On the Pervasiveness of
Home Market Effects∗

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Abstract

Krugman’s (1980) model of trade predicts that the country with the relatively large
number of consumers is the net exporter and hosts a disproportionate share of firms in
the increasing returns sector. He terms these results “home market effects.” This paper
analyzes three additional models featuring increasing returns, firm mobility, and trade costs
to assess the robustness of home market effects to alternative modeling assumptions. We
find strikingly similar results for two of the models that relax assumptions about the nature
of demand, competition, and trade costs. However, a model that links varieties to nations
rather than firms can generate opposite results.

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Does a large home market confer an advantage to the firms that produce there? Krugman’s trade model of monopolistic competition yields two related predictions regarding the effects of market size asymmetries on the geographic distribution of industry activity. First, Krugman (1980) demonstrates that the country with the larger number of consumers of an industry’s goods will run a trade surplus in that industry. Further development of the model in Helpman and Krugman (1985) shows that the larger country’s share of firms in the increasing returns industry exceeds its share of consumers. Helpman and Krugman (1985, p. 209) recognize that their demonstration of these so-called home market effects relies on specific functional form assumptions: Dixit-Stiglitz (1977) preferences, firms that are small relative to the size of the market, and “iceberg” transport costs—but suggest that the results may well have greater generality:

We have been able to work only with a highly specialized example; it is probable, however that “home market effects” of the kind we have illustrated here are actually quite pervasive.

Determining whether home market effects generalize beyond Helpman and Krugman’s “example” is important for three reasons. First, if home market effects are pervasive in models with increasing returns and transport costs, then they can be used as a means to discriminate empirically against alternate models based on constant or decreasing returns. This line of reasoning has been pursued in empirical work by Davis and Weinstein (1998, 1999). Second, as Krugman (1980) shows, imposing balanced trade in equilibrium on industries that would otherwise exhibit home market effects requires the small country to have lower factor prices. This raises the concern that trade liberalization with a larger partner might lower wages in the small country. Finally, as noted in Fujita, Krugman, and Venables (1999, pp. 57-59), the home market effect provides a “building block” for a theory of economic geography. To the extent that workers are
better off in the larger market, there will tend to be a cumulative process of migration leading
to the “core-periphery” pattern.

This paper explores the pervasiveness of home market effects by analyzing models of im-
perfect competition, increasing returns, and firm mobility that offer alternative assumptions on
the nature of demand, transportation costs and competition. The model of Ottaviano, Tabuchi,
and Thisse (forthcoming) maintains the assumption of monopolistic competition but employs
linear demand and per-unit transport costs. In this model, unlike the Helpman-Krugman model,
the FOB price is sensitive to the number and geographic distribution of firms as well as trans-
port costs. The Cournot, segmented markets model analyzed in Brander (1981) relaxes the
assumption maintained in both the Helpman-Krugman and Ottaviano-Tabuchi-Thisse models
of non-strategic firms. In the Brander model, firms producing homogeneous goods choose out-
puts knowing the effect of their actions on the payoffs of competitors. A fourth model retains
Cournot competition but adopts Markusen and Venables’ (1988) specification of linear demands
for varieties differentiated by the nation of production. It employs a parameter measuring the
degree of differentiation that admits the Brander model as a special case.

The Helpman-Krugman model generates simple and unambiguous results. Specifically, a
country’s share of firms in the increasing returns sector is a linear function of its share of con-
sumers with a slope exceeding one. Since symmetric countries will have equal production shares,
this slope implies that the large country will host a disproportionate share of firms. We find
precisely the same result in the Ottaviano-Tabuchi-Thisse and Brander models. Moreover, the
Helpman-Krugman, Ottaviano-Tabuchi-Thisse, and Brander models predict a positive relation-
ship between net exports and the share of consumers in the increasing returns sector. Thus, all
three generate home market effects. The Markusen-Venables model, however, exhibits differ-
ent characteristics. Except for the case of zero differentiation, there is a nonlinear relationship
between shares of firms and consumers. More importantly, when national varieties are poor sub-
stitutes, there are reverse home market effects: the large country hosts a less than proportionate share of the firms and is a net importer in the increasing returns industry.

Two recent papers show that reverse home markets can arise from some departures from the Helpman-Krugman modeling assumptions. Head and Ries (forthcoming) obtain reverse home market effects in a model featuring perfect competition and national product differentiation. Feenstra, Markusen and Rose (forthcoming) develop a Cournot, segmented markets framework with Cobb-Douglas demand curves for homogeneous goods and free entry. They demonstrate a home market effect by starting from symmetric demand and cost conditions and showing that reallocation of demand to one country makes that country become a net exporter. They also show that the result depends crucially on assumptions about entry. If the number of firms is set equal to one in each country, reverse home market effects occur.

Our objective is to provide an integrated derivation of home market effects for different models of imperfect competition with endogenous firm location. The following section lists the common elements of the models and develops a general framework for deriving home market effects in terms of firms’ location decisions. It focuses on the tradeoff between the advantage of locating close to customers and the disadvantage of proximity to competitors. In sections 2, 3, and 4, we derive the equilibrium share of firms and net exports for three models that generate home market effects: Helpman-Krugman, Ottaviano-Tabuchi-Thisse, and Brander. Section 5 expresses the home market effects that arise in the models in terms of figures showing the relation between a country’s share of consumers and its share of firms as well as its trade balance. Section 6 presents the Markusen-Venables model where imperfect competition, increasing returns, and firm mobility are insufficient to guarantee home market effects.
1 A General Framework

We follow the literature in assuming two sectors. The sector of interest is characterized by plant-level fixed costs that give rise to increasing returns to scale (IRS) and imperfect competition. When necessary for clarity, we will refer to it as the IRS sector. The other sector is left in the background. It has constant returns to scale (hence, termed the CRS sector), perfect competition, and zero trade costs.

The purpose of the CRS sector in this literature is to allow for factor price equalization as well as to offset trade imbalances that emerge in the IRS sector. Davis (1998) argues that the assumption of zero trade costs in the CRS sector is not innocuous. Indeed, sufficiently high CRS trade costs can neutralize the home market effect. Our interest here lies in exploring how the assumptions made about the IRS sector affect whether one obtains home market effects or not. Assuming a CRS sector with zero trade costs is useful for that purpose. However, one should recognize that the “incipient” home market effects in the models we analyze may not actually manifest themselves under alternative assumptions about the CRS sector.

We consider a two-stage game where firms in the IRS sector first locate a single plant in one of two countries (indexed $H$ for home and $F$ for foreign) and then choose prices (Helpman-Krugman and Ottaviano-Tabuchi-Thissee) or outputs (Brander and Markusen-Venables).

We employ a common notation in analyzing the three models:

- $M$ is total number of identical consumers, of which an exogenous share $x$ reside in country $H$.

- $N$ is the total number of firms, of which endogenous share $s^*$ locate in country $H$.

- $\tau$ is the trade cost which takes either the iceberg (ad-valorem) or specific (per-unit) form.

- $\omega$ is the constant marginal costs of production and $K$ is the plant-level fixed cost.
• $q_{ij}$ is the amount an individual firm in country $i$ (the origin) sells to each individual consumer in country $j$ (the destination).

We focus on whether the equilibrium share of firms increases disproportionately with the share of consumers, i.e., whether $ds^*/dx$ exceeds one. This is the relationship expressed in Helpman and Krugman (1985). Since symmetry implies that equal sized countries have equal shares of firms, $ds^*/dx > 1$ means that the large country hosts a disproportionate share of firms. Moreover, this condition implies that the large country will run a trade surplus in the increasing returns sector under fairly general conditions. To see this, we express the trade balance (in quantity units) as

$$B = MN[(1 - x)q_{HF} - x(1 - s)q_{FH}].$$

Rearranging, we observe $B > 0$ if and only if

$$\frac{s}{x} \frac{1 - x}{1 - s} > \frac{q_{FH}}{q_{HF}}.$$

Since the large country has a disproportionate share of firms, both fractions on the left side of the $> in the condition above are greater than one. Finally, in the Helpman-Krugman, Ottaviano-Tabuchi-Thisse, and Brander models $q_{FH} < q_{HF}$ when $x > 1/2$. Intuitively, firms export less to markets where there are more competitors. Therefore, the large country runs a trade surplus in the increasing returns sector.

We analyze each model from the perspective of the representative firm’s location decision. First we determine the prospective profits in the two locations as a function of the share of firms, $s$, and the share of demand, $x$. We must begin with the second stage, solving for the prices and quantities as a function of the exogenous parameters and the geographic distribution of firms determined in the first stage ($s$).
This leads to four “individual” profit functions \( \pi_{ij}(s) \) that are defined as the profit an individual firm from country \( i \) earns from selling to an individual consumer in country \( j \). As shown in the appendix, these functions do not depend directly on the distribution of consumers, \( x \). When trade costs take the iceberg form \( \pi_{ij}(s) = (p_{ij}(s)/\tau_{ij} - \omega)q_{ij}(s) \), where \( \tau > 1 \). Otherwise, \( \pi_{ij}(s) = (p_{ij}(s) - \tau_{ij} - \omega)q_{ij}(s) \). The total profits a representative firm in each location would earn are given by

\[
\Pi_H(s, x) = M[x\pi_{HH} + (1 - x)\pi_{HF}] - K,
\]

\[
\Pi_F(s, x) = M[x\pi_{FH} + (1 - x)\pi_{FF}] - K.
\]

We assume that plant-level fixed costs in the IRS sector, \( K \), are high enough to ensure that each firm chooses to produce in only one of the two markets. Without this assumption, firms could serve each market with a local plant and the relative size of the two markets would not affect the distribution of plants. Furthermore it is precisely the assumption of sizeable fixed costs that identifies this sector as the one with increasing returns.

Define \( G(s, x) = \Pi_H(s, x) - \Pi_F(s, x) \) as the gain in profits from relocating to country \( H \). We use the locational equilibrium concept that \( s^* \) is an equilibrium if no individual firm can raise its profits by relocating. Thus, \( G(s^*, x) = 0 \). Now totally differentiate this expression with respect to \( s \) and \( x \). We have

\[
dG = \frac{\partial G}{\partial x} dx + \frac{\partial G}{\partial s} ds^* = 0.
\]

This can be solved to obtain the slope of the implicitly defined share function which we shall refer to as \( h \):

\[
h \equiv \frac{ds^*}{dx} = \frac{\frac{\partial G}{\partial x}}{\frac{\partial G}{\partial s}}.
\]

We will refer to \( \frac{\partial G}{\partial x} \) as the “demand effect” and denote it \( d \) and refer to \( \frac{\partial G}{\partial s} \) as the “competition” effect and denote it \( c \). Then we obtain the most important expression in the paper: \( ds^*/dx > 1 \)
if and only if the sum of the demand and competition effects is positive, i.e. $d + c > 0$.

Since the prices do not depend directly on the distribution of demand, (i.e. $\frac{\partial p_{ij}}{\partial x} = 0$ in all the models we consider),

$$d \equiv \frac{\partial G}{\partial x} = M\left[(\pi_{HH} - \pi_{FH}) + (\pi_{FF} - \pi_{HF})\right].$$

As long as there are trade costs, there will be a wedge between what a firm earns from local sales and what it earns from exporting. Hence, $\pi_{HH} > \pi_{FH}$ and $\pi_{FF} > \pi_{HF}$, leading to a positive demand effect, $d > 0$. Intuitively, as long as trade costs limit the access of exports to the market and give local producers an advantage, increasing the size of the home market will make it more attractive, other things equal, for firms to choose that location.

Turning to the competition effect, a change in $s$ will affect profits through both direct effects and induced changes in the action variables (prices or quantities). Denoting derivatives of the maximized profit functions, $\pi_{ij}(s)$, with respect to $s$ as $\pi'_{ij}$, we can represent the “competition effect” as

$$c \equiv \frac{\partial G}{\partial s} = M \left[ x (\pi'_{HH} - \pi'_{FH}) + (1 - x) (\pi'_{HF} - \pi'_{FF}) \right].$$

We expect each of the two terms in parentheses above to be negative. The intuition behind this is that when a firm moves from $F$ to $H$ it lowers $\pi_{HH}$ and $\pi_{FH}$ because the $H$ market now has more local suppliers. Correspondingly $\pi_{FF}$ and $\pi_{HF}$ rise because the $F$ market now has fewer local sellers. This suggests that the sign of each difference is ambiguous. However, since local firms have higher profits on their local sales, we expect them to incur the greater losses from competition. Hence the $\pi'_{HH}$ and $\pi'_{FF}$ should dominate the terms they are paired with above. Indeed, the competition effect is negative in each of the cases we analyze.

The existence of home market effects hinges on whether the positive demand effect is large
enough to offset the negative competition effect. As we shall illustrate at the end of the paper, this will not always be the case. However, we show in the next sections that the Helpman-Krugman, Ottaviano-Tabuchi-Thisse, and Brander models do in fact predict that $h = d/(-c) > 1$. Indeed for all three of these models $h$ does not depend on $x$, i.e. the share function is linear in the distribution of consumers:

$$s^* = g + hx. \quad (1)$$

Given the symmetry in preferences and costs that we assume, $s^* = 1/2$ when $x = 1/2$. Therefore when $h > 1$, it must also be that $g < 0$. Thus, an important corollary of the slope exceeding one is that there will be a critical level of the share of demand that, if exceeded, causes all firms to concentrate in one country. Specifically, all firms locate in H when $x \geq (1 - g)/h$ whereas all firms will locate in F when $x \leq -g/h$. For those ranges, the slope of the equilibrium share equation is zero.

The following three sections express the linear share function for the Helpman-Krugman, Ottaviano-Tabuchi-Thisse and Brander models and show that the slope, $h$, exceeds one. For each of these models, we also show that the trade balance is monotonically increasing in a country’s share of demand consumers for interior values of $s$. For brevity and clarity, these sections exclude most of the computations that generate the equations. The appendix provides the full set of equilibrium prices and outputs for each of these models.

2 Monopolistic Competition with CES Demand

We begin with a model derived from the widely used Dixit and Stiglitz (1977) monopolistic competition framework, applied by Krugman (1980) to international trade. Our treatment follows that of Helpman and Krugman (1985) except that we obtain our solution by equating profits in the two locations rather than assuming that free-entry sets profits equal to zero in
both countries.

Each of the identical $M$ consumers has an expenditure on the differentiated good normalized as 1. Consumer preferences exhibit a constant elasticity of substitution, $\sigma$, between varieties. The individual demand functions from countries $F$ and $H$ for a representative variety produced in each country are given by:

$$q_{ij} = \frac{p_{ij}^{-\sigma}}{P_j^{-\sigma}} \tau_{ij}$$

where $P_j \equiv \frac{N}{1/(1-\sigma)} \left[ sp_{Hj}^{1-\sigma} + (1-s)p_{Fj}^{1-\sigma} \right]^{1/(1-\sigma)}$. (2)

The $p_{ij}$ are delivered prices to consumers in $j$ for varieties produced in $i$ and $P_j$ is the price index for market $j$. Cross-border trade entails an “iceberg” transport cost of $\tau$. For each unit consumed, the consumer must order $\tau > 1$ units since a share $\tau - 1$ of the units “melt” en route. However, $\tau_{ij} = 1$ for $i = j$.

In monopolistic competition models, the firm maximizes profits taking the price indexes, $P_F$ and $P_H$, as given. Solving for optimal prices and making substitutions back into the profit equation we obtain the difference in profits as

$$G = \frac{M}{\sigma N} \left[ \frac{s(\rho - 1) - \rho + x(\rho + 1)}{s(1-\rho)(1-s) + \frac{\rho}{1-\rho}} \right],$$

where $\rho \equiv \tau^{1-\sigma} < 1$. The competition effect, $c$, and demand effect, $d$, are shown below:

$$d = \frac{M(1+\rho)}{\sigma N \left[ s(1-\rho)(1-s) + \frac{\rho}{1-\rho} \right]} > 0, \quad c = -\frac{M(1-\rho)^2}{\sigma N \left[ s(1-\rho) + \rho \right]^2} + \frac{(1-x)}{[s(\rho - 1) + 1]^2} < 0.$$  

Thus, firms prefer to locate where there are few competitors and many consumers. Setting the difference in profits to zero in order to find the location equilibrium of the game, we obtain

$$s^* = -\frac{\rho}{1-\rho} + \frac{1+\rho}{1-\rho} x.$$

(4)
Since $0 < \rho < 1$, $h = (1 + \rho)/(1 - \rho) > 1$ and $g < 0$. Denote $p = \sigma \omega/(\sigma - 1)$ as the mill price. Then we can express the trade balance as

$$B = \frac{M}{p} [s - x] = \frac{M}{p} \left[ \frac{\rho(2x - 1)}{1 - \rho} \right].$$

Net exports are therefore a linear function of $x$, positive for $x > 1/2$, and negative for $x < 1/2$. The derivative of the trade balance with respect to $x$ (taking into account induced changes in $s^*$) can be expressed as

$$\frac{dB}{dx} = \left( \frac{M}{p} \right) \left( \frac{ds^*}{dx} - 1 \right).$$

For interior equilibria, $\frac{ds^*}{dx} > 1$ and $\frac{dB}{dx} > 0$. When all firms are located in a single country ($s = 0$ or $s = 1$), $ds^*/dx = 0$ and the derivative $\frac{dB}{dx}$ is negative. When production is totally concentrated in the large country, trade occurs in a single direction. A reallocation of consumers to the large country reduces exports, resulting in decreases of the trade balance.

Helpman and Krugman’s derivation of home market effects employs a number of restrictive assumptions:

- Preferences exhibit a constant elasticity of substitution (CES) between varieties.
- Each variety is made by a unique firm.
- Firms are so small that they disregard the influence of their actions on their competitors. This assumption, combined with the above two, results in prices that are a fixed markup over marginal costs. Thus, they do not depend on the proximity of competitors.
- Trade costs take the iceberg form: Only a fraction of the goods exported actually arrive in the destination market.

Fujita, Krugman and Venables (1999) refer to these “peculiar assumptions of the Dixit-
Stiglitz model” as “modeling tricks” necessary to “respect the effects of increasing returns at the level of the firm without getting bogged down in them” (p. 6). Later in the book (p. 45) they continue, “Dixit-Stiglitz monopolistic competition is grossly unrealistic, but it is tractable and flexible; as we will see, it leads to very special but very suggestive set of results.” We will now investigate the extent that Helpman and Krugman’s results depend on “special” assumptions. In the next section we abandon the CES iceberg assumptions but retain assumptions 2 and 3 above. Then we will work with the Brander model which removes all four assumptions.

With CES demand, there will always be intra-industry trade if there are firms in both countries ($0 < s < 1$). To establish home market effects in models with linear demand curves, we assume that trade costs are low enough such that consumers in each country purchase from firms located in the other country. In our notation this means that for any interior distribution of firms, the $q_{ij}(s)$ in the appendix exceed zero, i.e., there is two-way trade in the IRS industry. Following terminology in the spatial competition literature (Anderson, de Palma and Thisse, 1992, p. 334), we refer to this as the overlapping markets condition.

3 Monopolistic Competition with Linear Demand

The model of monopolistic competition presented by Ottaviano, Tabuchi, and Thisse (forthcoming) builds on a different specification of utility (quasi-linear with quadratic subutility) that yields individual linear demand functions. As shown in the appendix, we can choose units for prices and quantities so as to reduce the set of parameters in the individual demand curve to just $\theta$, a measure of substitutability between varieties analogous to $\sigma$ in the Helpman-Krugman model. Individual demand curves are given by

$$q_{ij} = 1 - (1 + \theta N)p_{ij} + \theta P_j,$$

where $P_j = N[s p_{Hj} + (1 - s)p_{Fj}]$. 

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Ottaviano et al. replace the iceberg assumption with constant per-unit transport costs (that we continue to denote as \( \tau \), however now \( \tau < 1 \)). As with the Dixit-Stiglitz model of monopolistic competition, firms choose prices to maximize their profits while neglecting the effect of individual price changes on the price index \( P_j \). Unlike the Helpman-Krugman model, it matters in the Ottaviano-Tabuchi-Thisse framework whether price discrimination is permitted. We work here with the model in which firms can set different prices for each market. The resulting prices in this model, in contrast to Dixit-Stiglitz, have the desirable feature that they are affected by the number of firms and their location choices.

After solving for prices and quantities, the difference in profits equation is given by

\[
G = \frac{(1 + \theta N)\tau M}{2(2 + \theta N)} \left[ -\tau \theta N s + (2(1 - \omega) - \tau)2x - 2 + \frac{\tau}{2(2 + \theta N) + 2\omega} \right].
\] (5)

The competition effect is clearly negative, i.e. \( c = \frac{\partial G}{\partial s} < 0 \).

For the location equilibrium, we obtain

\[
s^* = \frac{2(1 - \omega) - (2 + \theta N)\tau/2}{\tau \theta N} + \frac{2(2(1 - \omega) - \tau)}{\tau \theta N} x.
\] (6)

In order for the export price to cover transport costs and marginal costs, it must be that \( \tau(2 + \theta N) < 2(1 - \omega) \). Thus, the overlapping markets condition is sufficient to guarantee \( h > 1 \) and \( g < 0 \). Indeed, the condition is sufficient to set \( h > 4 \).

Net exports are

\[
B = \frac{NM(1 + \theta N)}{2(2 + \theta N)} [2(1 - \omega - \tau)(s - x) + s(1 - s)N\theta \tau(2x - 1)].
\]

\(^1\)The firms in Helpman-Krugman perceive the same elasticity of demand in each market and set export prices (net of transport costs) equal to their domestic prices.
Note that \( B = 0 \) if \( x = 1/2 \); trade is balanced when countries are of equal size. When the majority of consumers is located in H, we know from the derivations above that the share of firms in H exceeds the share of consumers \( (s^* > x) \). Hence all terms are positive and the large country is a net exporter of the product. Conversely, when \( x < 1/2 \), we have in equilibrium \( s^* < x \) and thus H is a net importer of the good when it has a smaller share of consumers than F.

Taking derivatives with respect to \( x \) yields

\[
\frac{dB}{dx} = \frac{NM(1 + \theta N)}{2(2 + \theta N)} \left\{ 2(1 - \omega - \tau) \left( \frac{ds^*}{dx} - 1 \right) + N\theta \tau \left[ 2s^*(1 - s^*) - \frac{ds}{dx} (1 - 2s^*)(1 - 2x) \right] \right\}.
\]

When firms concentrate in one country \( (s^* = 1 \text{ or } s^* = 0) \) \( ds^*/dx = 0 \) and the derivative is negative. For interior values of \( s \), \( ds^*/dx = h > 1 \) and the only negative term in the expression is \( -h(1 - 2s^*)(1 - 2x) \). To sign the derivative for interior values of \( s \), first consider values in the range \( 1/2 \leq x \leq (1 - g)/h \). The term \( (1 - 2s^*)(1 - 2x) \) is uniformly increasing in both \( s^* \) and \( x \). Note also that the only other term in the expression that is a function of \( x \), \( s^*(1 - s^*) \), is at its lowest value (zero) when \( s^* = 1 \). Thus, if the derivative is positive when \( s^* \) reaches 1, it will be positive for all \( x \geq 1/2 \) for all interior equilibria. We therefore substitute \( x = (1 - g)/h \) into the preceding equation where \( s^* = 1 \) to obtain

\[
\frac{dB}{dx} = \frac{NM(1 + \theta N)}{2(2 + \theta N)} [2(1 - \omega - \tau)(h - 1) + N\theta \tau(h + 2g - 2)].
\]

Equation (6) implies that \( h + 2g = 1 \), yielding

\[
\frac{dB}{dx} = \frac{NM(1 + \theta N)}{2(2 + \theta N)} [2(1 - \omega - \tau)(h - 1) - N\theta \tau].
\]

The overlapping markets condition, \( \tau \theta N < (1 - \omega - \tau) \), implies \( h > 4 \) and establishes that
net exports are monotonically increasing in \( x \) for \( 1/2 \leq x \leq (1 - g)/h \). In our two-country model, the large country’s trade surplus is the small country’s trade deficit. This implies that the derivative is also positive for \(-g/h \leq x \leq 1/2\).

The analysis in this section shows that the assumptions of CES preferences and iceberg transport costs are not important in generating home market effects. We now make a more radical change in assumptions: we abandon monopolistic competition and its assumptions of differentiated products and firms that believe they are too small to affect the market price indexes.

4 Cournot Oligopoly with Homogeneous Goods

We now examine the oligopoly model introduced by Brander (1981). Unlike the monopolistic competition models considered previously, firms in the Brander model recognize in their maximization problems the impact of their actions on market prices. We assume that each of \( M \) identical consumers have individual demand curves given by \( 1 - P^{2} \). This implies inverse demand curves of

\[
P_j = 1 - Q_j = 1 - N(sq_{Hj} + (1 - s)q_{Fj}),
\]

where \( Q_j \) is the total quantity sold to an individual consumer in country \( j \) consisting of quantities produced by identical firms located in country H \( (q_{Hi}) \) and country F \( (q_{Hi}) \). The profit equations are the same as in the previous model. We also allow for price discrimination in the sense that firms choose amounts to ship to each market independently and therefore the export price (net of transport costs) need not equal the price charged to the domestic consumers. This segmented markets assumption is necessary to obtain overlapping markets in the homogeneous

\[\text{As detailed in the appendix, with the appropriate choice of units for prices and quantities, this can represent any linear demand function.}\]
goods Cournot model. After solving for equilibrium quantities and prices (provided in the appendix), the difference in profits can be expressed as

$$G = \frac{2M\tau}{N+1} \{ -N\tau s + 2[1 - \omega - \tau/2]x - [1 - \omega - (N+1)\tau/2] \}.$$  (7)

As with Ottaviano-Tabuchi-Thisse’s model, the competition effect under Cournot is negative and proportional to $\tau^2$. The higher are transport costs, the more important it is to avoid locating near one’s competitors.

Setting equation (7) equal to zero and solving for an interior $s$ yields

$$s^* = \frac{1 - \omega - (N+1)\tau/2}{N\tau} + \frac{2(1 - \omega - \tau/2)}{N\tau} x.$$  (8)

A home market effect, $h > 1$ and $g < 0$, will obtain whenever the overlapping market condition, $\tau(N+1) < (1 - \omega)$, holds. Indeed, that condition is sufficient to set $h > 2$.

A common feature of each of the models presented so far is that the slope of the share equation flattens as transport costs rise. In the Ottaviano-Tabuchi-Thisse and Brander models, an increase in the number of firms also flattens the slope of the share equation (the slope is independent of $N$ in the case of Helpman-Krugman). Together, these last two observations suggest that increases in trade barriers and competition dampen home market effects.

The equilibrium balance of trade is given by

$$B = \frac{NM}{N+1}[(1 - \omega - \tau)(s^* - x) + s^*(1 - s^*)N\tau(2x - 1)].$$

Again, as in the other models, trade is balanced when $x = 1/2$. Market size asymmetries result in the large country being a net exporter of the industry’s goods. The derivative of the net
export equation is

$$\frac{dB}{dx} = \frac{NM}{N+1} \left\{ \left( 1 - \omega - \tau \right) \left( \frac{ds^*}{dx} - 1 \right) + N\tau \left[ 2s^*(1 - s^*) - \frac{ds^*}{dx} (1 - 2s^*) (1 - 2x) \right] \right\}.$$  

As before, the slope is negative when $s^* = 1$ or $s^* = 0$. To sign the derivative for interior values of $s^*$ we following the approach we employed in investigating this derivative in the Ottaviano-Tabuchi-Thisse model. Namely, we evaluate the expression at $x = (1 - g)/h$, the value of $x$ where the derivative is smallest for $1/2 \leq x \leq (1 - g)/h$. This yields

$$\frac{dB}{dx} = \frac{NM}{N+1} \left[ (1 - \omega - \tau)(h - 1) + N\tau(h + 2g - 2) \right].$$

Equation (8) gives $h + 2g = 1$, and thus

$$\frac{dB}{dx} = \frac{NM}{N+1} \left[ (1 - \omega - \tau)(h - 1) - N\tau \right]$$

The overlapping markets condition, $\tau(N+1) < (1-\omega)$, yields $h > 2$ and is sufficient to establish that the derivative is positive. Thus, we demonstrate that net exports are uniformly increasing in $x$ for $(1 - g)/h \geq x \geq 1/2$. As is the case for the Ottaviano-Tabuchi-Thisse model, symmetry implies that the derivative is also positive for $-g/h \leq x \leq 1/2$.

5 Unifying Figures

In this section, we present graphs of the share equation and trade balance equation for the Ottaviano-Tabuchi-Thisse and Brander models. To do so, we select parameter values of $M = 1$, $\omega = 0.4$, $\tau = 0.1$, $N = 5$ and $\theta = .5$. These settings make the overlapping market condition bind at $s = 0$ and $s = 1$. As a result, they lead to the smallest home market effect $h$ that is
consistent with overlapping markets. We omit the Helpman-Krugman relationships because of the problem of selecting comparable parameter values. The shapes of the Helpman-Krugman equations resemble the plots of the Ottaviano-Tabuchi-Thisse model.

All three models have in common the feature that the share of firms is a linear function of the share of demand with a slope greater than one and a negative intercept. Since there cannot be negative shares or shares greater than one, this implies that globally $s$ is a piecewise linear function of $x$:

$$
s = \begin{cases} 
0 & \text{if } x < -\frac{g}{h} \\
1 & \text{if } x > \frac{(1 - g)/h}{\text{otherwise}} \\
g + hx & \text{otherwise}
\end{cases} \quad (9)
$$

Figure 1 plots the relationship between the home country’s share of firms and its share of consumers for the Brander and Ottaviano-Tabuchi-Thisse models. The 45-degree line in Figure 1 indicates the values for which the distribution of firms mimics the distribution of
consumers. The piecewise linear relationship is apparent as is the result that there are ranges of high and low values of $x$ where firms completely concentrate in a single country. As can be seen in Figure 1, the home market effect in Ottaviano-Tabuchi-Thisse is much more pronounced than that in Brander (a slope of 8.8 versus 2.2). This is related to the trade-off between demand and competition effects discussed in the general framework section. Competition is fiercer in the Brander model because firms produce identical products. This results in a much lower coefficient $h$.

Figure 2 plots the trade balance against the share of consumers in country H. The upward sloping sections of the lines represent ranges of $x$ where, in equilibrium, firms locate in both countries. This demonstrates the home market effect in terms of the relationship between net exports and country size. When production concentrates completely in the large country, the

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3Recall that in our representation of the demand systems of the Ottaviano-Tabuchi-Thisse and Brander models, we chose price and output units to normalize coefficients to one. When plotting the trade balance function, we adjust units in the Brander model to make the two models comparable.
slope is negative. As described previously, in this situation trade occurs in a single direction and shifting consumers to the large country reduces its exports.

6 Cournot Oligopoly with National Product Differentiation

Having shown the striking degree of similarity of the first three models, we now turn to a model developed by Markusen and Venables (1988) that can produce very different results. First, for some parameter values, the model predicts reverse home market effect even though it retains some the main features of the earlier models: imperfect competition, endogenous firm location, and transport costs. Second, the tractability of the previous models that yielded a linear share function disappears in this model.

The Markusen-Venables specification removes the Helpman-Krugman model assumption that links varieties to firms. Instead, we assume that products are differentiated according to nations, an idea often referred to as the Armington (1969) assumption. In this model, a firm’s choice of location determines the variety that it sells. This assumption is reasonable if the characteristic of the good depends on an immobile factor of production. For example, a wine producer in Germany or France must produce different varieties due to the differing climate and soil conditions in each country. We will also assume linear demand and Cournot competition with segmented markets, as considered in Markusen and Venables (1988). By choice of units, we reduce the demand curves to the following equations:

\[ P_{ij} = 1 - Q_{ii} - \nu Q_{ji}. \]

When \( \nu = 0 \), product differentiation is so large that the demands for varieties \( H \) and \( F \) are independent. When \( \nu = 1 \), products are homogeneous and thus the Markusen-Venables model collapses to the Brander model. Section 4 establishes that home market effects obtain for this
extreme value of $\nu$. We now show that reverse HMEs can result from smaller values of the product differentiation parameter.

The difference in profits equation is

$$G(s, x) = M[x(q_{HH}(s)^2 - q_{FH}(s)^2) + (1 - x)(q_{HF}(s)^2 - q_{FF}(s)^2)],$$

where the $q_{ij}(s)$ are provided in the appendix. Setting this equation equal to zero and solving for equilibrium $s^*$ gives the share equation as in the other spatial competition models analyzed in sections 2, 3, and 4. While the share equation turns out to be linear in those three models, it is nonlinear in the Markusen-Venables model and too unwieldy to reproduce here. We therefore move directly to the somewhat simpler equation of the derivative of the equilibrium share of firms with respect to share of consumers in country $H$. This equation is presented with the marginal cost of production ($\omega$) set to 0, in order to abbreviate the expression:

$$\frac{ds^*}{dx} = \frac{(2 - \tau)\tau \left[(\nu^2 - 1)[2s^*(s^* - 1) + 1]N^2 - 2(N + 1)\right]^2}{2N [(\nu^2 - 1)s^*(s^* - 1)N^2 + N + 1] \left[\left((\tau - 1)^2 + 1\right)\left[2 + N(\nu^2 + 1)\right] + 4(\tau - 1)(N + 1)\nu\right]}.$$  \hspace{1cm} (10)

When $\nu = 1$, the Brander model obtains where $\frac{ds^*}{dx} = (2 - \tau)/N\tau > 1$. However, the introduction of national product differentiation can strengthen competition effects enough to outweigh the positive demand effect. When a firm moves from $F$ to $H$, not only does it represent an additional competitor in country $F$ but, due to national product differentiation, it also switches from being an imperfect competitor to being a perfect competitor for firms in country $H$. This tends to increase the disincentives for additional firms to move to country $H$. This additional influence on the competition effect is highest when products exhibit maximal differentiation, i.e. when demands for the two varieties are independent ($\nu = 0$). We can study this other polar case by

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4The Maple file containing the share equation and its derivation is available at [http://economics.ca/keith/lab.mws](http://economics.ca/keith/lab.mws).
setting $\nu = 0$ in (10) and evaluating $ds^*/dx$ around the symmetry point of $x = s^* = 1/2$:

$$h \equiv \frac{ds^*}{dx} = \frac{(N + 2)\tau(2 - \tau)}{2N[(\tau - 1)^2 + 1]}.$$ 

In this case, a reverse home market effect obtains, $h < 1$, when

$$N > \frac{2\tau(2 - \tau)}{3(\tau - 1)^2 + 1}. \quad (11)$$

The expressions for the equilibrium quantities reveal that $\tau$ has to be strictly less than one for trade to occur. This implies that the right-hand side of this equation is an increasing function of $\tau$, obtaining its highest value when $\tau$ is at its maximal value of just below one. When $\tau = 1$, the right-hand side equals 2. Thus, when $\tau$ is strictly less than one, the critical value of $N$ for $h < 1$ is strictly below 2. Since there must be at least two firms in the model, we have found a case of a reverse home market effect. There is a reverse home market effect in the Markusen-Venables model around the symmetry point when product differentiation is maximal.

We have shown that for one extreme value of product differentiation ($\nu = 1$), the model yields a home market effect, while the other extreme value ($\nu = 0$) it generates a reverse home market effect around the symmetry point $x = 1/2$. We now show graphically what happens for intermediate values of $\nu$ between zero and one.

Figure 3 depicts a graph displaying the share of consumers choosing to locate in country H, $(x)$, and the associated equilibrium share of firms located in H, $(s^*)$\textsuperscript{5}. The figure shows that the two polar cases yield opposite results in terms of the home market effect, with a slope greater than one for $\nu = 1$ and less than one when $\nu = 0$. As $\nu$ rises (i.e., and the goods become closer substitutes), $h$ rise from less than one to greater than one. We also provide a figure portraying

\textsuperscript{5}We use the same parameters as Figure 1.
predictions of the model for how the trade balance varies with a country’s share of consumers. Figure 4 indicates that the large country is a net exporter when $\nu = 1$ and a net importer when $\nu = 0$ and that these balances are lowered for intermediate values of $\nu$.

The Markusen-Venables model indicates that imperfect competition, transport costs, and firm mobility, common ingredients to the widely used models presented in the previous sections, are not sufficient to yield a home market effect. Rather, a model with increasing returns and national product differentiation can yield a less then proportional equilibrium relation between shares of activity and demand hosted in a country. This finding has important implications for the recent empirical literature trying to use the existence of HME as a way to discriminate between CRS and IRS industries. Our results show that IRS industries can exhibit reverse home market effects. Hence, it appears that IRS is a necessary but not sufficient condition for HMEs.
7 Conclusion

The analysis shows that three alternative models of imperfect competition yield remarkably similar predictions regarding the effects of market size asymmetries on a country’s share of firms and its net exports in an increasing returns industry. These effects are known as home market effects. Using a location choice framework, we argue that home market effects emerge when the positive demand effect from locating in the larger of two markets overwhelms the negative competition effect of having more firms nearby.

We show that several assumptions that Helpman and Krugman justified on the grounds of tractability rather than realism are not necessary conditions for their results. First, product differentiation is not required since the homogeneous goods Brander model exhibits home market effects. Second, we show that the result is also robust to relaxing the assumption that transport costs take the iceberg form. Finally, we find that home market effects do not hinge on the
Dixit-Stiglitz model’s lack of price responsiveness to the proximity of competitors.

We find, however, that the Markusen-Venables model in which varieties are linked to nations—rather than firms—can yield reverse home market effects. This result is consistent with those found in Head and Ries (forthcoming) and Feenstra, Markusen, and Rose (forthcoming) who also consider varieties tied to nations. Unlike these earlier papers, however, the Markusen-Venables model analyzed here maintains the Helpman-Krugman assumptions of imperfect competition with an endogenous number of firms in each location.

The home market effects found by Helpman and Krugman are surprisingly pervasive, given the restrictive assumptions they employed. For two other important models of trade with imperfect competition—Brander and Ottaviano-Tabuchi-Thisse—the results hold. Moreover, even the tractability of the Helpman-Krugman model persists. However, they are not common to every model featuring increasing returns, imperfect competition, and trade costs. Armington-type assumptions can cause their reversal even in models with firm mobility. Whether or not increasing returns industries exhibit home market effects appears to depend on whether varieties are linked to firms or the nation of production.

Appendix: Preferences and Equilibrium Quantities

This appendix lists utility functions and reduced-form equilibrium output and price equations for each model.

**Helpman-Krugman**

Helpman and Krugman’s model assumes that the representative consumer has a utility function of

\[ U = A^{1-\mu} \left( \int_{k=0}^{N} D(k) \frac{\sigma-1}{\sigma} dk \right)^{\frac{\mu}{\sigma-1}} \]

Maximization subject to an income of \( y \) results in the consumer spending \( \mu y \) on varieties \( k = 1 \) to \( N \) with the share spent on each variety given by \( p_k^{1-\sigma} / \sum_{k=1}^{N} p_k^{1-\sigma} \). We normalize \( \mu y = 1 \) in order to pose the model in terms of \( M \) consumers who spend one dollar each on the differentiated product sector.

Using demand and profit functions given in section 2, we find the usual optimal price for each producer: \( p_{ij} = \sigma \omega \tau_{ij} / (\sigma - 1) \) with \( \tau_{ij} = 1 \) for \( i = j \) and \( \tau_{ij} = \tau \) for \( i \neq j \). Let
\(p \equiv \sigma \omega / (\sigma - 1)\) and \(\rho \equiv \tau^{1-\sigma}\). Plugging into the quantities equations you get the following equilibrium quantities:

\[
q_{HF} = \frac{\rho}{s \rho + (1 - s) \frac{1}{Np}} \quad \text{and} \quad q_{FF} = \frac{1}{s \rho + (1 - s) \frac{1}{Np}}
\]

\[
q_{FH} = \frac{\rho}{s + (1 - s) \frac{1}{Np}} \quad \text{and} \quad q_{HH} = \frac{1}{s + (1 - s) \rho \frac{1}{Np}}
\]

**Ottaviano-Tabuchi-Thisse**

Let individual consumption of variety \(k\) be given by \(D(k)\). The Ottaviano-Tabuchi-Thisse utility function for the representative consumer is given by

\[
U = A + \alpha \int_0^N D(k)dk - \frac{\beta - \gamma}{2} \int_0^N D(k)^2 dk - \frac{\gamma}{2} \left( \int_0^N D(k)dk \right)^2,
\]

where there are \(N\) varieties and \(A\) is consumption of the numeraire good. Ottaviano, Tabuchi, and Thisse (forthcoming) derive the standard demand curves for these preferences for the representative individual as

\[
D(k) = \frac{\alpha}{\beta + (N - 1) \gamma} - \frac{1}{\beta + (N - 1) \gamma} p(k) + \frac{\gamma}{(\beta - \gamma)(\beta + (N - 1) \gamma)} \int_0^N [p(\ell) - p(k)]d\ell.
\]

We choose to measure quantities in units of \(\alpha / (\beta + (N - 1) \gamma)\) and prices in units \(1/\alpha\). After redefining \(D\) and \(p\) in terms of these new units, we re-express the demand curve as

\[
D(k) = 1 - p(k) + \theta \int_0^N [p(\ell) - p(k)]d\ell,
\]

where \(\theta \equiv \gamma / (\beta - \gamma)\). The demand equation in the body of the paper is obtained by rearranging, imposing symmetry, and substituting in the formula for the price index.

Using demand and profit functions, the equilibrium prices and quantities can be shown to be equal in this model to:

\[
p_{FF} = \frac{2(1 + \omega(1 + \theta N)) + \tau \theta s N}{2(2 + \theta N)} \quad \text{and} \quad p_{HH} = \frac{2(1 + \omega(1 + \theta N)) + \tau \theta (1 - s) N}{2(2 + \theta N)}
\]

\[
p_{FH} = p_{HH} + \tau / 2 \quad \text{and} \quad p_{HF} = p_{FF} + \tau / 2
\]

\[
q_{HH} = (p_{HH} - \omega)(1 + \theta N) = \frac{1 + \theta N}{2(2 + \theta N)}[2(1 - \omega) + \tau \theta (1 - s) N]
\]

\[
q_{FF} = (p_{FF} - \omega)(1 + \theta N) = \frac{1 + \theta N}{2(2 + \theta N)}[2(1 - \omega) + \tau \theta s N]
\]

\[
q_{HF} = (p_{HF} - \omega - \tau)(1 + \theta N) = \frac{1 + \theta N}{2(2 + \theta N)}[2(1 - \omega) - \tau(2 + \theta(1 - s) N)]
\]

\[
q_{FH} = (p_{FH} - \omega - \tau)(1 + \theta N) = \frac{1 + \theta N}{2(2 + \theta N)}[2(1 - \omega) - \tau(2 + \theta s N)].
\]

The overlapping markets condition can therefore be stated as \(\tau < \frac{2(1 - \omega)}{2 + \theta N}\). This ensures that
\((p_{ij} - \omega - \tau)\) is positive and independent of the geographic distribution of firms, thereby guaranteeing that both exports and price net of transport and production costs are positive.

**Brander**

The preferences for Brander may be obtained as a restricted form of those for Ottaviano-Tabuchi-Thisse. The assumption is that a single variety, \(D\), is produced and that \(\beta = \gamma\). In that case the representative consumer’s utility function is

\[U = A + \alpha D - (\gamma/2)D^2.\]

This implies a standard demand curve of \(D = \alpha/\gamma - P/\alpha\). We now choose to measure quantities in units of \(\alpha/\gamma\) and prices in units of \(1/\alpha\). This gives rise to the individual demand curve involved in the text of \(D = 1 - P\). Note that while we measure Brander and Ottaviano-Tabuchi-Thisse prices in the same units, the units for quantity are larger in Brander. Hence, whenever we want to compare results involving quantities across the two models, we scale up Brander results by factor \(1/\theta + N\).

Solving for equilibrium quantities in the Cournot subgame yields the following shipments to each market for a firm deciding to locate in country \(F\):

\[
q_{FF} = \frac{1 - \omega + sN\tau}{N + 1}, \quad q_{FH} = \frac{1 - \omega - \tau - sN\tau}{N + 1}.
\]

Equilibrium quantities shipped to each market by a firm producing in country \(H\) are given by

\[
q_{HH} = \frac{1 - \omega + (1 - s)N\tau}{N + 1}, \quad q_{HF} = \frac{1 - \omega - \tau - (1 - s)N\tau}{N + 1}.
\]

Equilibrium prices are thus decreasing functions of the number of firms in the considered country:

\[
P_H = \frac{1 + N(\omega + (1 - s)\tau)}{N + 1} \quad \text{and} \quad P_F = \frac{1 + N(\omega + s\tau)}{N + 1}.
\]

The overlapping markets condition, \(\tau(N + 1) < (1 - \omega)\), can be obtained by setting \(q_{FH} = 0\) at \(s = 1\).

**Markusen-Venables**

We assume consumers have the following utility arising from the consumption of the CRS good, \(A\), and two varieties of the IRS good, \(D_H\) and \(D_F\):

\[U = A + \alpha(D_H + D_F) + \frac{\beta - \gamma}{2}(D_H^2 + D_F^2) + \frac{\gamma}{2}(D_H + D_F)^2.\]

This structure of utility yields individual inverse demand functions of the form \(P_{ij} = \alpha - \beta Q_{ii} - \gamma Q_{ji}\) for the IRS good, where \(i\) is the country of production of the good and \(j\) its country of consumption. By choosing units such that prices are expressed in units of \(\alpha\) and quantities in units of \(\beta/\gamma\), we obtain \(P_{ij} = 1 - Q_{ii} - \nu Q_{ji}\), where \(\nu = \gamma/\beta\) measures the degree of product differentiation.

Profit maximization yields the following equilibrium quantities for the representative firm:

\[
q_{HH} = \frac{(1 - \omega)[(1 - s)N + 1] - \nu(1 - s)N(1 - \omega - \tau)}{(1 - \nu^2)\nu^2(1 - s)N^2 + (N + 1)},
\]

26
\[ q_{FH} = \frac{(1 - \omega - \tau)(sN + 1) - \nu sN(1 - \omega)}{(1 - \nu^2)s(1 - s)N^2 + (N + 1)}, \]
\[ q_{FF} = \frac{(1 - \omega)(sN + 1) - \nu sN(1 - \omega - \tau)}{(1 - \nu^2)s(1 - s)N^2 + (N + 1)}, \]
\[ q_{HF} = \frac{(1 - \omega - \tau)[(1 - s)N + 1] - \nu(1 - s)N(1 - \omega)}{(1 - \nu^2)s(1 - s)N^2 + (N + 1)}. \]

Equilibrium prices are:

\[ P_{HH} = \frac{(1 - sN\omega)[(1 - s)N + 1] - \nu(1 - s)N[1 - \omega(1 - sN\nu) - \tau]}{(1 - \nu^2)s(1 - s)N^2 + (N + 1)}, \]
\[ P_{FH} = \frac{[1 + (1 - s)N(\omega + \tau)](sN + 1) - \nu sN[1 - \omega + (1 - s)N\nu(\omega + \tau)]}{(1 - \nu^2)s(1 - s)N^2 + (N + 1)}, \]
\[ P_{FF} = \frac{[1 + (1 - s)N\omega](sN + 1) - \nu sN[1 - \omega(1 - (1 - s)N\nu) - \tau]}{(1 - \nu^2)s(1 - s)N^2 + (N + 1)}, \]
\[ P_{HF} = \frac{[1 + sN(\omega + \tau)][(1 - s)N + 1] - \nu(1 - s)N[1 - \omega + sN\nu(\omega + \tau)]}{(1 - \nu^2)s(1 - s)N^2 + (N + 1)}. \]

References


