Job Contact Networks*

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Abstract

Many workers hear about or obtain their jobs through friends and relatives. Networks of personal contacts transmit employment opportunities through word-of-mouth communication and constitute an alternative source of employment information to more formal methods. The aim of this paper is twofold. First, we investigate what job contact networks are reasonable to expect by focusing on individual incentives and strategic issues of network formation. Second, we establish a relationship between the social structure of bilateral contacts and the job- nding process. To this purpose, we develop a model specifying at the individual level both the decision to establish or to maintain social ties with other agents, and the process by which information about jobs is obtained and transmitted.

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1 Introduction

It is widely known and documented that many workers hear about or obtain their jobs through friends and relatives: about half of all jobs are filled through contacts. Networks of personal contacts mediate employment opportunities which flow through word-of-mouth. They constitute an alternative source of employment information to more formal methods −such as advertisement and employment agencies− and play a central role in many labor markets by matching job seekers with vacancies. The prevalent social contacts and labor market connections thus strongly determine, or at least influence, economic success of individuals.

The aim of this paper is twofold. First, we investigate what labor market or job contact networks are reasonable to expect by focusing on individual incentives and strategic issues of network formation. In particular, we examine how labor market conditions influence both network size and players interconnections. Second, we establish a relationship between the social structure of interorganizational contacts and the job-finding process. The analysis focuses on the impact an endogenous determination of job contact networks has on the aggregate unemployment level. As expected, the agents reliance on word-of-mouth information gathering has important implications for the aggregate behavior of the economy that we discuss.

Our model specifies at the individual level both the decision to establish or to maintain social ties with other agents, and the process by which information about jobs is obtained and transmitted. Following an early contribution by Boorman (1975), we model the structure of social contacts by an undirected network where symmetric binary ties connect individuals. Those ties represent direct communication channels created by mutual consent of the two linked individuals and through which job information may flow. The labor market we consider follows Diamond (1981) in that all jobs are subject to an exogenous risk of breakdown, somehow counterbalanced by an exogenous arrival rate of job opportunities. Departing from this paper, we assume here that employed workers who learn of a job vacancy may transmit this information to their unemployed friends, if any, or relay it to some of their employed relatives. Agents thus partly rely on friends or relatives to gather information about potential jobs, and our model explicitly deals with the manner in which information is diffused in the network of relationships.

Given a network of contacts, we first explore the relationship between the prevailing social structure and the effectiveness of information diffusion.

Because job information may be transmitted along bilateral ties, direct neighbors are perceived by players as possible sources of job information. Networking can be interpreted here as a passive job search where players establish connections in order to broaden their available employment channels. The effectiveness of such passive job search is measured by the frequency with which unemployed individuals are taught about a vacant job through word-of-mouth communication, which depends on their relative social embeddedness that itself derives from the whole structure of relationships. When networks are symmetric, meaning that all agents have the same number of direct acquaintances, we nd that the individual probability of nding a referred job exhibits dismishing returns to network size. Moreover, this probability may decrease with network size due to congestion externalities that arise when the pattern of interconnections is too dense and that limit access to word-of-mouth employment opportunities.

In our context, bilateral contacts provide insurance to individuals about potential job loss. Players enter ties with others and set contacts in whom they invest time to mutually help them cope with the job uncertainty due to the labor market turnover. Assuming that such social relations result from the pursuit of self-interest by strategic individuals, we investigate what job contact networks are reasonable to expect by focusing on individual incentives and strategic issues of network formation. To characterize endogenously created networks, we use the concept of pairwise stability introduced by Jackson and Wolinsky (1996) and later discussed by Dutta and Mutuswami (1997). By de nition, a pairwise stable network is such that no player bene ts by unilaterally severing an existing link, and no two players mutually gain by establishing a new connection. We rst obtain a full characterization of the set of pairwise stable contact networks and illustrate how the associated pattern of social links heavily depends on the labor market and social conditions. In particular, the easier it is to establish a link, the richer the set of relationships any player possesses in an endogenously created network. We then show that there always exists a stable and (almost) symmetric job contact network where all players have the same number of contacts, unique up to the addition or the deletion of one link to all the players in the population.

Once we have examined the determinants of the neighborhood size and the players interconnections in endogenously emerging labor market networks, we establish a relationship between social structure and labor market outcomes. More concretely, we compute the aggregate unemployment level generated by a pairwise stable job contact network. Not surprisingly, networks that mediate job information through word-of-mouth uently and that match those vacancies with job seekers effectively result in a low unemployment rate, inversely related to the network aggregate effectiveness of passive job search. In sparse networks, more social links lead to a more uent transmission of information about employment opportunities and reduce the corresponding aggregate level of unemployment. In dense networks, though, congestion externalities slow down the diffusion of job information and unemployment is higher. The unemployment also depends on the prevalent labor market conditions: lolow risk of job breakdown yields naturally to low unemployment levels whereas a high labor market turnover generates high unemployment rates.

We then characterize efficient networks, which rst requires de ning and computing a welfare measure. Indeed, some con gurations of bilateral ties may achieve a widespread transmission of job information at low cost, while other networks may either be costly to maintain or poorly transmit this information, or both. We measure the efficiency of a job contact network by determining how the corresponding graph performs the job matching process relative to the aggregate cost of network creation and maintenance. We rst show that, given a labor market, there exists a unique (almost) symmetric and efficient network whose size always lies below the congestion threshold value where more contacts result in a better transmission of information about job openings, thus reducing unemployment. We then compare the aggregate unemployment level corresponding to either endogenous contact networks arising from a decentralized process of link formation where players establish and sever connections according to their own incentives, or to efficient labor market networks. We can distinguish two regimes. First, when social connections are too costly to obtain (networking is difficult), players set links cautiously and the unemployment level of the endogenously created labor market network is lower than the unemployment rate corresponding to the efficient network. Second, when costs of forming and maintaining links are low, stable networks are over-connected from an efficient viewpoint and yield a higher aggregate unemployment level than efficient ones. Congestion externalities are now strong enough so that decentralized creation of social ties generates an unemployment premia with respect to the unemployment level obtained with efficient labor market networks. Our labor market model with passive job search through word-of-mouth information gathering thus exhibits a crucial trade-off between stability and efficiency due to the presence of negative externalities in dense networks.

The remaining of the paper is organized as follows. Section 2 presents the model of labor market. Section 3 deals with the endogenous formation of job contact networks and network stability. Section 4 analyzes the corresponding aggregate employment level, addresses the question of network efficiency and examines the relationship between endogenously created labor market networks and efficient ones. Finally, section 5 generalizes our model by relaxing the assumptions on the length of information transmission, and relates our paper to previous work. All proofs are presented in appendix.

2 Job transmission on contact networks

2.1 The job contact network

Let $N = \{1, \ldots, n\}$ be the nite set of players ($n \geq 2$). We only consider undirected graphs constituted of symmetric binary ties connecting individuals. Binary ties represent direct communication channels created by mutual consent of the two linked individuals who both share the costs and the bene ts associated to this connection. The complete graph, denoted g^N, is thus the set of all subsets of N of size 2, and corresponds to a situation where everybody can directly communicate with anybody. The set of all possible graphs on N is $\mathcal{G} \equiv \left\{ g \mid g \subseteq g^N \right\}$. Let $g \in \mathcal{G}$. A link in g between two players i and j is denoted by $ij \in g$. We introduce the following de nition:

De nition 1 *For all $g \in \mathcal{G}$ and for all $i \in N$, i s neighborhood denoted by $N_i(g)$ is the set of players in N directly connected to i when the contact network is g that is, $N_i(g) = \{j \in N \setminus \{i\} \mid ij \in g\}$.*

Note that by de nition $i \notin N_i(g)$, and $i \in N_j(g)$ is equivalent to $j \in N_i(g)$. In words, the neighborhood relationship is non re exive and symmetric. We denote by $n_i(g)$ the cardinality of $N_i(g)$ that is, the number of nodes adjacent to i or, equivalently, the number of links that are incident with i. To simplify notations, we denote $N_i(g)$ by N_i (resp. $n_i(g)$ by n_i) when no confusion is possible. Also, to avoid trivialities we restrict throughout the analysis to $\mathcal{G}^* = \{g \in \mathcal{G} \mid n_i(g) \geq 1, \forall i \in N\}$ that is, to those networks where all players have at least one neighbor and no individual is totally isolated.

2.2 The model of job transmission

Our model is built on Boorman (1975) s model for transmission of job information through contact networks and on Diamond (1981) s model of labor market network.[1] The prevailing employment situation of the labor market is fully described by the two following parameters:

De nition 2 *Any individual in the population loses his job with probability b and hears directly of a vacant job with probability a.*[2]

Fix a job contact network $g \in \mathcal{G}^*$. If some player hears of a vacant job and is *unemployed* (which happens with probability ab), he immediately takes the job. If on the contrary some individual hears of a vacancy while he is *employed* (which happens with probability $a(1-b)$), he maintains his current position and transmits this job information to any of his unemployed neighbors, if any. In other words, employed workers who learn of a job opportunity inform one of their unemployed friends of its existence.[3] In this latter case, we assume that neighbors are treated on an equal footing, meaning that unemployed neighbors all have the same probability of being informed.[4] Also, if an unemployed worker hears of two (or more) vacancies

[1] For previous analysis of word-of-mouth communication see for instance Ellison and Fudenberg (1995).

[2] As in Diamond (1981), we are thus assuming that all jobs are subject to a risk of breakdown at the constant breakup rate b (representing exogenous factors). Also, players (both employed and unemployed) learn about job opportunities with an exogenous arrival rate a. For sake of simplicity we take it as xed, thus independent of players behavior. We depart from this paper in at least two respects: rst, we allow for word-of-mouth job information transmission; second, we consider a static model of labor market. For a dynamic model of labor market networks see for instance Calvó-Armengol (2000).

[3] According to Granovetter (1995): For 57.9 percent of the individuals nding their new job through contacts, the interaction during which job information was passed was, in fact, initiated by the contact. In another 20.9 percent of the instances, the respondent contacted his friend, asked him if he knew of anything, and was told about the job he subsequently took. (p. 33) Our assumption of job information being passed as an initiative of the informed and employed neighbor seems thus quite realistic.

[4] Contrarily to Boorman (1975) that makes a sharp distinction between strong and weak contacts −thus introducing a priority ranking over neighbors−, we assume in our model that pairwise links are all of the same kind, which motivates this equal-treatment-of-neighbors assumption. Montgomery (1991) also provides a stylized an elegant model that makes a distinction between strong and weak ties. In particular, he examines how a change in the composition of social interaction affects the steady-state unemployment equilibrium in a dynamic labor market.

either directly or through word-of-mouth, we assume that he selects one job randomly, the other job(s) being then lost. Note that, so far, job information can only ow through word-of-mouth from a worker to an unemployed that is, between players with different employment status. In section 5 we relax this assumption and allow for a more general information transmission protocol where job information can be relayed from employed worker to employed worker. In our model, players rely (partly) on friends or relatives to gather information about potential jobs, meaning that they are engaged in passive job search. The individual probabilities of getting a job through contacts measures the effectiveness of this passive search.[5]

Proposition 1 *The probability that player* $i \in N$ *gets a job through contacts is* $P_i(g) = 1 - Q_i(g)$*, where* $Q_i(g) = \prod_{j \in N_i(g)} \left[1 - a(1-b)\frac{1-(1-b)^{n_j(g)}}{bn_j(g)}\right]$.

For all player $i \in N$, $Q_i(g)$ denotes the probability that player i does actually *not* nd a job through contacts when the referral network is g. When there is no ambiguity, we simply denote this probability by Q_i. The probability P_i with which a player i is taught about a vacant job through word-of-mouth communication depends on his (relative) social embeddedness or structure of relationships. Indeed, all players in direct contact with i contribute to P_i by an amount that depends itself on these players relative locations in the labor market network. We also deduce from the expression of P_i given above that adding or severing a link between two players only affects their individual probabilities of obtaining a referred job and that of their direct neighbors, while it has no effect out of the joint neighborhood of the two players. As it will become clear in the next sections, this externality implies that endogenously created labor market networks do not generally correspond to efficient outcomes. The following Lemma clari es the relationship between social location and job-matching.

[5] An alternative and equivalent job information transmission protocol is the following. We assume again that any worker hears directly of a vacant job with probability a. As before, when the directly informed worker is unemployed, he takes the job. If on the contrary the directly informed worker is currently employed, we assume now that he makes a public announcement of the available job to all his direct friends. Employed friends, if any, then ignore this job announcement (they already have a job) while all unemployed friends, if any, postulate for the vacancy which is randomly assigned to one of them. As before, vacancies remain un lled when an unemployed worker receives multiple job offers.

Lemma 1 *For all player $i \in N$ and for all direct neighbor $j \in N_i$, P_i increases with n_i and decreases with n_j.*

In words, the probability P_i of player i getting a referred job increases with the size of his neighborhood and decreases with the size of this direct neighbors neighborhood. The relationship between network size and job- nding is straightforward. The better some player $i \in N$ is connected, the higher his probability to obtain a job through contacts. Indeed, neighbors constitute as many different possible sources of job information. Increasing one s neighborhood size thus expands the available information channels yielding to a higher probability of getting a job through contacts or effectiveness of passive job search. In other words, more contacts provide job seekers with a higher likelihood of receiving information about job openings. But the better i's neighbors are connected, the lower this probability. Indeed, when player i s neighbors have a large set of acquaintances, the number of potential unemployed relatives that can bene t from their job information is high. Consequently, the probability of player i being selected to be taught about a job vacancy by his neighbors is low.

Example 1 *If $g \in \mathcal{G}^*$ is symmetric, meaning that all neighbors have the same neighborhood size μ, $P = 1 - Q$ where $Q = \left[1 - a\left(1 - b\right)\frac{1-(1-b)^{\mu}}{b\mu}\right]^{\mu}$.*

Symmetric networks where all players have the same neighborhood size μ form a particular class of labor market networks. According to the previous Lemma, increasing the number of contacts in a symmetric job network has both a direct and an indirect effect. Indeed, increasing μ expands the neighborhood size of some given player and thus increases his probability P of nding a job through contacts (direct effect). But increasing his neighbors neighborhood size reduces this probability P of nding a referred job (indirect effect). In fact, the direct effect prevails for low values of the network size μ, while the indirect effect overweights the direct effect in networks of big size.

Lemma 2 *In a symmetric network, P is strictly concave, has a global maximum $\bar{\mu} > 1$ and reaches its global minimum at $\mu = 1$.*

Remember that networking can be interpreted as a passive job search: players establish connections in order to broaden their available employment channels that is, the potential sources of information about vacant jobs.

Link building or socializing thus corresponds to a passive search strategy. According to the previous Lemma, the effectiveness of such passive job search, measured by the individual probability P of nding job through contacts in a symmetric network, exhibits *disminishing returns to network size*. Likewise, traditional (active) search models with random sampling and reservation wages often exhibit disminishing returns to search intensity.[6]

Also, P increases on $[1, \bar{\mu}]$ and decreases on $[\bar{\mu}, +\infty)$. In words, the individual probability of getting a referred job increases with the symmetric network size μ in sparse networks, while it decreases with μ in densely connected labor market networks. How job information is disseminated and the resulting job- nding behavior thus heavily depends on the prevailing structure of social contacts that determines their course and result. Moreover, the network of personal ties may either exacerbate or constrain the diffusion of job information through word-of-mouth communication. Indeed, more contacts are usually thought of as an advantage since there are more network members who can potentially connect job seekers with vacancies. But, in fact, this positive impact of one s neighborhood size only holds when the network size is not too high and stays below the threshold value $\bar{\mu}$. Network size may also have a negative impact on job information acquisition, and it is worth asking under what circumstances some of one s neighbors hold a better relative location in the contact network to provide information than others.

The intuition behind the positive and negative role of network size is the following. Increasing the size of a symmetric network expands players neighborhoods. Individuals become better connected, and more connections increase the potential job information available to any player. Indeed, the probability that at least one neighbor is informed about job openings is $1 - (1 - a)^{\mu} \uparrow 1$ as $\mu \to +\infty$. But neighbors being themselves better connected, the information they possess is now shared among a bigger group of players. One s direct neighbors are now likely to be in contact with many other unemployed workers in the population, and the job information to which one may have acceeded privately is now likely to reach someone else instead. Indeed, the individual probability that some particular unemployed worker is informed about the vacancy a direct and employed neighbor of him is aware of is $\frac{1-(1-b)^{\mu}}{b\mu} \downarrow 0$ when $\mu \to +\infty$. We then deduce from $\lim\limits_{\mu \to +\infty} \frac{1-(1-b)^{\mu}}{b\mu} [1 - (1 - a)^{\mu}] = 0$ that the constraints imposed in symmetric

[6] See Barron and Gilley (1981), for instance.

networks of high size by information sharing and job rationing (the labor market turnover being determined by a and b) reduces access to job information despite the huge number of employment channels available to every player.

When the set of interpersonal contacts is dense (the symmetric network size μ is higher than $\bar{\mu}$), positive decisions of one worker alone (expanding his set of relatives to increase his probability of a getting a referred job) has a negative effect on others (it reduces their probability of getting a job through referrals). The random assignment of jobs inherent to our information transmission protocol yields to coordination failures.[7] When network interconnections are dense (network size is high), mismatches between employment opportunities and job-seekers arise. They can be interpreted as information transmission frictions that limit access to word-of-mouth employment opportunities and take the form of *congestion externalities*.[8] We can thus think of $\bar{\mu}$ as a *congestion threshold value* delimitating at her left-hand side the domain where the individual probability P of gathering job information through personal contacts increases with the symmetric network size, and at her right-hand side the domain where P decreases with μ. This congestion threshold value $\bar{\mu}$ only depends on the breakup rate b and the arrival rate a, and is given by the unique solution to $P'(\mu) = 0$.

Now consider again a general job contact network $g \in \mathcal{G}^*$ that can be either symmetric or asymmetric.

Lemma 3 *For all $i \in N$, P_i decreases with the breakup rate b and increases with the arrival rate a.*

The probability that some of i s neighbors hears of a vacancy and needs it is ab. This probability increases with the breakup rate b. Therefore, the higher b, the lower the probability of hearing of a vacancy from a neighbor because it is more likely that the neighbor herself is jobless and lls the vacancy he hears about directly. Also, the probability that some of i s neighbors hears of a vacancy and does not need it is $a(1-b)$. Thus, the higher the arrival rate a, the higher this probability and the more player i is likely

[7] Coordination failures in decentralized matching processes are frequent. Think for instance on the job market for economists where each year two or even three rounds are required to clear the market.

[8] For another labor market model with congestion externalities in referral search, see for example Kugler (1997).

of being informed by a friend about a job vacancy that this relative herself does not need.

2.3 The individual payoffs

According to Boorman (1975): individuals set contacts in whom they invest time and from whom jobs may ow (p. 228). When a worker is hired, he receives and exogenous wage denoted by $w > 0$. In our model wage determination is not an issue. We assume indeed that all jobs are identical and that salaries are xed and equal to w. For simplicity, any match between an employer and an employee gives an output (the match rent) of one. This assumption is consistent with the model of (passive) information gathering about job opportunities through contacts developed here. Indeed, the search for information in any market has both an extensive and an intensive margin.[9] In our case, players (passively) search job information at the extensive margin by establishing an additional connection with one more player thus (potentially) increasing their access to this information. Searching at the intensive margin would rather consist on getting additional information concerning an offer already received. The extensive margin prevails when information is highly standardized, as it is the case in our model where jobs and wages are identical.

Consistent with Boorman, we also assume that each direct link ij results in a cost $c > 0$ to both individuals i and j, equal across individuals players and independent of the number of existing links.[10] This cost of forming a link with another player may for instance be interpreted as the time, the effort or the money a player must spend in order to maintain an active connection. It can also be interpreted as the effort made to strenghten existing links in order to guarantee word-of-mouth information transmission when employment opportunities are available to neighbors. Note that social networks need not be strictly geographic and may be strati ed along physical or socio-demographical dimensions such as distance, race or ethnicity, religious

[9] This idea is developed by Rees (1966).

[10] We could normalize $w \equiv 1$ or, equivalently, $c \equiv 1$ without loss of generality. Indeed, it is the rate of the wage w to the per-capita cost of a link c that matters when characterizing both stable and efficient job networks. We will, though, keep both (redundant) parameters w and c throughout to clearly differentiate gains from costs when considering the individual incentives to form and to cut links. We could also allow for a more general per-capita link cost function: the implied differences are provided in footnotes when required.

affiliation, language, education, age, etc. In any case, the cost of developing or establishing links should presumably be an increasing function of the social and economic distance metrics in the corresponding abstract space of link formation, and not a xed value c as assumed in our model.[11] It is nonetheless a recognized (and empirically documented) fact that agents are likely to establish connections mostly with people very close to themselves. In other words, there exists a strong degree of homogeneity in social networks (often referred to as assortive matching or inbreeding bias in the literature) thus supporting our assumption of a constant per-capita link cost.

Given a referral network $g \in \mathcal{G}^*$, the expected net payoff $Y_i(g)$ of some (initially employed) player $i \in N$ is given by:

$$Y_i(g) = w \left\{ \underbrace{(1-b)}_{i \text{ keeps job}} + \underbrace{b \left[\underbrace{a}_{direct} + \underbrace{(1-a)\, P_i(g)}_{referred} \right]}_{i \quad \text{red and reemployed}} \right\} - cn_i(g).$$

These individual payoffs de ne a mapping $Y : \mathcal{G}^* \to \mathbb{R}^n_+$ that gives the *ex ante* distribution of payoffs, equal to the players expected bene ts less their costs of link formation. $Y(g)$ describes how individual (expected) payoffs are distributed given any network g connecting players.

3 Stable job contact networks

So far, we have analyzed the relationship between (passive) job search effectiveness and social embeddedness by determining how the individual probabilities of getting a job through friends depend on players interconnections. We now investigate what job contact networks are reasonable to expect by focusing on individual incentives and strategic issues of network formation.

[11] Conley and Topa (1999) examine possible costs of interaction in detail including physical distance, travel time and differences in ethnic and occupational distribution between locations. They study the spatial patterns of unemployment in Chicago between 1980 and 1990 and analyze unemployment clustering with respect to different social and economic distance metrics that re ect the structure of agents social networks.

3.1 Definitions

To characterize endogenously created networks, we use the concept of pairwise stability introduced by Jackson and Wolinsky (1996). It states that a network is pairwise stable if no player would benefit by unilaterally severing an existing link, and no two players would benefit by forming a new link. Given a network g, $g - ij$ (resp. $g + ij$) denotes the network deduced from g bt cutting (resp. adding) the link ij. More formally,

Definition 3 *The network $g \in \mathcal{G}^*$ is pairwise stable if and only if both:*

(a) *for all $ij \in g$, $Y_i(g) \geq Y_i(g - ij)$ and $Y_j(g) \geq Y_j(g - ij)$;*

(b) *for all $ij \notin g$, $Y_i(g + ij) > Y_i(g)$ implies $Y_j(g + ij) < Y_j(g)$.*

Stable networks, thus, are such that no player gains by altering the current configuration of links, neither by adding a new connection nor by eliminating an existing one. This notion of stability is an appropriate equilibrium concept to characterize endogenously created networks in a context where players set up connections in a decentralized manner while pursuing their own self interest that is, increasing their individual expected outcome accruing from the prevailing labor market network. Pairwise stability allows only link severance by individuals and link formation by pairs. It is just a concept among many other possible notions but provides sharp results for our analysis.[12] Moreover, the class of pairwise stable networks can be easily interpreted as the limiting graphs of a dynamic procedure of endogenous network formation. Indeed, consider a dynamic process were players myopically add or sever links based on the improvement the resulting network offers relative to the current network, with each network differing from the previous only by one link. Clearly, if this formation process converges, the networks

[12] One variation of this stability notion would be to strenghten the concept of pairwise stability by allowing for side payments between two players who deviate by constructing a new link. Another variation allows for a network to be immune to deviations by more than two players simultaneously (in the case of link formation), and to be immune to the severance of more than one link by a single player. Also, stable networks can be defined as the Nash equilibria (and its refinements when required) of a one-shot strategic link formation game where players strategies consist on the set of players with whom they want to form a link. We refer the reader to Dutta and Mutuswami (1997) for a thorough analysis of the notion of (coalitional) stability as the Nash equilibria of a strategic form game when players are graph-connected.

ultimately reached are pairwise stable.[13] It seems quite reasonable to assume that networks in real-life arise as the long-run outcome of a dynamic process of adjustment as the one described above.

3.2 Stable job contact networks: characterization

In this section we address the question of what pairwise stability predicts concerning which networks might form. To do so, we rst establish a criterium for pro table unilateral deviations. To simplify notations, denote by $\alpha \equiv a(1-b)$ the probability of not being red and of hearing directly of a vacant job, and let $\beta \equiv b(1-a)$ be the probability of being red and of not hearing directly of a vacancy. In other words, α is the probability with which an employed worker has a job slot to allocate to any of her unemployed neighbors, while β measures to what extend an unemployed player relies on (needs) her contacts to recover a job. The individual probability Q of some player i *not* nding a job through contacts in a symmetric network of size μ can also be written $Q(\mu) = q(\mu)^{\mu}$, where $q(\mu) = 1 - \alpha\frac{1-(1-b)^{\mu}}{b\mu}$ denotes the probability of player i not hearing of an employment opportunity from some particular neighbor with neighborhood size μ.[14] We now state a technical Lemma crucial to conduct the stability analysis.

Lemma 4 *$Y_i(g+ij) > Y_i(g)$ if and only if $w\beta\left[1 - q\left(n_j(g)+1\right)\right] Q_i(g) > c$.*

The term on the right-hand side of the inequality is the cost c of an additional link. The term on the left-hand side gives the expected payoff from an additional link established by player i with some newly connected neighbor j. Indeed, $Q_i(g)$ is the probability of uplayer i *not* nding a referred job when the contact network is g, while β gives the probability of becoming jobless and not hearing directly of a new vacancy, thus needing the help of a friend or a relative to become employed. The term $\left[1 - q\left(n_j(g)+1\right)\right] Q_i(g)$ corresponds then the probability of player i nding a job through word-of-mouth thanks to, and exclusively from, the newly connected neighbor j.[15] The inequality in Lemma 4 just states that it is worth for player i

[13] Jackson and Watts (1998) for the case of undirected graphs and Bala and Goyal (2000) for the case of directed graphs directly tackle the question of network formation in a dynamic framework.

[14] It is easy to check that q increases, with minimum $q(1) = 1 - \alpha$ and supremum $\lim_{x\to+\infty} q(x) = 1$.

[15] We can directly check that $\left[1 - q\left(n_j(g)+1\right)\right] Q_i(g) = P_i(g+ij) - P_i(g)$.

establishing links with those neighbors whose expected contribution in case of being unemployed does compensate for the cost of being connected.

Corollary 1 *If $c > w\alpha\beta$, it is always pro table to cut a link unilaterally. Consequently, no graph is stable.*

Therefore, when the per-capita cost of link formation is too high, no player has incentives to establish any connection with other players in the population. Players remain isolated from one another and passive job search through friends or relatives is not an issue. We denote by $c_{\max} \equiv w\alpha\beta$. Suppose that all players are initially employed and isolated, meaning that no link has yet been formed. Then, $w\alpha\beta - c$ is the individual expected payoff from creating a pair by linking an isolated and currently employed worker like oneself. Indeed, after the job turnover, the player I am forming a pair with will be endowed with an extra job slot payed w with probability α while I will need this job with probability β. The maximal cost I am thus willing to pay to create such a pair is $w\alpha\beta = c_{\max}$. We now characterize the class of pairwise stable (endogenously created) job contact networks.

Proposition 2 *A network $g \in \mathcal{G}^*$ is pairwise stable if and only if both:*

(*a*) $c \leq w\beta \min\left\{\left[1 - q\left(n_j\left(g\right)\right)\right] Q_i\left(g - ij\right); \left[1 - qn_i\left(g\right)\right] Q_j\left(g - ij\right)\right\}$, $\forall ij \in g$;

(*b*) $c < w\beta\left[1 - q\left(n_j\left(g\right) + 1\right)\right] Q_i\left(g\right)$ *implies* $c > w\beta\left[1 - q\left(n_i\left(g\right) + 1\right)\right] Q_j\left(g\right)$, $\forall ij \notin g$.

The probability $Q_i(g)$ of some player $i \in N$ *not* nding a job through contacts when the referral network is g only depends on neighborhood sizes and on the exogenous parameters a and b characterizing the prevailing labor market. Therefore, Proposition 2 provides quite a simple characterization of stable graphs just in terms of neighborhood sizes.[16] In particular, stable networks need not be symmetric: in a pairwise stable job con guration of links, some players may be very well-connected whilst others nearly or totally isolated. In fact, players relative locations create a (negative) externality for

[16] This Proposition provides a degree type of result where a class of networks satisfying a certain property (here, pairwise stability) is fully characterized only by imposing conditions on the degrees of the network that is, the number of links adjacent to the nodes of the graph.

other players, and this externality may lead to multiple equilibria with both symmetric and asymmetric stable graphs.

Although providing a full characterization, the criteria for pairwise stability stated above (that characterizes endogenously created networks) yields, in practice, to a tedious and systematic link-by-link examination. A more direct characterization of some appealing stable networks is nonetheless possible. Indeed, the mapping $Y(g)$ is anonymous in that changing the names of the individuals while keeping the geometry of the network connecting them does not change the allocations they receive. In other words, the only information required to determine $Y(g)$ is the particular shape of g, and not the label of players. All players being ex ante identical in the situation considered, it is natural to concentrate on symmetric referral networks where all players have a common neighborhood size denoted by μ that is, $\mu = n_i(g)$, $\forall i \in N$. The probability of not nding a job through contacts in a symmetric network of size μ for any player is then $Q(\mu) = 1 - P(\mu) = q(\mu)^{\mu}$.

Proposition 3 *A symmetric network with neighborhood size μ is pairwise stable if and only if $w\beta q(\mu)^{\mu}[1 - q(\mu+1)] < c \leq w\beta q(\mu)^{\mu-1}[1 - q(\mu)]$.*

Recall that $q(\mu)$ gives the probability of *not* hearing of a vacancy from some neighbor with neighborhood size μ. The complementary probability $1 - q(\mu)$ thus gives the frequency with which information about an available job opportunity is transmitted by such a neighbor possessing μ different connections. Also, and by de nition, a pairwise stable contact network is such that no player bene ts by severing an existing link, and no two players bene t by establishing a new connection.

The bene ts of an existing link in a $\mu-$symmetric contact network corresponds to the probability of being informed about a new job with wage w thanks to this link. It is thus equal to $w\beta q(\mu)^{\mu-1}[1 - q(\mu)]$. The condition $c \leq w\beta q(\mu)^{\mu-1}[1 - q(\mu)]$ in Proposition 3 simply states that in a $\mu-$symmetric stable network any player has incentives to maintain any of her μ existing connections whose potential rewards compensate for the required cost to form and maintain these links. Stability also requires that no two players not yet connected have strict incentives to form a direct link with each other thus increasing their neighborhood sizes by one, from μ connections to $\mu + 1$. The bene t of this additional link depends on the probability $\beta q(\mu)^{\mu}[1 - q(\mu+1)]$ that none of the μ current neighbors (with μ connections each) is helpful to hear about job vacancies when needed, whereas the

$(\mu+1)-$th newly contacted player provides information about such vacant jobs. The condition $w\beta q(\mu)^{\mu}[1-q(\mu+1)] < c$ in Proposition 3 simply states that in a $\mu-$symmetric stable network no player has incentives to establish a new connection because the corresponding cost does not compensate for the bene t of this additional link. The effect of an additional link does not overweight the corresponding cost of link formation.

When the network connecting players is the complete graph g^N, every player can directly accede to any other. This situation re ects full job information sharing among the population. We show that full communication is more likely to occur in small populations.

Corollary 2 *The complete network g^N is pairwise stable if and only if:*
$w\beta q(n-1)^{n-1}[1-q(n)] < c \leq w\beta q(n-1)^{n-2}[1-q(n-1)]$.

We know that $q(n) \uparrow 1$ when $n \rightarrow \infty$. Namely, in large populations the individual contribution of any particular neighbor to one s effectiveness of passive search vanishes. Therefore, when the population size n is high, both the left-hand side and the right-hand side term of the inequality tend to zero. Consequently, the complete graph is stable only for very low connection costs c or, equivalently, for very high wages w. With a large population, endogenously created networks tend thus to be sparse rather than fully intraconnected.

3.3 Stable job contact networks: existence

We say that a neighborhood size μ stabilizes a network if $\mu-$symmetric networks are pairwise stable. Proposition 3 characterizes all neighborhood sizes μ stabilizing symmetric networks. In this section we deal with the existence of values satisfying the conditions stated in this Proposition and with the existence of geometric networks stabilized by those values (that is, with the possibility of connecting players by bilateral links in such a way that these ties constitute a symmetric network of a given size). Recall that, by de nition, $c_{\max} = w\alpha\beta$.

Proposition 4 *If $c \leq c_{\max}$ and $\alpha \leq b$, there exists at least one and at most three successive neighborhood sizes $\{\mu_s - 1, \mu_s, \mu_s + 1\}$ stabilizing a symmetric network. If $\alpha > b$ these results hold as long as n is not too high.*

In words, in all labor markets where some mild conditions on the parameters characterizing the employment situation are satis ed, there always exists at least one integer value such that networks of this size are pairwise stable. The previous Proposition, though, does not state uniqueness of such a stabilizing network size. Indeed, if some network where all players have a given number of contacts is stable, the networks deduced from it either by adding or by substracting a link to every player may still be stable. Nonetheless, stabilizing neighborhood sizes are unique up to such an addition or severance of one link. In particular, if the population n is big enough, any stabilizing neighborhood size (either $\mu_s - 1$, μ_s or $\mu_s + 1$) gives a good idea of the order of magnitude of stable network sizes.[17]

Example 2 *Stabilizing neighborhood sizes when the arrival rate is* $a = 0.25$.[18]

b	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.9*
$c = 0.8w\alpha\beta$	*1*	*1*	*1*	*1*	*1*	*1*
$c = 0.6w\alpha\beta$	*2*	*2*	*2*	*2*	*2*	*1*
$c = 0.4w\alpha\beta$	*4*	*4*	*4*	3^+	*3*	*2*
$c = 0.2w\alpha\beta$	*7*	*8*	*9*	*8*	*7*	*5*
$c = 0.1w\alpha\beta$	*13*	*17*	18^+	*17*	*15*	*10*

As expected, the stabilizing neighborhood size μ_s depends negatively on the per-capita link cost c or, equivalently, μ_s increases with w. In other words, the easier it is to establish a link, the richer the set of relationships any player possesses in an endogenously created graph. The following result clari es this point.

Corollary 3 *Under the conditions stated above, μ_s is a decreasing function of c with supremum $\lim_{c \to 0} \mu_s = n - 1$ and minimum $\mu_s(c_{\max}) = 1$.*

As expected, low per-capita costs of link formation lead to dense endogenously created symmetric networks. Also, when creating links becomes costless players tend to connect with every other $n - 1$ player in the population. At the limit $c \to 0$, the complete network where full communication occurs arises as the unique stable social con guration of links.

[17] This *quasi*-uniqueness result somehow validates the (pairwise) stability notion used throughout. Indeed, any other stronger stability concept strenghtens the results obtained so far by reducing the set of stable graphs, thus only leading to the possibility of non existence of such stable job contact networks.

[18] A neighborhood size with a superscript $^+$ (such as μ^+) means that both μ and $\mu + 1$ are stabilizing values.

The structure of a symmetric network is de ned by its network size (i.e. the cardinality of the set of direct neighbors, common to all players) and the structure of relationships (bilateral ties) among players. Given a stabilizing network size μ_s, we now ask whether there always exists a pattern of links connecting players such that they all have exactly μ_s neighbors. The following Proposition gives a partial answer.

Proposition 5 *If n or μ_s are even, μ_s- symmetric networks always exist.*

Although the existence of symmetric networks is always ensured when either n or μ_s are even, if both n and μ_s are odd, symmetric graphs of size μ_s connecting the n players may fail to exist. Suppose that indeed both n and μ_s are odd. If there exists two successive stabilizing values $\{\mu_s, \mu_s + 1\}$, the network size μ_s+1 is even. According to Proposition 4 we can thus construct a stable $(\mu_s + 1)$ $-$symmetric network. Similarly, if there exists three successive stabilizing values $\{\mu_s - 1, \mu_s, \mu_s + 1\}$, we can construct two stable $(\mu_s - 1)$ and $(\mu_s + 1)$ $-$symmetric networks with both even values $\mu_s - 1$ and $\mu_s + 1$. Assume now that both n and μ_s are odd and that μ_s is the unique stabilizing neighborhood value. In words, the networks of size $\mu_s - 1$ and or $\mu_s + 1$ obtained by adding or severing one link to all players are not pairwise stable anymore. In that case, the existence of a symmetric and stable network is not guaranteed. We can nonetheless construct a pairwise stable and *almost*-symmetric network that differs from a symmetric network only by one link.

Proposition 6 *If n and μ_s are odd, a pairwise stable almost-symmetric network where $n - 1$ players have a neighborhood of size μ_s and 1 player has a neighborhood of size $\mu_s - 1$ always exists.*

Strictly speaking, the existence of pairwise stable symmetric networks is not always guaranteed. Indeed, we can (nearly always) compute a stabilizing network size, but we may not be able to connect players in a symmetric network of such size. Nonetheless, we can always approximate such a symmetric and pairwise stable network up to one link. Also, stable symmetric networks need not be unique. Still, we can associate a unique stable and symmetric (or sometimes *almost*-symmetric) network to any prevailing labor market condition up to one common link addition or severance. Therefore, under very mild conditions imposed on the labor market characteristics, there always exists a unique and stable symmetric job contact network in order of magnitude terms.

4 Job contact networks and unemployment

4.1 The aggregate unemployment level

So far, we have examined the determinants of the neighborhood size and the pattern of social links in endogenously emerging labor market networks. We have shown that the network size μ_s of stable and symmetric configurations of links heavily depends on the labor market conditions summarized by the breakup rate b, the arrival rate of employment opportunities a, the exogenous wage w and the cost c of forming and maintaining links. In particular, low values of c or, equivalently, high values of w yield to densely connected stable networks with high neighborhood size μ, the complete graph being obtained in the limiting case where $c \to 0$.[19]

In this section we establish a relationship between social structure and labor market outcomes. More concretely, we analyze the impact an endogenous determination of (stable) networks has on the aggregate unemployment level and then characterize efficient networks. Giving a job contact network $g \in \mathcal{G}^*$, we denote by $u(g)$ the corresponding unemployment rate.

Proposition 7 *The unemployment rate is* $u(g) = \beta \left[1 - \frac{1}{n} \sum_{i \in N} P_i(g) \right]$.

Given a network of contacts g, the vector $[P_i(g)]_{i \in N}$ determines how information about employment opportunities flows through word-of-mouth. It thus captures the population effectiveness of passive job search when the pattern of social ties among individuals is given by the labor market network g. These individual probabilities of finding a job through friends depend on two network attributes: its size, and the pattern of social links among players. Networks that mediate job information fluently and that match those vacancies with job seekers effectively are such that the aggregate probability $\sum_{i \in N} P_i(g)$ of finding a job through contacts is high. These networks result in a low unemployment rate $u(g)$, negatively related to this aggregate probability. On the contrary, networks where the informal job search method relying

[19] Allowing for a more general class of per-capita cost functions $C : \mathbb{N} \to \mathbb{R}_+$, where $C(m)$ gives the cost of forming or maintaining m links, complicates the analysis without providing further nontrivial valuable insights. In particular, it is easy to see that a concave (resp. convex) cost function $C(\cdot)$, that captures decreasing (resp. increasing) marginal costs of link formation, yields to more densely connected (resp. more sparsely connected) stable networks.

on contacts and friends is not effective −low values of $\sum_{i\in N} P_i(g)$− result in a high unemployment rate. Unemployment also depends on the labor market considered. In particular,

Lemma 5 *The unemployment rate $u(g)$ increases with the breakup rate b.*

Not surprisingly, when the labor market turnover is high the unemployment is high.[20] When the network is symmetric, the unemployment rate u takes a particularly simple form that depends on the common neighborhood or network size μ.

Corollary 4 *The unemployment rate in a symmetric network with neighborhood size μ is $u(\mu) = \beta[1 - P(\mu)]$.*

We deduce from the above expression that increasing the probability P of nding a job through contacts reduces the unemployment rate. Note also that in the absence of a labor market network mediating employment information, the equilibrium unemployment rate would be $b(1-a)$. Therefore, allowing for passive job search through contacts reduces the unemployment level by an amount depending on the network size or density of social links μ. We then derive from Lemma 2 that the unemployment level u in a symmetric network is strictly convex with a global minimum at $\overline{\mu} > 1$ and a global maximum at $\mu = 1$. Network size may have either a positive or a negative effect: an increase in μ decreases the unemployment level if and only if the structure of contacts is no too dense. In sparse networks, more social links lead to a more uent transmission of information about employment opportunities and reduce the corresponding aggregate level of unemployment. In dense networks, though, congestion externalities (i.e. the fact that the indirect negative effect of other players establishing a new link overweights the direct positive effect of some given player also building a link) lead to an increase of the unemployment rate when players get better connected. Because the diffusion of job information is slowed, we get a higher equilibrium unemployment rate. We denote by $u_{\min} \equiv u(\overline{\mu})$ the minimum achievable unemployment level in a symmetric network.

[20] The comparative statics with respect to the exogenous arrival rate a are ambiguous. In fact, a more realistic model should posit a as an endogenous variable decreasing with the amount of vacancies lled or, similarly, increasing with the unemployment rate.

4.2 Efficient job contact networks

Characterizing efficient networks rst requires de ning and computing a welfare measure. In the labor market we analyze, individual economic outcomes vary with players accessibility to information about job vacancies, which itself depends on the existing pattern of social contacts among them. In particular, some con gurations of bilateral ties may achieve a widespread transmission of job information at low cost, while other networks may either be costly to maintain or poorly transmit this information, or both. We can measure the efficiency of a job contact network by determining how the corresponding graph performs the job-matching process relative to the aggregate cost associated to network creation and maintenance. We thus take as a welfare measure of a network $g \in \mathcal{G}^*$, denoted by $W(g)$, the total productivity of the labor market (recall that, by assumption, the match rent of one job is taken equal to one for simplicity) minus the total costs of forming and maintaining bilateral ties among players.

De nition 4 *The welfare measure is* $W(g) = [1 - u(g)]\, n - c \sum_{i \in N} n_i(g)$.

In symmetric networks, this welfare measure takes a particularly simple form given by $\frac{1}{n} W(\mu) = [1 - u(\mu)] - c\mu$. While the network size is not too high (below the congestion threshold value $\overline{\mu}$), u is a decreasing function of μ. In that case, increasing the network size μ reduces the occupation rate $[1 - u(\mu)]$ but increases the cost of network creation captured by $c\mu$. The welfare function W then re ects a trade-off between obtaining a more effective passive job search and supporting the costs associated to the contact network underlying it. On the other hand, we know that in symmetric networks of high size (above the congestion threshold value $\bar{\mu}$) the unemployment rate u increases with the network size μ due to congestion externalities. Therefore, in densely connected networks, the welfare measure is a decreasing function of μ. Congestion externalities thus yield to two types of inefficiencies. First, the excess number of bilateral ties perturbs information transmission. Second, this excessive number of links is costly to maintain. We now characterize efficient networks maximizing this welfare measure.

Proposition 8 *There exists a unique efficient network size* μ_e. *This neighborhood size* μ_e *always lies below the congestion threshold value of P that is,*

$\mu_e < \bar{\mu}$.[21]

Whatever the labor market conditions, there exists a unique efficient network size μ_e that always lies below the congestion threshold value $\bar{\mu}$ where increasing the neighborhood size μ leads to a higher probability of nding a job through friends, i.e. a better transmission of information about job opportunities thus reducing unemployment. This efficient symmetric network size μ_e depends on the per-capita link cost c.

Corollary 5 *The efficient network size μ_e is a decreasing function of c with supremum $\lim_{c \to 0} \mu_e = \bar{\mu}$ and minimum $\mu_e(c_{\max}) = 1$.*

As expected, low per-capita costs of link formation lead to a high network size in efficient symmetric networks. Also, when creating links becomes costless players tend to be better connected. At the limit $c \to 0$, the efficient network size coincides with the congestion threshold value $\overline{\mu}$ and the minimum achievable unemployment level $u_{\min}$ is reached.

4.3 Endogenous job network formation and efficiency

So far, we have analyzed how labor market conditions in uence network size and players interconnections and the corresponding job- nding process, both for endogenously created and for efficient job contact networks. We now compare the aggregate unemployment level obtained with either pairwise stable labor market networks or efficient ones.

We have seen that the (almost) unique network size μ_s of a (pairwise stable) endogenously created labor market network is a decreasing function of the per-capita link formation cost c. In particular, $\mu_s(c) \to n-1$ when $c \to 0$. In words, when establishing contacts is costless, the complete graph where everybody is directly connected to everybody forms. When the population size is large enough (formally, $n-1 > \overline{\mu}$), there exists a unique cost value $\overline{c}$ such that $\mu_s(\overline{c}) = \overline{\mu}$. The stabilizing network size μ_s then lies below the congestion threshold value for low enough per-capita link formation costs ($c < \overline{c}$) while it is always higher than this threshold value $\overline{\mu}$ as soon as

[21] Here again (as in Proposition 5 and in Proposition 6), there does not always exist a pattern of links connecting players such that they all have exactly μ_e neighbors. In some cases, we can only construct an *almost*-symmetric efficient network where all players but one have μ_e neighbors whereas one player has only $\mu_e - 1$ neighbors.

$c > \overline{c}$. Given a link cost c, individuals trade off the costs and bene ts of link formation and organize themselves into a symmetric network of size $\mu_s(c)$ to which we can associate an aggregate unemployment level $u(\mu_s)$. This unemployment rate generated by a symmetric and pairwise stable network thus depends on c. We denote it by $u_s(c) \equiv u[\mu_s(c)]$.

Similarly, maximizing the welfare function W that re ects both the effectiveness of the job-matching process induced by the network of relationships and the aggregate cost of link formation yields to an efficient network size denoted by μ_e. As before, μ_e is a decreasing function of the individual per-link cost c and $\mu_e(c) \to \overline{\mu}$ when $c \to 0$. In words, the efficient symmetric network size approaches from below the congestion threshold value as socializing becomes easier. We denote by $u_e(c) \equiv u[\mu_e(c)]$ the unemployment level generated with the efficient symmetric network when the per-capita link cost is c.

It is easy to see that both u_e and u_s achieve their global maximum at $c = c_{\max}$ where every player only maintains one active connection with some other player in the population (players are thus matched by pairs, when n is even). Also, both u_e and u_s reach the minimum achievable unemployment level $u_{\min}$ on $[0, c_{\max}]$, respectively for $c = 0$ and $c = \overline{c}$. Moreover,

Proposition 9 *The unemployment rate u_e of the efficient network increases with the per-capita link cost c on $[0, c_{\max}]$, whereas the unemployment rate u_s of the stable network decreases on $[0, \overline{c}]$ and increases on $[\overline{c}, c_{\max}]$. Moreover, there exists a unique $c^* < \overline{c}$ such that $u_s > u_e$ if and only if $c < c^*$.*

In words, when it becomes more difficult to establish social connections (that is, c increases) the unemployment rate of the efficient network increases. The unemployment level obtained with endogenously created symmetric networks shows a similar feature as long as the per-capita link cost lies above $\overline{c}$ for which the minimum achievable unemployment value $u_{\min}$ is obtained. In any case, the unemployment level of emerging social structures is unlikely to be efficient.

If we compare the aggregate unemployment level corresponding to either endogenous contact networks arising from a decentralized process of link formation where players establish and sever connections according to their own incentives, or to efficient labor market networks, we can distinguish two regimes. First, when the per-capita link cost c lies above a certain threshold value c^*, social connections are too costly to obtain (networking is difficult) and players set links cautiously. As a result, the unemployment level obtained with the endogenously created labor market network is lower than the unemployment rate corresponding to the efficient network. And this is also true with $c \in (c^*, \overline{c})$, where the endogenous network size μ_s lies above the congestion threshold value $\overline{\mu}$ ($\mu_s(c) > \overline{\mu}$), and thus is higher than the efficient network size μ_e (recall that $\mu_e < \overline{\mu}$, $\forall 0 < c \leq c_{\max}$). Second, when costs of forming and maintaining links lie below the threshold value c^*, stable networks are over-connected and yield a higher aggregate unemployment level than efficient ones. Congestion externalities are now strong enough so that decentralized creation of social ties generates an unemployment premia with respect to the unemployment level obtained with efficient labor market networks. Our labor market model with passive job search through word-of-mouth information gathering thus exhibits a crucial trade-off between stability and efficiency due to the presence of externalities in networks of high size, where the dense pattern of relationships yields to sharp interdependence of individual decisions and outcomes in con ict with the aggregate efficiency.

5 Discussion

In this section we discuss the assumptions on information transmission of our model and relate the main theoretical results to previous work.

5.1 The role of the information transmission protocol

So far, we have assumed that the information about job vacancies cannot be relayed further away than the direct neighborhood of the informed player. In our setting, when an employed individual hears of a job vacancy, either he is directly connected to some unemployed neighbor and informs him of the vacancy, or no player in his direct vicinity is out of job, in which case the job slot remains un lled. In particular, the informed worker cannot transmit this information to any other employed neighbor that may then relay it to some unemployed individual in his neighborhood, if any. With these restrictions on information transmission, more contacts provide job seekers with a higher probability of receiving information about job openings through contacts as long as the network is not too dense. Because information is never relayed, the amount of information about potential jobs received by some employed worker does not depend on his set of acquaintances. Moreover, the better some player is connected, the higher the potential number of unemployed individuals in direct contact with him that can potentially receive the information he holds about available jobs. Unemployed thus prefer a poorly connected neighbor than a well-connected one to release the constraints of information sharing with a potentially bigger set of information recipients. In other words, well-connected neighbors generate a negative externality over their direct neighborhood. Lemma 1 clari es this relationship between social embeddedness and job-matching effectiveness.

Assume now that information about potential jobs can be relayed from worker to worker. To keep things tractable, we just allow for one relay (the general case is sketched as a footnote).[22] Now, when some worker hears of a job and none of his direct friends needs it (all direct neighbors are employed), the informed worker transmits this information to any of his employed neighbors (with uniform probability). The newly informed worker may now allocate the job slot he is aware of to any of his unemployed direct contacts, if any. If not, we assume that the job is lost.

Given a network of contacts g, we denote by $\widetilde{P}_i(g)$ the individual probability of getting a job through friends with this new information transmission protocol. We rst introduce some useful notations. For all pair of players $i, j \in N$, $d_{ij}(g)$ denotes the (geodesic) distance from i to j according to g that is, the number of links in the shortest path on g from i to j. In particular, $d_{ij}(g) = 1$ is equivalent to $ij \in g$.[23]. Also, we assume that relayed information is correctly transmitted with some probability $\delta \in [0, 1]$. Information may thus be lost during word-of-mouth transmission from worker to worker with probability $1 - \delta$. For all $ij \in g$, de ne:

$$\widetilde{a}_{ij}(g) = a + \delta(1-a)\left[1 - \prod_{k \in N_j(g), d_{ik}(g)=2} \left(1 - a\frac{(1-b)^{n_k(g)}}{n_k(g)}\right)\right]$$

We then have the following result:

Lemma 6 *The probability that player $i \in N$ gets a job through contacts is $\widetilde{P}_i(g) = 1 - \widetilde{Q}_i(g)$, where $\widetilde{Q}_i(g) = \prod_{j \in N_i(g)} \left[1 - \widetilde{a}_{ij}(g)(1-b)\frac{1-(1-b)^{n_j(g)}}{bn_j(g)}\right]$.*[24]

[22] According to Granovetter (1995), though, information transmission with at most one relay accounts for 84.4% of the cases encountered: Chains of length [one] accounted for 39.1 percent of the cases; 45.3 percent had length [two] (p. 57).

[23] We refer the reader to Bala and Goyal (2000) for a more formal de nition.

[24] Assume more generally that information can be relayed through word-of-mouth with

Note that when $\delta = 0$ we are back to the previous case where information cannot be diffused further away than the direct neighborhood of the initially informed player. Also, it is easy to check that $\widetilde{P}_i(g) \geq P_i(g)$, and that $\widetilde{P}_i(g)$ increases with n_i. In words, allowing for job information to be relayed from worker to worker increases the effectiveness of passive job search which is itself an increasing function of one s direct neighborhood size. Now, the relationship between job-matching and direct neighbors connectedness, though, is not clear. Indeed, for all player $i \in N$, direct neighbors matter in de n-ing opportunities (they may transmit valuable information) and constraints (this information must be shared among all their unemployed acquaintances). More concretely, if some neighbor $j \in N_i$ of player i becomes better connected, the probability $\widetilde{a}_{ij}(g)$ that player j is informed (either directly or by some of his neighbors) of some vacant job that player i may potentially ll increases. Therefore, player i s opportunities are enhanced when his direct neighbors expand their neighborhoods. On the other hand, the larger player j s direct neighborhood, the bigger the number of unemployed direct neighbors of player j (including player i) that may potentially ll the vacant job their direct neighbor player j is aware of. Therefore, the information sharing constraints faced by player i are exacerbated with his direct neighbors becoming better connected. Enriching direct neighbors connectedness thus has both a direct and an indirect effect. The following Lemma clari es this point.

Lemma 7 *For all player $i \in N$ and for all direct neighbor $j \in N_i$, if player*

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no restrictions whatsoever on the length of transmission. Given a network g, a path of length l between two players $i, j \in N$ is a sequence $\{i_0, \ldots, i_l\}$ such that $i_0 = i$, $i_l = j$ and $i_p i_{p+1} \in g$, $\forall 0 \leq p \leq l-1$. Given a player $i \in N$, a radius in g accruing from i is a maximal shortest path for the inclusion ordering on sets. We denote by $R_i(g)$ the set of radii accruing from player i. For all $\{i_0, \ldots, i_l\} \in R_i(g)$, let $\widetilde{a}_{i_{l-1}i_l} = a$ and for all $0 \leq p \leq l-2$ de ne recursively:

$$\widetilde{a}_{i_p i_{p+1}} = a + \delta(1-a)\left[1 - \prod_{k \in N_{i_{p+1}}(g), d_{i_o k}(g) = p+1} \left(1 - \widetilde{a}_{i_{p+1} i_{p+2}} \frac{(1-b)^{n_k(g)}}{n_k(g)}\right)\right]$$

The individual probability of player i getting a job through contacts when information can be relayed from player to neighbor without restrictions is then:

$$\widetilde{P}_i(g) = 1 - \prod_{j \in N_i(g)} \left[1 - \widetilde{a}_{ij}(g)(1-b)\frac{1-(1-b)^{n_j(g)}}{b n_j(g)}\right].$$

j is sufficiently well-connected, $\widetilde{P}_i$ increases with n_j as long as the players newly connected by j are not themselves too well-connected. If player j is poorly connected, $\widetilde{P}_i$ decreases with n_j unless the players newly connected by j are themselves very poorly connected. Also, $\widetilde{P}_i$ decreases with n_j when the players newly connected by j are in the direct neighborhood of player i.

The intuition behind this qualitative result is the following. Suppose rst that player $j \in N_i$, direct neighbor of player i, is very well-connected. If this is the case, player i is already harmed by the information sharing constraints imposed by player j and his large neighborhood. The only value of the link ij to himself is the opportunities it offers in terms of word-of-mouth information gathering. Therefore, the better player j is connected, the higher j s probability of knowing of a vacant job through relayed information by any of his neighbors. The effectiveness of player i s passive job search thus increases when his neighbors expand their neighborhood as long as the newly connected players constitute fruitful information sources with no risk of their information being too spread away (such players need not be too richly connected). On the other hand, when the direct neighbor j of player i is poorly connected the inverse is true. Now, the opportunities the neighbor j offers to player i in terms of employment information relayed from elsewhere are insigni cant. Player j, though, is an extremely valuable source of information to player i because player j shares this information with a small set of potential competitive recipients. It is thus of interest to player i that his neighbor j does not enhance his neighborhood thus exerting a sharper constraint in terms of information sharing, unless the newly contacted player is so poorly connected that it constitutes an extremely reliable (thus highly valuable) source of relayed information.

We now restrict to symmetric networks with common neighborhood (network) size μ. Within this class of networks, the impact an additional link has on the individual probability $\widetilde{P}(\mu)$ of nding a job through word-of-mouth communication is ambiguous. We can just infer from the previous Lemma that $\widetilde{P}$ increases for very low values and for intermediate values of the network size. The following Lemma clari es the asymptotic behavior of $\widetilde{P}$.

Lemma 8 *In a symmetric network, $\widetilde{P}$ is asymptotically decreasing.*

Even though job information may now be relayed through word-of-mouth from employed worker to employed worker before reaching an unemployed individual, congestion externalities still arise in densely connected networks. In other words, the emergence of such congestion externalities is immune to the restrictions imposed on the length of information transmission. In fact, in networks of high size, the probability that two players with a common neighbor are themselves directly connected is $\sim \frac{\mu}{n}$, thus directly related to the network size.[25] The higher the network size (relative to the population size), the higher the probability that two indirectly connected players are in fact directly connected. Therefore, if some player is unemployed, it is unlikely that the available information about job vacancies is relayed to him through intermediary workers. Rather, because the probability of the initially recipient of this information being directly connected to the unemployed is high, the unemployed workers hear of vacant jobs through direct contacts and without the intervention of any middle mediator. Therefore, relaxing the restrictions imposed on the length of information transmission does not alter the pattern of information diffusion in densely connected networks. We nonetheless lose concavity at the expenses of analytical tractability.[26]

5.2 Related literature

This paper attempts to bring together several disparate literatures. Sociologists and labor economists have produced a broad empirical literature on labor market networks. The seminal contributions of Rees (1966), Rees and Schultz (1970) and Reynolds (1951) describe and reveal the role of communication among social contacts in a labor market context. Granovetter (1995) documents how individuals in a white-collar labor market frequently locate their positions of employment through distant social acquaintances that he denominates weak ties, in opposition to strong ties constituted by close friends or relatives. Later contributions by Barron and Gilley (1981),

[25] Strictly speaking, the (average) probability that two vertices of a graph are connected given that they have a mutual friend (sometimes called the clustering coefficient) depends upon the local characteristics of the network. The value $\frac{\mu}{n}$ is just the asymptotic approximation of this probability for a random graph where $\frac{\mu n}{2}$ out of all possible $\frac{n(n-1)}{2}$ links are chosen at random and with equal probability. See Bollobás (1985) for more details.

[26] Note also that allowing for information to be relayed through word-of-mouth with no restrictions on the length of transmission complicates sharply the analysis. Indeed, when job slots smay be relayed further away than one s neighbors neighbors, the local topology of the network becomes relevant, and who is connected to whom as well as relative local position in the graph matter. Two networks of same size but with different topology may now perform information transmission very differently.

Corcoran *et al.* (1980), Datcher (1983) and Holzer (1996) among others explore the determinants of search method choices, including referral networks, and their different effects on employment outcomes of individuals. They also illustrate the pervasiveness of job contact networks and their relative effectiveness among sex, race and age groups.[27] More recently, Conley and Topa (1999) and Topa (1999) show that the observed spatial distribution of unemployment in Chicago is consistent with a model of local interactions and information spillovers, and may thus be generated by agent s reliance in informal methods of job search such as networks of personal contacts. In particular, the information spillover obtained varies with local urban characteristics, decreasing with the education level and the crime rate, and increasing in the percentage of minorities.

While the pivotal role of labor market networks has long been known to economists, there are few formal models that incorporate them. Some notable exceptions are Mortensen and Vishwanath (1994), Montgomery (1991) and Kugler (1997) that contribute to the theoretical literature on equilibrium wage determination in search markets.[28] Mortensen and Viswanath (1988) analyze an equilibrium search model with wage posting where the wage distribution is a mixture of the wages offered by employers and the wages earned by the worker s personal contacts. The distribution of wages that any particular worker actually receives thus depends on the worker s information mix. At equilibrium, despite all workers being equally productive, wages earned by personal contacts are more likely to be higher than those obtained through direct applications. In contrast, Montgomery (1991) assumes that workers are heterogeneous in productivity and develops an adverse selection model where contacts provide information about the productivity of poten-

[27] Indeed, the use of contact networks varies with the social group considered. For instance, it is high in European countries with higher shares of long-term unemployed and higher replacement ratios (54% of the nish workers and 63% of italian workers nd their jobs using connections and help from friends, relatives or former employees). On the contrary, employee referrals only play a minor role in the United States (between 35% and 40% nd their jobs through referrals), the United Kingdom (35%) and Japan (27%). Kugler (1997) documents these features and provides and exhaustive survey of the related literature. She also shows that workers nding jobs through contacts are usually payed higher wages, have higher productivity and higher tenure than those nding jobs through more formal search methods.

[28] Other theoretical (and sometimes also empirical) works focusing on the choices and effects of both formal and informal search methods are for instance Holzer (1988), Montgomery (1992, 1994) and Saloner (1985).

tial workers to the firm. Assuming that high-ability workers are more likely to refer individuals like themselves (a property often called assortive matching), at equilibrium both workers and firms prefer hiring through referrals rather than through formal channels. Moreover, workers with more contacts receive higher wages and firms hiring through contacts make higher profits. Finally, Kugler (1997) considers a matching model where heterogeneous employers hire workers either through referrals or via formal methods, while heterogeneous workers search for jobs using either formal methods or referrals. At equilibrium, the matching process sorts firms and workers into two distinct groups: referrals match good jobs to good workers, while formal methods match less attractive jobs to less productive workers. Moreover, this equilibrium search model replicates the same relationship between wage premia and the use of networks empirically documented in the first part of the paper.

Contrarily to those papers, wage determination is not an issue in our model. Our focus is rather on the micro-determinants of labor networks (emerging pattern of bilateral relationships and network size), the resulting (endogenous) returns of passive job search through personal contacts[29] and the associated unemployment rate in the economy. Social networks of job information diffusion constitute by themselves a labor market institution whose behavior and efficiency properties are better understood within our framework where social relations result from the pursuit of self-interest by strategic individuals.

Finally, recent contributions have directly tackled the issue of noncooperative or cooperative endogenous network formation in a general context or in specific situations, both in a static and a dynamic framework, and with networks modeled either as directed or undirected graphs. Examples of this flourishing new strand of research on network formation are Bala and Goyal (2000), Dutta and Mutuswami (1997), Dutta and Jackson (2000), Jackson

[29] The theoretical literature on equilibrium wage determination in search markets scarcely addresses the issue of endogenous determination of the gross return of the different information sources (formal and informal) available to the worker. An exception is for instance Seatter (1979) that derives a simple vacancy contact function exhibiting disminishing returns to search intensity for spatial reasons. Following an approach similar in spirit to our paper, Calvó-Armengol and Zenou (2000) derive an explicit micro scenario (involving a day-to-day process in which currently vacant jobs are posted by firms and currently active job seekers apply for these vacancies) in a partial equilibrium setup that leads to a well-defined job matching function, where the role of social networks and word-of-mouth information transmission is made explicit.

and Wolinsky (1996) and Jackson and Watts (1998). Our paper is more closely related to Jackson and Wolinsky (1996) that de ne and discuss the concept of pairwise stability that provides us with an appropriate equilibrium concept to characterize endogenously created job contact networks.

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A Proofs

A.1 Proof of Lemma 2

First note that the function Q can be written $Q(x) = [q(x)]^x$, where $q(x) = 1 - a(1-b)\frac{1-(1-b)^x}{bx}$, $\forall x \in [1, +\infty)$. We prove this lemma in four steps.

Claim 1 The function q is strictly concave on $[1, +\infty)$.

Proof. Proving that $q(x) = 1 - a(1-b)\frac{1-(1-b)^x}{bx}$ is strictly concave is equivalent to showing that $g(x) = \frac{1-(1-b)^x}{x}$ is strictly convex. To simplify notations, let $d = 1 - b$. Differentiating twice we get $g''(x) = \frac{1}{x^3}h(x)$ where $h(x) = 2(1-d^x) + 2(\ln d)xd^x - (\ln d)^2 x^2 d^x$. We now prove that $h(x) > 0$, $\forall x \in [1, +\infty)$. Differentiating once gives $h'(x) = -(\ln d)^3 x^2 d^x > 0$. Therefore, h is an increasing function with minimum $h(1) = 2(1-d) + 2d\ln d - d(\ln d)^2$. We are thus left to show that for all $d \in (0,1)$, $t(d) \equiv h(1)$ is positive. Differentiating gives $t'(d) = -(\ln d)^2$. Therefore t decreases with minimum $t(1) = 0$. ■

Claim 2 The function Q is strictly convex on $[1, +\infty)$.

Proof. First note that if f is a strictly convex and decreasing function ($f' < 0$ and $f'' > 0$) and g is strictly concave ($g'' < 0$), then $f \circ g$ is strictly convex. Indeed, $(f \circ g)'' = f''(g) \times [g']^2 + f'(g) \times g'' > 0$. Moreover, for all $0 < d < 1$ the function $x \mapsto d^x$ is strictly convex and decreasing while $x \mapsto q(x)$ is strictly concave (from the previous claim) and takes values on $(0,1)$. Therefore, $x \mapsto Q(x) = [q(x)]^x$ is strictly convex. ■

Claim 3 $Q'(1) < 0$.

Proof. From $Q(x) = [q(x)]^x$ we deduce that $Q'(x) = \frac{Q(x)}{q(x)}H(x)$ where $H(x) = q(x)\ln q(x) + xq'(x)$. Therefore $Q'(1) < 0 \Leftrightarrow H(1) < 0$. Direct computation gives $q'(x) = \frac{\alpha}{b}\left[\frac{1-(1-b)^x}{x^2} + \ln(1-b)\frac{(1-b)^x}{x}\right]$. From $q(1) = 1 - \alpha$ and $q'(1) = \alpha\left[1 + \frac{1-b}{b}\ln(1-b)\right]$ we get $H(1) = (1-\alpha)\ln(1-\alpha) + \alpha\left[1 + \frac{1-b}{b}\ln(1-b)\right]$. We denote by $v_b(\alpha)$ this last expression, where $\alpha \in (0, 1-b)$. Establishing that $Q'(1) < 0$ is thus equivalent to proving that $v_b(\alpha) < 0$ on $(0, 1-b)$, $\forall b \in (0,1)$. Fix $b \in (0,1)$. Differentiating once we get $v_b'(\alpha) = -\ln(1-\alpha) + \frac{1-b}{b}\ln(1-b)$ and $v_b''(\alpha) = \frac{1}{1-\alpha} > 0$. Therefore, v_b is convex implying that v_b' increases on

$(0, 1-b)$ with maximum $v_b'(1-b) = \frac{1}{b}[(1-b)\ln(1-b) - b\ln b]$. We now prove that $v_b'(1-b) < 0$. It is straightforward to see that $x \mapsto (1-x)\ln(1-x) - x\ln x$ is worth 0 at $x = 0$, $\frac{1}{2}$ and 1, takes positive values on $\left(0, \frac{1}{2}\right)$ and negative values on $\left(\frac{1}{2}, 1\right)$. We distinguish two cases:

(*i*) $b \geq \frac{1}{2}$. Then $(1-b)\ln(1-b) - b\ln b \leq 0$ implying that $v_b'(1-b) \leq 0$. Moreover, v_b' increases on $(0, 1-b)$. Therefore, $v_b'(\alpha) < 0$, $\forall \alpha \in (0, 1-b)$ implying that v_b decreases on $(0, 1-b)$. But $v_b(0) = 0$. Hence, $v_b(\alpha) < 0$, $\forall \alpha \in (0, 1-b)$;

(*ii*) $b < \frac{1}{2}$. Now $(1-b)\ln(1-b) - b\ln b > 0$ implying that $v_b'(1-b) > 0$. We already know that $v_b'(0) < 0$ and that v_b' increases. Therefore, there exists some $\alpha_b^* \in (0, 1-b)$ such that v_b decreases on $(0, \alpha_b^*)$ and increases on $(\alpha_b^*, 1-b)$. As $v_b(0) = 0$ it thus suffices to show that $v_b(1-b) < 0$ to conclude that $v_b(\alpha) < 0$ on $(0, 1-b)$. We have $v_b(1-b) = b\ln b + (1-b)\left[1 + \frac{1-b}{b}\ln(1-b)\right]$. We know that $b\ln b < (1-b)\ln(1-b)$ when $b \in \left(0, \frac{1}{2}\right)$. Therefore, $v_b(1-b) < \frac{1-b}{b}[b + \ln(1-b)]$. It is easy to check that $x \mapsto x + \ln(1-x)$ is negative on $(0, 1)$. Indeed, this function is worth 0 at $x = 0$ and decreases on $(0, 1)$. Hence, $v_b(1-b) < 0$ implying that $v_b(\alpha) < 0$, $\forall \alpha \in (0, 1-b)$. ■

Claim 4 For high values of $x \in [1, +\infty)$, the function Q increases towards its limit $\lim_{x \to +\infty} Q(x) = \exp\left(-\frac{\alpha}{b}\right)$, with $Q(1) > \exp\left(-\frac{\alpha}{b}\right)$.

Proof. We know that $Q'(x) = \frac{Q(x)}{q(x)} H(x)$ where $H(x) = q(x)\ln q(x) + xq'(x)$ and $q'(x) = \frac{\alpha}{b}\left[\frac{1-(1-b)^x}{x^2} + \ln(1-b)\frac{(1-b)^x}{x}\right]$. When $x \to +\infty$ we thus have $xq'(x) \sim \frac{a(1-b)}{bx}$, $q(x) \sim 1 - \frac{a(1-b)}{bx}$ and $\ln q(x) \sim -\frac{a(1-b)}{bx}$. Hence, $H(x) \sim \left[\frac{a(1-b)}{bx}\right]^2$ meaning that $Q'(x) > 0$ for high values of x. Therefore, Q increases towards its limit $\exp\left(-\frac{\alpha}{b}\right)$ when $x \to +\infty$. We now show that $Q(1) > \exp\left(-\frac{\alpha}{b}\right) \Leftrightarrow z(a) > 0$, $\forall a \in (0, 1)$ with $z(a) = b\ln(1 - a(1-b)) + a(1-b)$. Differentiating gives $z'(a) = (1-b)\left[1 - \frac{b}{1-a(1-b)}\right] > 0$, $\forall a \in (0, 1)$. Therefore z increases on $(0, 1)$ with $z(0) = 0$, implying that $z(a) > 0$, $\forall a \in (0, 1)$. ■

A.2 Existence of stable networks: proofs

A.2.1 Proofs of Proposition 4 and Corollary 3

For all $x \geq 1$, let $R(x) = q(x)^{x-1}[1 - q(x)]$ and $L(x) = q(x)^x [1 - q(x+1)]$. We rst establish the following Lemma.

Lemma 9 *If $\alpha \leq b$, R is strictly decreasing on $[1, +\infty)$. If $\alpha > b$, there exists some $\widehat{n} \geq 1$ such that for all $n \leq \widehat{n}$, R is strictly decreasing on $[1, n-1]$.*

Proof. After some algebra we get

$$R'(x) = R(x) \ln q(x) + q'(x) q(x)^{x-2} [x - 1 - xq(x)].$$

We know that q increases, implying that $q'(x) > 0$, $\forall x \geq 1$. Moreover, $q(x) < 1$ implying that $\ln q(x) < 0$, $\forall x \geq 1$. Therefore, to prove that $R'(x) < 0$ it suffices to show that $x - 1 - xq(x) < 0 \Leftrightarrow q(x) > 1 - \frac{1}{x}$. But $q(x) = 1 - \alpha \frac{1-(1-b)^x}{bx}$ by de nition. This last inequality is thus equivalent to $a(1-b)[1-(1-b)^x] < b$. The function $l(x) = a(1-b)[1-(1-b)^x]$ de ned on $[1, +\infty)$ is strictly increasing with minimum $l(1) = a(1-b)b < b$ and supremum equal to $\lim_{x \to +\infty} l(x) = a(1-b)$. We now distinguish two cases:

(i) $a(1-b) \leq b$: then for all $x \geq 1$, $l(x) < b$ implying that R decreases on $[1, +\infty)$;

(ii) $a(1-b) > b$: let then $\widehat{x} > 1$ such that $l(\widehat{x}) = b$. Then clearly $l(x) < b$, $\forall x < \widehat{x}$ implying that R decreases on $[1, \widehat{x})$. We then set $\widehat{n} = \lceil \widehat{x} \rceil$. Clearly, R strictly decreases on $[1, n-1]$, $\forall n \leq \widehat{n}$. ■

Proof of Proposition 4. Proposition 3 states that a symmetric network with neighborhood size μ is pairwise stable if and only if $L(\mu) < \frac{c}{w\beta} \leq R(\mu)$. We now prove that there exists at least one solution μ_s to this system of inequalities. Lemma 9 establishes conditions such that R is a decreasing function. Assume that these conditions hold. Then R decreases and in particular $R(x) \leq R(1) = \alpha$, $\forall x \geq 1$. Hence, if $\frac{c}{w\beta} \leq \alpha$ there exists at least one μ such that $L(\mu) < \frac{c}{bw} \leq R(\mu)$. We now prove that there exists at most three successive neighborhood sizes satisfying $L(\mu) < \frac{c}{w\beta} \leq R(\mu)$. We rst note that $R(x+1) = q(x+1)^x [1 - q(x+1)]$ and $q(x+1) > q(x)$. Therefore, $R(x+1) > L(x)$, $\forall x \geq 1$. Let μ_s such that $L(\mu_s) < \frac{c}{w\beta} \leq R(\mu_s)$. We distinguish three cases:

(i) if $R(\mu_s+1) < L(\mu_s-1)$, μ_s is the unique stabilizing neighborhood size when $\frac{c}{w\beta} \in (R(\mu_s+1), L(\mu_s-1)]$ whereas both μ_s and μ_s+1 (resp. μ_s and μ_s-1) are stabilizing when $\frac{c}{w\beta} \in (L(\mu_s), R(\mu_s+1)]$ (resp. $\frac{c}{w\beta} \in (L(\mu_s-1), R(\mu_s)]$);

(ii) if $R(\mu_s+1) > L(\mu_s-1)$, μ_s-1, μ_s and μ_s+1 altogether are stabilizing neighborhood sizes when $\frac{c}{w\beta} \in (L(\mu_s-1), R(\mu_s+1)]$ whereas only μ_s and μ_s+1 (resp. μ_s and μ_s-1) are stabilizing when $\frac{c}{w\beta} \in (L(\mu_s), L(\mu_s-1)]$ (resp. $\frac{c}{w\beta} \in (R(\mu_s+1), R(\mu_s)]$);

(iii) if $R(\mu_s+1) = L(\mu_s-1)$, μ_s and μ_s+1 (resp. μ_s and μ_s-1) are stabilizing when $\frac{c}{w\beta} \in (L(\mu_s), L(\mu_s-1)]$ (resp. $\frac{c}{w\beta} \in (R(\mu_s+1), R(\mu_s)]$). ■

Proof of Corollary 3. We know from Lemma 6 that the continuous function R is strictly decreasing on $[1,+\infty)$ (under certain conditions), implying that μ_s is an increasing function of c. By de nition of $c_{\max}$, $\mu_s(c_{\max}) = 1$. Moreover, $\lim_{x\to+\infty} R(x) = 0$, implying that $\lim_{c\to 0} \mu_s(c) = n-1$. ■

A.2.2 Proofs of Propositions 5 and 6

We now have to establish under what conditions, if any, it is possible to connect n players in such a way that they all have μ different neighbors. To this purpose, we use the following general existence theorem on graph theory due to Erdös and Gallai [see Berge (1970)]:

Theorem 1 *Let $d_1 \geq d_2 \geq \cdots \geq d_n$ a sequence of n integers such that $\sum_{i=1}^{n} d_i$ even. The two following conditions are equivalent:*

(a) *$\exists g \in G$ such that $n(g,i) = d_i$, $\forall i \in N$;*

(b) $\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{j=k+1}^{n} \min\{k, d_j\}$, $\forall k \in \{1,\ldots,n\}$.

We use this theorem to establish Propositions 5 and 6.

Proof of Proposition 5. Let $d_1 = \cdots = d_n = \mu_s$. If n is even or if μ_s is even, $\sum_{i=1}^{n} d_i = n\mu_s$ is also even. Proving that a μ_s−symmetric network exists is thus equivalent to showing that condition (b) of Theorem 1 holds. This condition is equivalent to:

$$k\mu_s \leq k(k-1) + (\mu_s - k - 1)k + (n - \mu_s + 1)\mu_s, \forall k \in \{1, \ldots, \mu_s - 2\} \quad (1)$$

$$k\mu_s \leq k(k-1) + (n-k)\mu_s, \forall k \in \{\mu_s - 1, n\} \quad (2)$$

Condition (1) is equivalent to $2k \leq \mu_s(n - \mu_s + 1), \forall k \in \{1, \ldots, \mu_s - 2\}$. But as $k \leq \mu_s$, it suffices to show that $2\mu_s \leq \mu_s(n - \mu_s + 1) \Leftrightarrow \mu_s \leq n - 1$ which is true.

Let $u_k = k(k-1) + (n-k)\mu_s - k\mu_s$. Condition (2) is equivalent to $u_k \geq 0, \forall k \in \{\mu_s - 1, \ldots, n\}$. Let $f(x) = x(x-1) + (n-x)\mu_s - x\mu_s$ de ned on $\mathbb{R}$: $f'(x) = 2x - 1 - 2\mu_s$ and $f''(x) = 2$. Thus f is strictly convex and reaches its (global) minimum at $x^* = \mu_s + \frac{1}{2}$. Hence, $\min_{k \in \{\mu_s - 1, \ldots, n\}} u_k = \min\{u_{\mu_s}, u_{\mu_s+1}\} = \mu_s(n - 1 - \mu_s) \geq 0$. ■

Proof of Proposition 6. Let $d_1 = \cdots = d_{n-1} = \mu_s$ and $d_n = \mu_s - 1$. If n and μ_s are odd, $\sum_{i=1}^{n} d_i = (n-1)\mu_s + \mu_s - 1$ is even. Proving that a pairwise stable architecture with neighborhood sizes d_i exists is thus equivalent to showing that condition (b) of Theorem 1 holds. This condition is equivalent to:

$$k\mu_s \leq k(k-1) + (\mu_s - k - 1)k + (n - \mu_s + 1)\mu_s, \forall k \in \{1, \ldots, \mu_s - 2\} \quad (3)$$

$$k\mu_s \leq k(k-1) + (n - k - 1)\mu_s + \mu_s - 1, \forall k \in \{\mu_s - 1, \ldots, n - 1\} \quad (4)$$

$$(n-1)\mu_s + \mu_s - 1 \leq n(n-1) \quad (5)$$

Condition (3) coincides with condition (1), already established in the previous proof. Condition (5) is equivalent to $n(n - 1 - \mu_s) \geq -1$, which is true. We are thus left with condition (4).

Let $u_k = k(k-1) + (n-k)\mu_s - k\mu_s$. Condition (4) is then equivalent to (after some algebra) $\min\limits_{k\in\{\mu_s-1,\dots,n-1\}} u_k = \mu_s(n-1-\mu_s) \geq 1$. The complete graph g^N is such that $\mu_s = n-1$, and its existence is not problematic. Therefore, $\mu_s < n-1$ and condition (4) is equivalent to $\mu_s \geq 1$ which is true because μ_s is odd.

■

A.3 Other proofs

Proof of Proposition 1. Assume that some player $j \in N_i$ in the neighborhood of player i hears of a vacant job and does not need it. The (conditional) probability that player j transmits this job information to player i is given by $(1-b)^{n_j-1} + \sum\limits_{k=1}^{n_j-1} \binom{n_j-1}{k} (1-b)^{n_j-k-1} b^k \left(\frac{1}{k+1}\right) = \frac{1-(1-b)^{n_j}}{bn_j}$. Therefore, the probability that i does not actually nd a job thanks to j is $1 - a(1-b)\frac{1-(1-b)^{n_j}}{bn_j}$, which gives the desired result. ■

Proof of Lemma 1. First note that $\frac{\partial P_i}{\partial n_i} > 0$ (resp. $\frac{\partial P_i}{\partial n_j} < 0$) is equivalent to $\frac{\partial Q_i}{\partial n_i} < 0$ (resp. $\frac{\partial Q_i}{\partial n_j} > 0$). Increasing n_i by 1 is equivalent to adding a new neighbor k to the current i s neighborhood N_i, which in turn implies multiplying Q_i by the additional term $0 < 1 - a(1-b)\frac{1-(1-b)^{n_k}}{bn_k} < 1$. Clearly, Q_i decreases. Let now $g(x) = (1-b)^x$. The function g is convex (resp. strictly convex) for all $x \geq 1$ (resp. $x > 1$). Therefore, $\frac{(1-b)^x-1}{x} = \frac{g(x)-g(0)}{x-0}$ increases on $(1, +\infty)$, implying that $1 - a(1-b)\frac{1-(1-b)^{n_j}}{bn_j}$ is an increasing function of n_j. Hence, Q_i increases with n_j, $\forall j \in N_i$. ■

Proof of Lemma 3. We know that $P_i = 1 - Q_i$, where $Q_i = \prod\limits_{j\in N_i}\left[1 - a(1-b)\frac{1-(1-b)^{n_j}}{bn_j}\right]$. For all $i \in N$ and $j \in N_i$, let $q_{ij} = 1 - a(1-b)\frac{1-(1-b)^{n_j}}{bn_j}$ denote the probability of player i not nding a job thanks to his neighbor j. We have $\frac{\partial q_{ij}}{\partial a} < 0$ implying that $\frac{\partial Q_i}{\partial a} < 0 \Leftrightarrow \frac{\partial P_i}{\partial a} > 0$. Also, rearranging terms gives $q_{ij} = 1 - a\left(\frac{1}{b} - 1\right)\frac{1-(1-b)^{n_j}}{n_j}$ where $\frac{\partial}{\partial b}\left[-a\left(\frac{1}{b}-1\right)\right] > 0$ and $\frac{\partial}{\partial b}\left[\frac{1-(1-b)^{n_j}}{n_j}\right] > 0$. Therefore, $\frac{\partial q_{ij}}{\partial b} > 0$ implying that $\frac{\partial Q_i}{\partial b} > 0 \Leftrightarrow \frac{\partial P_i}{\partial b} < 0$. ■

Proof of Lemma 4. Let $g \in G^*$ and let $i, j \in N$ such that $ij \notin g$. We want to check whether player i is better off adding the link ij or not. When the network is g, player i gets $Y_i(g) = w\{(1-b) + b[a + (1-a)P_i(g)]\} - cn_i(g)$. If the link ij is added, the network becomes $g + ij$ and player i gets $Y_i(g + ij) = w\{(1-b) + b[a + (1-a)P_i(g+ij)]\} - c[n_i(g)+1]$.

Therefore, $Y_i(g + ij) - Y_i(g) = w\beta[P_i(g+ij) - P_i(g)] - c$, and player i gains by adding the link ij if and only if: $P_i(g+ij) - P_i(g) > \frac{c}{w\beta} \Leftrightarrow Q_i(g) - Q_i(g+ij) > \frac{c}{w\beta} \Leftrightarrow [1 - q(n_j(g)+1)]\, Q_i(g) > \frac{c}{w\beta}$. ■

Proof of Corollary 1. We deduce from Lemma 4 that any player i bene ts from unilaterally severing an existing link with some other player j if and only if $1 - q(n_j(g)) < \frac{c}{w\beta Q_i(g-ij)}$. We know that for all $g \in G^*$ and for all $i \in N$, $0 < Q_i \leq 1$ implying that $\frac{c}{w\beta Q_i} \geq \frac{c}{w\beta}$. Moreover, $1 - q(x) \leq \alpha$, $\forall x \geq 1$. Therefore, when $\frac{c}{w} > \alpha\beta$ the inequality $1 - q(n_j(g)) < \frac{c}{w\beta Q_i(g-ij)}$ is satis ed $\forall g \in G^*$, $\forall i \in N$ and $\forall j \in N_i(g)$ meaning that unilateral severance of a link is always pro table for all players. Consequently, no graph is stable. ■

Proof of Proposition 2. Follows from Lemma 4 and the de nition of pairwise stability. ■

Proof of Proposition 3. Applying Lemma 4, a symmetric network of size μ is pairwise stable if and only if both:

(*a*) $\frac{c}{w\beta} \leq q(\mu)^{\mu-1}[1 - q(\mu)]$ (no player wants to cut one of the μ existing connections);

(*b*) $q(\mu)^{\mu}[1 - q(\mu+1)] < \frac{c}{w\beta}$ (no player wants to add a $(\mu+1)-$th connection). ■

Proof of Corollary 2. Straightforward from the previous proposition. ■

Proof of Proposition 7. The pool of unemployed is given by $nb\left[1 - a - (1-a)\sum_{i \in N} P_i(g)\right]$ which gives the desired result. ■

Proof of Lemma 5. We have $u(g) = b(1-a)\left[1 - \frac{1}{N}\sum_{i \in N} P_i(g)\right]$. From Lemma 3, $P_i(g)$ decreases with b, $\forall i \in N$ implying that $u(g)$ increases with b. ■

Proof of Corollary 4. Straightforward from Proposition 7. ■

Proof of Proposition 8. We know from Lemma 2 that P is a strictly concave function, implying that W is also strictly concave. The efficient network size μ_e is thus given by the unique solution to $W'(\mu) = 0$ that is, $b(1-a)P'(\mu) = c$ implying in particular that $P'(\mu_e) > 0$. Recall that the congestion threshold value $\bar{\mu}$ of P satis es $P'(\bar{\mu}) = 0$. Therefore, $\mu_e < \bar{\mu}$. ■

Proof of Corollary 5. We have $b(1-a)P'(\mu_e) = c$ and P' decreasing. Therefore, μ_e decreases with c. When $c \to 0$ we have $P'(\mu_e) \to 0$, implying that $\mu_e \to \bar{\mu}$. ■

Proof of Proposition 9. Straightforward from Corollaries 3 and 5. ■

Proof of Lemma 6. We need to compute the probability $\widetilde{a}_{ij}(g)$ of some player $j \in N_i$ in the neighborhood of player i hearing of a vacant job that he may transmit to his neighbor i. Player j can either receive this information directly (probability a) or relayed by any of his employed direct neighbors $k \in N_j$. For the relay to be effective, player k does not have to be in direct contact with player i (otherwise k transmits the information he holds directly to i) that is, $k \notin N_i$ equivalent to $d_{ik}(g) = 2$. Also, none of k direct neighbors has to be unemployed, which happens with probability $(1-b)^{n_k(g)}$. Then, $1 - a\frac{(1-b)^{n_k(g)}}{n_k(g)}$ gives the probability that player j does not receive relayed information from his informed direct neighbor k. Direct computation gives $\widetilde{a}_{ij}(g) = a + \delta(1-a)\left[1 - \prod_{k \in N_j(g), d_{ik}(g)=2}\left(1 - a\frac{(1-b)^{n_k(g)}}{n_k(g)}\right)\right]$, from which we derive the expression for $\widetilde{P}_i(g)$. ■

Proof of Lemma 7. Suppose that player $j \in N_i$ establishes a new link with some player h. The resulting graph is $g \cup jh$. We distinguish two cases:

(i) $h \in N_i$ that is, player h is player i s direct neighbor. Then $\widetilde{a}_{ij}(g + jh) = \widetilde{a}_{ij}(g)$ and $\frac{1-(1-b)^{n_j(g \cup jh)}}{n_j(g \cup jh)} = \frac{1-(1-b)^{n_j(g)+1}}{n_j(g)+1} < \frac{1-(1-b)^{n_j(g)}}{n_j(g)}$. Therefore, $\widetilde{P}_i(g + jh) < \widetilde{P}_i(g)$;

(ii) $h \notin N_i$ that is, player h is not in the direct neighborhood of player i. Then $\widetilde{a}_{ij}(g + jh) > \widetilde{a}_{ij}(g)$. By direct computation, $\widetilde{P}_i(g + jh) > \widetilde{P}_i(g) \Leftrightarrow \widetilde{a}_{ij}(g + jh)\frac{1-(1-b)^{n_j(g)+1}}{n_j(g)+1} > \widetilde{a}_{ij}(g)\frac{1-(1-b)^{n_j(g)}}{n_j(g)}$. When player j expands his set of connections by creating an additional link with player h, his neighborhood size increases by one link from $n_j(g)$ to $n_j(g) + 1$. As a result, the rst term in the expression $\widetilde{a}_{ij}(g)$ increases by an amount negatively related to $n_h(g)$ and positively related to $n_j(g)$. On the contrary, the second term in the expression $\frac{1-(1-b)^{n_j(g)}}{n_j(g)}$ decreases by an amount negatively related to $n_j(g)$ and independent of $n_h(g)$. One can check that the inequality $\widetilde{a}_{ij}(g+jh)\frac{1-(1-b)^{n_j(g)+1}}{n_j(g)+1} > \widetilde{a}_{ij}(g)\frac{1-(1-b)^{n_j(g)}}{n_j(g)}$ holds when $n_j(g)$ is high, as long as $n_h(g)$ is not too high, whereas it does not hold when $n_j(g)$ is low, as long as $n_h(g)$ is not too low, and reciprocally. ■

Proof of Lemma 8. In a symmetric network $\widetilde{P}(x) = 1 - \widetilde{Q}(x)$ with $\widetilde{Q}(x) = \left[1 - \widetilde{a}(x)\frac{1-(1-b)^x}{bx}\right]^x$ and $\widetilde{a}(x) = a + \delta(1-a)\left\{1 - \left[1 - a\frac{(1-b)^x}{x}\right]^{x-1}\right\}$. We can check that $\widetilde{a}(x) = a + o(1)$ when $x \to +\infty$ implying that $\widetilde{P}(x) \underset{x\to+\infty}{\sim} P(x)$. The asymptotic properties of $\widetilde{P}$ are thus derived from those of P established in Lemma 2. ■