction allocates the contract at a price at most equal to $p^* = \min\{\bar{c}_2, \bar{p}\}$.*

Proof. see appendix.

Competition between firms drives prices down to levels that are unrelated to the reservation price, as long as costs are not close to that reservation price.

This is a feature of the allocation process that is particularly important to the government: as long as \bar{p} reflects the government's willingness to pay for the contract, the government secures a surplus at least equal to $\bar{p} - \bar{c}_2$. In particular, this surplus increases as the cost structure moves downward. In addition, the maximal efficiency loss that may arise with this allocation process is bounded above by $\bar{c}_2 - c$.

3 A model with corruption

The previous model assumes that the bureaucrat has no discretion over the allocation process. This precludes corruption: the bureaucrat cannot take bribes in exchange for a favor since there are no favors he can make. In the rest of this paper, we investigate models where the bureaucrat may affect the allocation process (and exchange favors against bribes).

Our first objective is to illustrate how corruption can alter the basic force of competition as described in the previous section. To this end, we start with the following form of discretion: the bureaucrat may offer a firm an opportunity to readjust its bid. More precisely, after having received the (secret) price bids of the firm, we assume that before the official opening of the envelopes, the bureaucrat follows the following procedure:

- 1) He discloses to the firms the value p^* of the lowest bid and asks for a (secret) bribe offer b_i ;
- 2) He allows one firm, say firm i , to submit a new price bid p'_i in exchange for the bribe offered.⁵
- 3) The official prices are (p'_1, \dots, p'_n) , where $p'_k = p_k$ for all k , except possibly for the firm allowed to resubmit. The bureaucrat then selects a firm who has the minimal official price.

We also assume that the bureaucrat is interested in the highest possible bribe, but that he may not accept a bribe level above some threshold \bar{B} , for example because above \bar{B} the probability of detection is too large.⁶ Given these preferences, the bureaucrat will allow a resubmission from the firms (or one of the firms) who made the highest bribe offer below \bar{B} . It will be convenient to denote by b^* that highest bribe offer.

⁵In comment 4 below, we investigate the case where the bureaucrat makes a take-it or leave-it offer to each of the firm, and obtain similar insights.

⁶We will discuss other possible interpretations of \bar{B} in the discussion Section.

In most countries, soliciting new price bids is not allowed. There are empirical evidence however that such manipulations do occur in reality even in developed economies⁷. For example, relying on the annual report (1995) from Conseil de la Concurrence (French Competition Authority), Cartier Bresson (1998) makes a case study of corrupt practices in the allocation of contracts for the construction of the TGV North (the French High Speed train) and describes how resubmissions took place.⁸

Formally, our game has four stages. In the first stage, each firm i sends a price offer p_i to the bureaucrat. In the second stage, the lowest price $p^* = \min_i p_i$ is revealed to the firms, and firms may then send a bribe offer b_i to the bureaucrat (the bribe b_i may depend on p^* , p_i and c_i). In the third stage, the bureaucrat allows one of the firm (one with maximal bribe offer among those below \bar{B}) to resubmit a price. In the last stage, the firm picked by the bureaucrat makes a new price offer; there is an official opening of envelopes and the bureaucrat next chooses a firm among the lowest official price bids.

We are interested in (perfect Bayesian) equilibria where competition collapses in the first round. Formally, we define a *collusive equilibrium* as an equilibrium strategy profile for the firms and the bureaucrat in which the firms choose the same price p in the first round (independently of their actual cost).

Our main result is the following:

Proposition 2 *Assume that $\bar{p} - \bar{c}_1 - \bar{B} > 0$ and $\bar{c}_2 - \underline{c} < \frac{1}{n}[\bar{p} - \underline{c} - \bar{B}]$. Then there exists a collusive equilibrium in which the contract is sold at the reservation price \bar{p} . If in addition, $\bar{c}_n < \bar{p} - \bar{B}$, the expected cost of the firm who gets the contract is at least equal to*

$$\frac{1}{n} \sum_{i=1}^n E c_i$$

In other words, under the condition of Proposition 2, competition loses its force, public spending may rise up to the reservation price, and the expected

⁷Bid manipulations do not actually always require illegal switching of envelopes. In complex contracts, the proposals may leave some ambiguities in the initial price offer thereby opening the way for effective price or quality readjustements of the initial offer.

⁸It is interesting to note that resubmission practices are not always condemned forcefully by politicians, in particular if there is no or limited evidence of corruption associated to it. One reason is that from an ex post point of view, the bureaucrat may be trying to strick a better deal.

cost of the winning firm may increase (compared to the case where there is no corruption) since $\frac{1}{n} \sum_{i=1}^n E c_i$ may be larger than \bar{c}_2 .

The intuition for the result is as follows. In the corruption stage (the second stage), firms compete in bribes. Competition in bribes however stops at \bar{B} , because of the constraint on the level of bribes faced by the bureaucrat. As a result, if firms only compete in bribes (and not in prices) they all make positive expected profits (as long as their cost parameter does not exceed $\bar{p} - \bar{B}$). Still, since firms do not get the contract with probability 1, some might wish to compete in prices in the first round so as to increase the probability that they get the contract. However, there is a high cost to doing so. For small price deviations, competition in bribes leads to ties in bribes (because many firms can afford to propose \bar{B} and still make positive profits), and the deviator need not be picked with a larger probability by the bureaucrat. Thus increasing the probability of winning would require decreasing the price bid to a level where other firms cannot match the price and still make positive profits. And this price level may be so low that each firm prefers to stick to the collusive outcome.

Proof. A strategy for firm i specifies a price p_i (which may depend on c_i) and a bribe b_i (which may depend on p_i, p^* and c_i). A strategy for the bureaucrat specifies a firm i^* that is allowed to resubmit (as a function of the bribe and initial price profiles), and a firm that gets the contract (as a function of the bribe, initial and official price profiles). We test whether submitting \bar{p} in the first stage is part of an equilibrium strategy.

Concerning the strategies of the firms, we propose that i) each firm i submits $p_i = \bar{p}$ in the first round; ii) if the realized minimum price $p^* = \min p_i$ satisfies $p^* - \bar{B} - \bar{c}_2 > 0$, then each firm i submits the maximum bribe below \bar{B} it can afford (that is, $b_i = \bar{B}$ if $p^* - c_i - \bar{B} > 0$, or $b_i = \max\{p^* - c_i, 0\}$ otherwise). iii) if $p^* > \bar{c}_2$ and $p^* \leq \bar{B} + \bar{c}_2$, players keep the same priors concerning firm 1 and 2's costs parameters, and we choose (any) continuation strategies forming a Bayesian equilibrium of the continuation game. iv) if $p^* \leq \bar{c}_2$, we choose any continuation strategies forming a Bayesian equilibrium of the continuation game. v) firm i , if selected by the bureaucrat, submits p^* as its official price.

Concerning the strategy of the bureaucrat, we propose that the bureaucrat selects the firm allowed to resubmit as follows. Let b^* denote the highest bribe offer below \bar{B} . Given the bureaucrat's preferences, it is sufficient to describe which firm he selects in case of ties. In this case, he selects each of them with equal probability unless one of them, say firm i , made a price

offer p_i strictly below the others. In the latter case, he selects firm i with probability $1/n$, and the other(s) with equal probability. Finally, in the last stage, the bureaucrat gives the contract to i^* (the firm selected in the bribe competition stage) if its official price is no larger than p^* , and to any other firm i such that $p_i = p^*$ otherwise.⁹

When firms conform to the above strategy, any firm with cost parameter c such that $\bar{p} - c - \bar{B} \geq 0$ obtains the contract with probability at least equal to $1/n$, hence makes an expected profit at least equal to

$$\frac{1}{n}[\bar{p} - c - \bar{B}].$$

Assume now that firm k deviates and choose a price $p_k < \bar{p}$. Then $p^* = p_k$, and firms next submit bribe offers. We distinguish three cases.

a) $p^* - \bar{c}_2 > \bar{B}$. Under the proposed continuation strategies, both firm 1 and firm 2 submit a bribe offer equal to \bar{B} , and firm k therefore obtain an expected profit at most equal to $\max\{\frac{1}{n}[p^* - c - \bar{B}], 0\}$, which is no larger than the profit he would have obtained by bidding \bar{p} in the first round (thus the deviation is not worthwhile). The proposed continuation strategies are in equilibrium, because any firm who submit bribe offers according to these strategies makes positive expected profits (even strictly positive for firm 1 and 2), and any firm who bids differently makes at most 0 profits (firm i is not selected if $b_i \neq \bar{B}$, and it makes negative expected profits if $b_i = \bar{B}$ and $p^* - c_i < \bar{B}$).

b) $0 < p^* - \bar{c}_2 \leq \bar{B}$. We show in the appendix the following claim

Claim A: Assume that $0 < p^ - \bar{c}_2 \leq \bar{B}$ and let $B^* \equiv p^* - \bar{c}_2$. Then in any continuation equilibrium, with probability 1, at least one firm submits a bribe offer at least equal to B^* .*

The intuition of this claim is that the subgame is a standard first price auction in bribes where the value of the object for firm i is distributed on $[\underline{b}_i, \bar{b}]$, with $\bar{b} = p^* - \underline{c}$ and $\underline{b}_i = p^* - \bar{c}_i$. For a reason similar to proposition 1, the equilibrium bid is at least equal to $\underline{b}_2 = p^* - \bar{c}_2 (= B^*)$.

Given claim A, any firm getting the contract in equilibrium must therefore pay a bribe at least equal to B^* , which yields a profit at most equal to

$$p^* - c - B^* = \bar{c}_2 - c.$$

⁹There are other strategies of the bureaucrat which also allow firms to sustain collusion. We choose this particular one because it facilitates computations.

Under the conditions of proposition 2, we have that $\bar{c}_2 - c < \frac{1}{n}[\bar{p} - c - \bar{B}]$, hence the deviation is not profitable.

c) $p^* \leq \bar{c}_2$. Then the gain from the deviation is at most equal to $\bar{c}_2 - c$, hence it is not profitable. ■

Comment 1: The equilibrium characterized in proposition 2 depicts a case of implicit collusion in prices between firms. The bureaucrat's discretion on the procedure is key because when a firm deviates with a lower price bid, the other firms are given a "second chance" to match the lower price and get the contract, thereby making the deviation not profitable. In other words, the corruption stage of our model serves as an enforcement mechanism for sustaining collusion in price between firms.

Comment 2: There are two others key ingredients in our model: limited competition in bribes and uncertainty about the bureaucrat's choice in case of ties (in bribes or in prices). The conjunction of these two features ensures that firms make positive profits in the collusive equilibrium, and as a result, even efficient firms do not have incentives to compete in prices.¹⁰ In contrast, in situations where the bureaucrat is known to favor a particular firm in case of ties (e.g. a local firm), then the other firms would have incentives to compete in prices (see the next Section for further details).

Comment 3: To capture further the role of *limited* bribe competition, consider the extreme case where bribes are not constrained ($\bar{B} = +\infty$). The next Proposition shows that when there is collusion in prices and competition in bribes, then equilibrium expected profits are identical to those obtained in an equilibrium of the game where the bureaucrat has no discretion (as in Section 2).

Proposition 3 *Assume $\bar{B} = +\infty$. Consider a collusive equilibrium in which the contract is sold at a price $\bar{p} > \bar{c}_2$. Let π denote the vector of expected equilibrium profits. The vector π is also a vector of expected equilibrium profits of the game with no discretion.*

Proof. Consider a collusive equilibrium σ^* in which the contract is sold

¹⁰Our analysis is similar in this respect to that of Mc Afee and Mc Millan (1992).

at price $\mathfrak{p} > \bar{c}_2$.¹¹ We use the strategy profile σ^* to construct a strategy profile for the game without corruption. We consider the strategy for firm i that consists in bidding $p_i = \mathfrak{p} - b_i$ whenever he would have bid b_i on the equilibrium path of σ^* . These strategies generate the same profile of expected profits. Note in particular that no firms make negative profits under this strategy profile (see also footnote 11). Firm i does not have an incentive to deviate to some other $p'_i \leq \mathfrak{p}$ because if this were the case, he would also have an incentive to deviate to $b'_i = \mathfrak{p} - p'_i$ in the game with corruption. He does not have an incentive to deviate to $p'_i > \mathfrak{p}$ either because he would obtain 0 profits by doing so. (Because other firms choose a price no larger than \mathfrak{p}). ■

Comment 4: The corruption stage has been described as an auction mechanism. We would like to emphasize however that other mechanisms would yield a conclusion similar to that of Proposition 2. As an illustration, we replace stage 2 and 3 of our game by the following procedure. The bureaucrat reveals the lowest bid $p^* = \min_i p_i$ to the firms. He then makes (simultaneously and secretly) a take-it-or-leave-it offer to all firms (except the lowest bidder if there is only one such lowest bidder): he offers an opportunity to resubmit a price bid in exchange for a bribe of \bar{B} . In case several firms accept, he selects one of them. We have the following Proposition:

Proposition 4 *Assume that $\bar{c}_2 - \underline{c} + \bar{B} < \frac{1}{n}[\bar{p} - \bar{B} - \underline{c}]$. Then under the alternative corruption stage described above, there exists a collusive equilibrium where the contract is sold at the reservation price \bar{p} .*

Proof. see appendix.

4 Unilateral controls

We have assumed so far that firms had unbounded bribing capacities. We now wish to investigate the case where *one* of the firms, say firm i , is constrained

¹¹Note that in a collusive equilibrium, even a firm i for which $c_i > \mathfrak{p}$ participates and bids \mathfrak{p} in the first round. They do not make negative profits however, because since $\mathfrak{p} > \bar{c}_2$, the highest bribe offer must be strictly positive, hence firm i may avoid getting the contract by making a bribe offer equal to 0 in the bribing stage. Also note that Proposition 3 would also hold if we assumed that in a collusive equilibrium in which the contract is sold at \mathfrak{p} , only firms for which c_i is no larger than \mathfrak{p} participate (or bid seriously).

in its ability to make illegal transfers. It can only pay \bar{b} , and \bar{b} is assumed to be strictly smaller than \bar{B} .

There are various interpretation for this threshold. One of them is unilateral controls: above the bribe level \bar{b} , fines or probability of detection turn out to be very high.¹² This captures the case when firm i comes from a country where the corruption of foreign civil servants is severely prosecuted as for instance in the US. Another interpretation is that the firms' bribe capacity reflects their connections in the host country. A firm that lacks connections(i.e., an outsider) has no effective bribing capacity. It may not know who are the real decision makers, or how to approach them, how to interpret corruption offers etc...

We start with the case of an outsider. We let c^{out} denote the cost for the outsider, and b^{out} his bribe capacity. We assume that c^{out} is drawn from $[\underline{c}, \bar{c}^{out}]$.

Proposition 5 *Assume that $b^{out} = 0$ and $\bar{c}^{out} < \bar{c}_1$. Then in any Bayesian equilibrium of the game, the price p^* at which the contract is sold is below \bar{c}_1 with probability 1.*

Proof. Consider a Bayesian equilibrium. We first show that if the outsider chooses a price $p^{out} > \bar{c}_1$, then he makes 0 profits. Indeed, when the outsider chooses $p^{out} > \bar{c}_1$ then:

i) either $p^{out} > \min \{p_i\}$ and the outsider cannot get the contract because he cannot participate in the corruption game.

ii) Or $p^{out} = \min \{p_i\}$. Consider the continuation game and assume that the outsider gets the contract with probability $q > 0$. Let Q denote the probability that $b_i = 0$ for all $i \neq 1$. We must have $q \leq Q$, since the outsider may only get the object when $b_i = 0$ for all i . By choosing $b_1 = 0$, firm 1 obtains at most $(Q - q)(p^{out} - c_1)$. By choosing $b_1 = \varepsilon > 0$, firm 1 obtains the contract with probability at least equal to Q , hence gets an expected profit at least equal to $Q(p^{out} - c_1 - \varepsilon)$, which is strictly larger than $(Q - q)(p^{out} - c_1)$ when ε is small enough (since $p^{out} > \bar{c}_1$). Hence choosing $b_1 = 0$ cannot be optimal for firm 1, contradicting the hypothesis $q > 0$.

So if $p^{out} > \bar{c}_1$, the outsider cannot get the contract in equilibrium, hence he makes 0 profits. Yet if the outsider bids $p^{out} \in (\bar{c}^{out}, \bar{c}_1)$, he can secure

¹²Alternatively, it could be the case that above \bar{b}_i , the probability of detection is positive and as a consequence, the bureaucrat would not want to accept from firm i a bribe level above \bar{b}_i .

himself a strictly positive expected profit (since with some positive probability no firm can match his bid in the bribe auction). It follows that in equilibrium at least one firm (the outsider) bids below \bar{c}_1 . ■

The intuition as to why competition is restored is that the outsider has no incentives to collude: He always loses when the price is high because he cannot compete in bribes. Therefore he competes in price, which drives down the equilibrium price. An interesting implication of Proposition 5 is the effect of entry on competition in a corrupt environment. The result suggests that in this context, promoting entry (and possibly subsidizing entry) of an outsider with poor connections could turn out to reduce substantially public spending.

We now consider the more general situation where a unilateral control limits firm i 's bribe capacity. We establish the following result:

Proposition 6 *Suppose that firm i has a bribe capacity $\bar{b} < \bar{B}$, and $\underline{c}_i < \bar{c}_1$. Also assume that other firms are not constrained. Then there cannot exist a collusive equilibrium where all firms submit a price that exceeds $\bar{c}_2 + \bar{b}$.*

Proof. see appendix

The intuition is that when the price is too high, firm i always loses in the bribe competition, because it is constrained to a low bribing level. As a consequence, it has an incentive to undercut other firm's offers in the first round. Unilateral controls thus restore competition in price.

It is interesting to note that unilateral controls have the effect of decreasing the controlled firm's expected profit. For example, in the case where $\bar{b} = 0$, Proposition 5 implies that the expected profits of the controlled firm are at most equal to $\bar{c}_2 - c_i$ (which in Proposition 2 was assumed to be strictly smaller than $\frac{1}{n}[\bar{p} - c_i - \bar{B}]$). Our model may thus explain why firms would oppose unilateral controls on their bribing behavior: unilateral controls may force firms to compete in prices in the first stage, which may reduce their expected profits.

This explanation however does not appear to be consistent with the standard motive for opposing unilateral controls. Firms often complain that controls exclude them for competing effectively for some contracts, while in our model, controlled firms may actually obtain contracts with higher probability (so long as they are more efficient, an assumption that complaining firms presumably make).

The following section introduces other forms of discretion, and we will see that the comparative statics with respect to controls may change.

5 Other forms of Discretion

A very common form of discretion is one where the bureaucrat is allowed to choose a firm even if it is not the lowest bidding firm. A common justification for such a practice is that there may be quality concerns over the way the contract will be handled, and that the bureaucrat may better assess the relative quality of each firm's offer.¹³

Our objective is to assess how the previous analysis is altered when the bureaucrat also has the discretion of not choosing the lowest bidding firm (in addition to providing the opportunity to resubmit).¹⁴

We model the new discretion as follows. When price offers are equal to (p_1, \dots, p_n) , the bureaucrat is free to choose any firm i for which $p_i \leq \min_j p_j + \Delta$ (and $p_i \leq \bar{p}$ as before). We say that the bureaucrat administers a "best offer" procedure. We assume that the corrupt bureaucrat follows the following procedure:

- 1) He discloses to the firms the value p^* of the lowest bid sent and asks for a (secret) bribe offer $b_i \leq \bar{B}$;
- 2) He may then allow one firm, say firm i , to submit a new price p'_i in exchange for the bribe offered.
- 3) The official prices are (p'_1, \dots, p'_n) , where $p'_k = p_k$ for all k , except possibly for the firm allowed to resubmit. The bureaucrat then selects a firm whose (possibly adjusted) official price is below $\min_k p'_k + \Delta$.

We first describe collusion possibilities under this extended form of discretion, and we show that unilateral controls on a firm may deteriorate efficiency, because the only effect of such controls may be to exclude that firm from being a winner.

Proposition 7 *Assume that $\Delta - \bar{B} > \bar{c}_3 - \underline{c}_1$ and $\bar{p} - \bar{B} - \bar{c}_3 > 0$. Then*

¹³Quality concerns will not be modelled here however, as we are mainly interested in how this additional discretion affect the previous analysis.

¹⁴This procedure should be distinguish from the procedure that selects the firm that scores best with respect to an ex-ante well-defined objective criteria including quality measures. In the latter case, and as long as the components of the criteria are objectively measurable, we are in a "first price auction" like situation i.e., *ex-post* the bureaucrat has no (additional)discretion.

there exists a collusive equilibrium where the contract is sold at price \bar{p} . And if $\bar{b}_i < \bar{B}$, firm i never gets the contract in equilibrium.

Proof. see appendix.

The intuition for the result is that when the bureaucrat has sufficient discretion in selecting a firm that is not the lowest bidder, the price that would prevent competition in bribes is so low that no firm would make any profit. In such a situation unilateral control on a particular firm only leads to that firm being excluded from the market.

Comment 1: It is easy to see that the result in Proposition 7 also applies to situations where Δ captures the value of favors the bureaucrat can provide to the winner. Such favors may be available for "trade" when the bureaucrat can affect implementation costs by say, awarding modifications in the contract during the course of completion. The result carries over immediately when, in exchange for the bribe \bar{B} , the bureaucrat gives a (promise of) cost reduction equal to Δ to the winning firm.

An interesting implication is that unilateral control are more likely to be ineffective and even hurt efficiency in contexts where the contract sold is complex, as more opportunities to secretly grant cost reducing modifications may then exist. On the opposite, unilateral controls are more likely to boost competition in the context of simple transactions such as the public purchase of standard goods. However, the Russian experience shows that even simple contracts can offer significant opportunities for favoritism in the course of implementation of the contract, as bureaucrats may affect the delay after which firms will be paid.

Comment 2: In order to better understand the role of resubmissions, we wish to investigate the case where resubmissions are not possible, that is, the case where choosing a firm whose bid is not too far above the lowest bid is the only form of discretion available to the bureaucrat. To this end, we replace stage 2 and 3 of the procedure described above by:

2') *The official prices p'_k are identical the initial prices, and the bureaucrat picks a firm who submits the highest bribe offer, among those who offered a price no larger than $\min_k p'_k + \Delta$.*

To simplify the analysis, we also assume that in case of ties in bribe offers, the bureaucrat selects each one of the firms he is allowed to choose from with equal probability. The following proposition shows that corruption at worst induces a bounded mark-up on the competitive price as opposed to a price unconnected with the cost structure as in proposition 2. We have the following result:

Proposition 8 *In any Bayesian equilibrium, the contract is sold at a price not exceeding $\hat{p} = \bar{c}_2 + \text{Max}\{5\Delta - \bar{B}, \Delta\}$*

Proof. see appendix

Proposition 8 implies that when the bureaucrat cannot offer a second chance to submit a price bid, competition in price is reintroduced. The reason is that similarly to the purely competitive case, reasonably small deviations (that is, comparable to Δ) from collusive bidding would be profitable. In contrast to the purely competitive case however, very small deviations from collusive bidding cannot be profitable: firms must reduce their bid by Δ at least in order to affect the probability of winning. Therefore competition is milder. Nevertheless, and to the extent that Δ is not too large, competition must drive the contract price away from \bar{p} .

6 Discussion

In this section, we summarize our theoretical results, discuss the main assumptions of our model and derive some policy implications concerning procedures, controls and entry.

The main insights. The first important insight from our theoretical results is that collusion in price between firms may become feasible when the procedures do not prevent resubmissions. Price competition is then replaced by bribe competition, which is limited by the amount of bribe the bureaucrat can reasonably receive. Controlling (even a few) firms may be very effective in this context. It forces the controlled firms to compete in prices, thereby restoring price competition.

The second important insight is that controls on few firms only may become ineffective when the bureaucrat may offer additional favors. Whether the additional favors affect the allocation process itself (favorable evaluation

of the firm's proposal) or the costs of implementation (later readjustment of the contract in a way favorable to the winning firm), the sole effect of controls may then be to exclude the controlled firms from competing.

Discussion of the main assumptions. Two ingredients appear to be key to our theoretical analysis: the possibility that a firm may be offered an opportunity to *readjust its bid*; the existence of *discretion* that can be abused to bias competition toward a particular subset of firms. We wish to discuss the relevance of these two ingredients in practice, as well as the extent to which existing laws prevent abuses. Our analysis also implicitly assumes that the reservation price \bar{p} is public and high compared to cost levels. We briefly address the empirical relevance of this assumption.

Bid re-adjustments. Manipulation of envelopes is an obvious way by which bids may be re-adjusted. There are easy ways to reduce this risk. One way is to have participants send their offers to two independent institutions which can cross check each other.

There are however many contexts where preventing readjustments of offers is difficult. One such context is complex contracts. Firms would (possibly voluntarily) leave some ambiguity in the description of their proposals. Before the bureaucrat decides upon which firm to allocate the contract, these ambiguities have to be resolved. At this stage, there are large opportunities to modify the initial offer.

Of course, legislation often prevents prices to be modified (although possible contradictions between the detailed price description and the global price may leave some room too). Nevertheless, in contexts where both the price bid and the quality of the offer are taken into account (such as construction contracts) readjustments still can take place. A firm asked to resolve some ambiguity in its proposal might take this opportunity to improve the quality of its proposal and thereby match other bidders offers. Thus, even if a firm were to deviate from a collusive agreement and bid a lower price, the other firms may still be given an opportunity to match the deviator's offer by properly adjusting the quality of their proposal.

The possibility that the bureaucrat discuss with one or several firms so that they may clarify the content of their offer is simply acknowledged by the French Code on Procurement, article 95 bis. Other codes include explicit procedures to be followed when dealing with ambiguities. In the Worldbank's

guidelines for International Competitive Bidding(ICB), all requests of clarifications as well as the bidders' response must be made in writing (art. 2.45). The same concerns with abuses with respect to clarifications can be found in the Russian Code which provides precise rules as to how to deal with arithmetical errors e.g., in cases where the total price does not correspond to the sum of the prices, the later prevails(art. 42, Prikaz n.117 1997 Ministerstvo Ekonomiki).

Sources of discretion: a) *Post-allocation favors*:

Many types of favors may be provided in the course of implementation of the contract. Contract modifications are an obvious candidate. More generally, we have in mind any decision taken by the bureaucrat after the allocation of the contract such that it leads to a significant cost reduction to the winner. Such cost reducing decisions are in principle forbidden by Law, at least when they occur shortly after the allocation of the contract (see for example Article 95 ter, Code des Marchés Publics). However, some cost reducing decisions can be difficult to detect: This includes for example the tightness of the controls concerning the realization of the contract, or the decision to reduce penalties for late completion when 'good' reasons can be found for this.

One remedy might be to separate allocation and implementation, so that the bureaucrat in charge of allocation cannot make any promise concerning implementation. We note however that separation may be undesirable as it prevents having a single bureaucrat accountable for the realization of the public project.

b) *Pre-allocation favors*.

People readily understand that when the evaluation of proposals leaves room for subjectivity, it also implies a risk for corruption. To reduce those risks the following measures have been advocated: i) a clear definition of the criteria to be used by the bureaucrat and the way evaluations according to each criterium are aggregated, ii) the use of objective criteria rather than subjective ones, iii) the use of independent panels of experts when subjective criteria seem necessary, and iv) fair appeal procedures.

It should be noted however that even in contexts where, prior to the auction, the criteria to be used as well as the weight given to each criterium are clearly defined, there still exist some room for corruption. We give several examples.

- The bureaucrat may ex-ante tailor the criteria to a particular firm's (or subset of firms') comparative advantages. Firms who cannot offer bribe will therefore be biased against in competition.

-Favors can also be given by a privately informed bureaucrat. He can choose to share his information with one firm, who will therefore have an edge over the other firms. ¹⁵

- If the firms leave some ambiguities in their proposals, the bureaucrat has the option of not asking them to clarify them. Ex post, if one such firm is selected, it will have the opportunity to take advantage of this lack of precision.

The reservation price. As in Mc Afee and Mc Millan (1992), the reservation price \bar{p} plays an important role in the analysis. We briefly address two assumptions of significance for our results: i) \bar{p} is significantly higher than the lowest cost (otherwise collusion is little disruptive); ii) \bar{p} is public information to the firms.

i) According to the Worldbank's ICB art. 2.61 (and to practice in the French Ministry of Equipement) a public buyer should consider rejecting all bids when the lowest bids significantly exceeds his own cost estimate (in France about +25%) which may leave still a reasonable margin for collusive gains (even if this estimate is actually correct).

ii) One interpretation of the reservation price is that it reflects the political willingness of the government to implement the project. In such a case, when firms have enough knowledge of the local political situation, they presumably have access to this information which may be available at little cost. Still, as noted in Mc Afee and Mc Millan(1992), reservation prices are, in practice, often kept secret. This procedure is sometimes explained as an anti-cartel device. In our context however, given that the agent is corrupt, this should not bother us. Indeed, the agent, has an incentive to reveal the

¹⁵To illustrate, consider the case where the government sells the right to supply two products A and B. Each firm bids a pair of unitary prices. The bureaucrat must select the firm that minimizes the criterium $p_A Q_A + p_B Q_B$, where Q_A and Q_B are estimates of the quantities that the winning firm will need to supply. The winner supplies the two products at unitary prices equal to the bids it made. If now the bureaucrat has more information about the true quantities that will be needed by the government, then he may give this information to a firm in exchange for a bribe. This firm may then use this information to bid more (less) aggressively on the product for which the demand has been relatively over-estimated (under-estimated).

reservation price to facilitate a cartel from which he receives some benefits through corruption.

7 Policy implications

We distinguish three types of policy implications: on procedures, controls, and entry policy.

On procedures: Our theoretical insights suggest that in designing procedures that promote competition, one should pay particular attention to: i) the opportunities for bid re-adjustments; ii) the possible sources of discretion for the bureaucrat (during and after the allocation process).

An obvious consequence is that one should look for procedures that preclude the manipulation of envelopes. The discussion above also suggest that the procedures dealing with ambiguities in the initial offers should receive particular attention. Our analysis supports the adoption of transparent procedures regulating the bureaucrats response to ambiguities in offers, in the spirit of the Worldbank's guidelines for ICB (cf. art. 2.45, 2.49).

On controls: An important outcome of our approach is to contrast the impact of controls on bureaucrats versus controls on firms. As Proposition 3 shows, controls on firms work when they prevent a rather efficient firm from competing in bribes; Then this firm is left with no choice but to compete in prices, thus forcing the others to compete in prices too.

Tighter controls on bureaucrats, in contrast, do not seem to be very effective in our context. Indeed, one interpretation of the threshold \bar{B} is that because of controls, the bureaucrat cannot take the risk of accepting a bribe larger than \bar{B} , or more generally, that the value to the bureaucrat of accepting a bribe b has a maximum at \bar{B} . Then reducing \bar{B} does not appear to reduce the firms ability to collude, but only to enlarge the total profits realized by firms, as the bribe level sufficient to sustain collusion is reduced.¹⁶

¹⁶We would not conclude however that weaker controls would diminish the scope for collusion. It is after all in the interest of the bureaucrat that collusion be sustainable. If asking the largest possible bribe were to destroy collusion (and ultimately the benefits of corruption for the bureaucrat) then the bureaucrat would have an incentives to commit to a reasonably low upper bound on bribes, or build a reputation for not accepting bribes that would be too large. This observation is actually consistent with the fact that in practice, the size of the the bribes seem to be governed by well-established rules rather than static maximization of the highest bribe one can extort.

Entrant and subsidy policies: Another way to alter the auction game and mitigate the impact of corruption is to affect the number and types of participants in the game. From our results (proposition 3), one way to break collusion and corruption is to introduce a low cost entrant whose bribe capacity is low. In other words, this suggests that promoting and even subsidizing the entry of an outsider who lacks connections to the local corruption network, can be quite efficient to secure competition in procurement contracts.

Obviously, a controversial aspect of such a policy is that it can be manipulated by governments for protectionist or mercantilist reasons, impeding thereby the smooth functioning of international trade transactions. One way to mitigate this is to make sure that dispute settlement procedures in the WTO over subsidy policies in international public markets allows for a fair and objective accounting of corruption risks. For instance WTO panels investigating a case can explicitly take into account features like the level of corruption in the host country (e.g., on the basis of Transparency International's ranking lists), and the type of the subsidized firm (new entrant or on the contrary established firm).

8 Conclusion

This paper has investigated some aspects of corruption in the allocation of public markets. We have shown that in an auction context, the issue of corruption is closely linked with that of collusion in price between competing firms. The purpose of this paper has been to shed some light on these links and to discuss the implications for policy making with respect to procedural design and control policies.

Let us summarize our three main insights: 1) Corruption may affect competition because resubmission opportunities given by a corrupt bureaucrat may provide firms with a mechanism to enforce collusion in price. Price competition is then replaced by bribe competition, which is limited by the amount of bribe the bureaucrat can reasonably receive. 2) Controlling (even a few) firms may be very effective in this context, because it forces the controlled firms to compete in prices, thereby restoring price competition; 3) When the bureaucrat may offer additional favors, either during the allocation process (by evaluating favorably the firm's proposal) or during implementation (by readjusting the contract in a way favorable to the winning firm), then controls on few firms only may become ineffective, as the sole effect of these

controls may be to exclude the controlled firms from competing.

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Appendix

Proof of proposition 1 Consider a (Bayesian) equilibrium σ . For any realization (p_1, \dots, p_n) of the price bids, we let $p^* = \min\{p_1, \dots, p_n, \bar{p}\}$. Our aim is to show that for any $\varepsilon > 0$, the probability

$$Q(\varepsilon) \equiv \Pr_{\sigma}\{p^* > \bar{c}_2 + \varepsilon\}$$

is equal to 0. Fix $\varepsilon > 0$ and suppose by contradiction that $Q(\varepsilon) > 0$. Before proceeding we define

$$\bar{p}_i = \sup\{p, \Pr_{\sigma_i}\{p_i > p\} > 0\}$$

The price \bar{p}_i can be interpreted as the highest price chosen by firm i in equilibrium. And we also define

$$Q_i = \lim_{p \nearrow \bar{p}_i} \Pr_{\sigma_i}\{p_i > p\}$$

We proceed in steps.

Step 1: Whatever its own cost c_i , each firm $i \in \{1, 2\}$ must make a profit at least equal to $\varepsilon Q(\varepsilon)$ in equilibrium. Indeed, since firm 1 and 2's costs are bounded by \bar{c}_2 , whatever its costs $c_i \leq \bar{c}_2$, each firm $i \in \{1, 2\}$ may secure an expected profit at least to $\varepsilon Q(\varepsilon)$ by bidding $\bar{c}_2 + \varepsilon$, hence her equilibrium profit is at least equal to $\varepsilon Q(\varepsilon)$ too (otherwise bidding $\bar{c}_2 + \varepsilon$ would be a profitable deviation).

Step 2: We must have $\bar{p}_1 = \bar{p}_2$ and $Q_i > 0$ for $i = 1, 2$.

Indeed, if $\bar{p}_1 < \bar{p}_2$, then firm 2 makes 0 profits when choosing $p > \bar{p}_1$. Since firm 2 is supposed to obtain at least $\varepsilon Q(\varepsilon)$ in equilibrium, the probability that firm 2 chooses a price strictly above \bar{p}_1 in equilibrium must be equal to 0, contradicting $\bar{p}_2 > \bar{p}_1$. By the same argument, we cannot have $\bar{p}_2 < \bar{p}_1$.

Now assume that $\bar{p}_1 = \bar{p}_2$ and $Q_1 = 0$. If firm 2 chooses a price $p \in (\bar{p}_1 - \eta, \bar{p}_2]$ then her expected profit is at most equal

$$[\bar{p}_2 - c] \Pr_{\sigma_1}\{p_1 > \bar{p}_1 - \eta\}.$$

Since $Q_1 = 0$, this profit gets arbitrarily small when η tends to 0, (hence smaller $\varepsilon Q(\varepsilon)$). It follows that there must exist $\eta > 0$ such that firm 2 never bid above $\bar{p}_1 - \eta$, implying that $\bar{p}_2 \leq \bar{p}_1 - \eta$, hence contradicting $\bar{p}_2 = \bar{p}_1$.

Step 3: We cannot have $\bar{p}_1 = \bar{p}_2$ and $Q_i > 0$ for $i = 1, 2$, in equilibrium.

Indeed, either firm 1 or firm 2, say firm 2, would strictly prefers to bid slightly below \bar{p}_1 , contradicting $Q_2 > 0$. ■

Proof of claim A: We show that in any continuation equilibrium, with probability one, at least one firm chooses a bribe offer at least equal to B^* . Choose any $\underline{b} < B^*$. Assume by contradiction that with positive probability, all firms choose a bribe below \underline{b} . Then by choosing a bribe offer b such that $\underline{b} < b < B^*$, any firm with a cost parameter $c \leq \bar{c}_2$ makes an expected profit *bounded away from zero* (since $p^* - b - \bar{c}_2 > 0$). In particular this is true for firm 1 and 2. Let σ_i denote the strategy followed by firm i and define:

$$\underline{b}_i = \inf\{b, \Pr_{\sigma_i}(b_i < b \mid p^*, p_i) > 0\}.$$

That is, \underline{b}_i may be interpreted as the lowest bribe offer made by firm i under σ_i . We also let

$$Q_i = \lim_{b \searrow \underline{b}_i} \Pr_{\sigma_i}(b_i < b \mid p^*, p_i)$$

By an argument similar to the one in proposition 1, we must have $\underline{b}_1 = \underline{b}_2 = \underline{b}$ and $Q_i > 0$, for $i = 1, 2$. (This is because if $\underline{b}_1 < \underline{b}_2$, or if $\underline{b}_1 = \underline{b}_2$ and $Q_2 = 0$, firm 1 would obtain profits close to 0 by choosing a bribe b close to \underline{b}_1 , contradicting the fact that firm 1 must get expected profits bounded away from 0 in the continuation equilibrium). But when $Q_1 > 0$ and $Q_2 > 0$, either firm 1 or firm 2 strictly prefers to make a slightly higher bribe offer, yielding a contradiction. We may thus conclude that with probability one, at least one firm chooses a bribe offer at least equal to \underline{b} . Since this is true for any $\underline{b} < B^*$, we get the desired conclusion. ■

Proof of Proposition 4. The proof follows the steps of Proposition 2. Concerning the strategy of the firms, the only difference is that each firm i may now either accept or reject the proposal of the bureaucrat (to pay \bar{B} in exchange for the right to resubmit). We propose that any firm who can afford to accept accepts. Concerning the strategy of the bureaucrat, he selects one firm from the pool of firms who have accepted (if at least one has accepted); and he selects any firm who has offered the minimum price offer p^* otherwise.

When firms conform to the above strategy profile, any firm with cost parameter c such that $\bar{p} - c - \bar{B} \geq 0$ obtains the contract with probability at least equal to $1/n$ (as in Proposition 2) hence makes an expected profit at least equal to

$$\frac{1}{n}[\bar{p} - c - \bar{B}].$$

Assume now that firm k deviates and choose a price $p_k < \bar{p}$. Then $p^* = p_k$, and the bureaucrat makes a take-it or leave-it to the firms. We distinguish two cases.

a) $p^* - \bar{c}_2 > \bar{B}$. Under the proposed continuation strategies, both firm 1 and firm 2 would accept the bureaucrat's proposal. Firm k therefore obtains a profit equal to 0. The proposed continuation strategies are in equilibrium because as soon as one firm i is willing to accept, any firm who can afford to accept prefers to accept.

b) $p^* - \bar{c}_2 \leq \bar{B}$. Then firm k makes a profit at most equal to $\bar{c}_2 + \bar{B} - c$, which is smaller than $\frac{1}{n}[\bar{p} - c - \bar{B}]$ by assumption.

In either case, the deviation is not profitable. ■

Proof of proposition 6 Assume by contradiction that there exists a collusive equilibrium where all firms submit a price $\bar{p} > \bar{c}_2 + \bar{b}$. Consider the subgame where each firm i has chosen \bar{p} , and assume that in the continuation equilibrium, there is a positive probability that the maximal bribe offer b^* is no larger than \bar{b} . Then both firm 1 and firm 2 would make positive expected profits, whatever their cost parameters $c_1, c_2 \leq \bar{c}_2$ are (because each one could choose a bribe slightly larger than \bar{b}). By the same argument as the one developed in Proposition 2, we obtain a contradiction. So in equilibrium, with probability 1, at least one firm must choose a bribe offer exceeding strictly \bar{b} . Hence firm i would obtain 0 profit in equilibrium.

However, firm i can secure a strictly positive expected profit by choosing p_i such that $c_i < p_i < \bar{c}_1$ (since with positive probability, no firm will then be able to match price p_i). Hence it cannot be that all firms choose contract prices above $\bar{c}_2 + \bar{b}$ with probability 1 in equilibrium. ■

Proof of proposition 7 We follow the steps of Proposition 2 and test whether $p_i = \bar{p}$ for all firm is part of an equilibrium. The strategies we propose for the firm are such that: i) all firms submit \bar{p} in the first round; ii) if the realized minimum price $p^* = \min p_i$ is no smaller than $\underline{c} > 0$, then each firm $j \in \{1, 2, 3\}$ submits the maximum bribe below \bar{B} it is allowed to offer. iii) a firm, if selected, resubmits a price $p'_i = \min\{p_i, p^* + \Delta\}$.

Concerning the strategy of the bureaucrat when selecting a firm allowed to resubmit, we propose the same strategy as the one proposed in the proof of Proposition 2. We also propose that the bureaucrat gives the contract to the selected firm if $p'_i \leq \min\{p_i, p^* + \Delta\}$.

When firms conform to the above strategy profile, firm i (the constrained firm) obtains 0 profits because it cannot match the bribes \bar{B} offered by at least two other firms; Any firm with cost parameter c such that $\bar{p} - c - \bar{B} > 0$ obtains the contract with probability at least equal to $1/(n-1)$, hence makes an expected profit at least equal to

$$\frac{1}{n-1}[\bar{p} - c - \bar{B}].$$

First observe that no firm may obtain strictly positive profits by choosing a price below or equal to \underline{c} . Also observe that in any continuation game where $p^* \in (\underline{c}, \bar{p})$, at least two firms among $\{1, 2, 3\}$ submit a bribe offer equal to \bar{B} (because at most one is constrained). And it is indeed optimal for an unconstrained to do so because it is her only chance to get selected, and in case she is, she obtains a profit at least equal to $\min\{p^* + \Delta, \bar{p}\} - \bar{B} - \bar{c}_3$, which is positive under the conditions of Proposition 7 since $p^* > \underline{c}$.

It follows from the above observations that no deviation of the constrained firm i can generate strictly positive profits (because she cannot match \bar{B}). And since for any $p^* \in (\underline{c}, \bar{p})$, there are always at least two firms offering \bar{B} , any unconstrained firm cannot improve the probability of being selected by deviating. Thus no unconstrained firm has a profitable deviation either. ■

Proof proposition 8 Let \bar{p}_i denote the highest price chosen by firm 1 in equilibrium, that is $\bar{p}_i = \sup\{p, \Pr_\sigma\{p_i > p\} > 0\}$, and let $\mathfrak{p} = \min \bar{p}_i$. We assume that $\mathfrak{p} > \hat{p} = \bar{c}_2 + \text{Max}\{5\Delta - \bar{B}, \Delta\}$ and show that we get a contradiction.

Step 1: We first show that for firm $i = 1, 2$, $\bar{p}_i \leq \mathfrak{p} + \Delta$, which means that \bar{p}_1 cannot differ from \bar{p}_2 by more than Δ . By choosing $p_i = \hat{p} - \Delta$, party i wins the auction in the event where $p_j > \hat{p}$ for all $j \neq i$. By assumption, this event has positive probability, so any firm $i \in \{1, 2\}$ secures strictly positive expected profits (as $\hat{p} - \Delta - \bar{c}_i > 0$). Besides, firms 1 and 2 never choose $p_i > \mathfrak{p} + \Delta$ in equilibrium. Hence this implies:

$$\bar{p}_i \leq \mathfrak{p} + \Delta.$$

Step 2: We derive an upper bound on player i 's equilibrium payoff for $i = 1$ or $i = 2$.

Consider the event $A_i = \{p_k \geq \mathfrak{p} - \Delta, \forall k \neq i\}$. We will show that player i 's equilibrium is bounded above by

$$v_i \equiv \frac{\Pr_\sigma\{A_i\}}{2}[\bar{p}_i - \bar{B} - c_i]$$

for either $i = 1$ or $i = 2$.

We distinguish two cases.

a) $\bar{p}_i > \bar{p}$ for $i = 1$ or $i = 2$, say for $i = 1$. We compute an upper bound on the payoff obtained by player 1 when he chooses $p_1 > \bar{p}$. Since player 1 indeed chooses such prices with positive probability in equilibrium, this will give us an upperbound on player 1's equilibrium payoff.

Consider the event $A_1 = \{p_k \geq \bar{p} - \Delta, \forall k \neq 1\}$. (Under the complement event, player 1 is excluded and makes no profits). Since $p_2 \leq \bar{p} + \Delta$, either $p_2 < p_1 - \Delta$ and party 1 is excluded, or $p_1 - \Delta \leq p_2 \leq p_1 + \Delta$, and both player 1 and 2 have a chance to win under event A_1 . Since choosing \bar{B} would secure each of them strictly positive expected profits, we must have $b_1 = b_2 = \bar{B}$ in the continuation equilibrium, and each party obtains the contract with probability 1/2. Thus when party 1 chooses $p_1 > \bar{p}$, he obtains a payoff at most equal to

$$v_1 \equiv \frac{\Pr_\sigma\{A_1\}}{2}[\bar{p}_1 - \bar{B} - c_1]$$

b) $\bar{p}_1 = \bar{p}_2 = \bar{p}$. We compute an upper bound on the payoff obtained by player 1 when he chooses $p_1 > \bar{p} - \varepsilon$, where $\varepsilon < \Delta$. Clearly, the analysis of case a) above applies because $p_2 \leq \bar{p} \leq p_1 + \Delta$. And the same bound v_1 on player 1's equilibrium value is obtained.

Step 3: We derive a lower bound on player i 's equilibrium payoff, $i = 1, 2$.

When party i chooses $p_i < \bar{p} - 2\Delta$, he is certain to obtain the contract under event A_i . Hence, choosing p_i arbitrarily close to $\bar{p} - 2\Delta$, party i secures an expected profit is at least equal to

$$\Pr\{A_i\}[\bar{p} - 2\Delta - c_i].$$

Finally, in equilibrium, for either $i = 1$ or $i = 2$, this latter expression must be smaller or equal than v_i , implying that

$$2[\bar{p} - 2\Delta - c_i] \leq \bar{p} + \Delta - \bar{B} - c_i$$

or equivalently,

$$\bar{p} \leq c_i + 5\Delta - \bar{B} \leq \hat{p}.$$

contradicting the assumption $\bar{p} > \hat{p}$ ■