

Bargaining with Stochastic Pairwise Meetings: Application to a Decentralized Market

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Abstract

We analyze a decentralized market where transactions are concluded at pairwise meetings of agents. We explicitly take into account the structure of trading relationships modeled by a stochastic matching process. This stochastic matching technology constitutes a rich framework embodying a huge variety of trading links, and is consistent with the variability inherent to agents' behavior in a decentralized market. It allows us both to specify and to specialize the way in which those who wish to buy are matched with those who wish to sell, the mechanism of price determination, and how transactions evolve through time. We analyze the impact of different types of market organization (corresponding to different stochastic technologies) on market variables, and investigate in particular the role of time (history) dependence of the matching process, the potential mutual benefits derived from the establishment of regular trading relationships, the effect of introducing a bias in the selection of the first proposer within a matched pair during the bargaining round and the impact of time-varying discount factors on negotiation outcomes.

Keywords: noncooperative bargaining, stochastic matching, Markov perfect equilibrium

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1 Introduction

In markets in which the trading process is decentralized and transactions are concluded at pairwise meetings, agents may interact in many different ways. In some markets, each buyer can access to each seller and vice-versa, whilst in others individuals are endowed with only a limited subset of potential trading partners. Also, buyers may be loyal to one shop or, on the contrary, visit all shops with equal probability while searching for attractive offers. In markets where transactions are frequent (e.g. markets for perishable goods) and trade is decentralized, different patterns of behavior, constituting as many types of trading relationships, often coexist.¹

The aim of this paper is to determine the impact of the market organization on market variables such as equilibrium prices and agents values. To this purpose, we explicitly take into account the structure of trading relationships modeled through a stochastic matching process. The stochastic matching technology provides a rich framework embodying a huge variety of trading links. Moreover, it is consistent with the variability inherent to agents' behavior in a decentralized market. It allows us both to specify and to specialize the way in which those who wish to buy are matched with those who wish to sell, and how transactions evolve through time. The overall market organization is then determined by (i) the nature of trading links modeled by an appropriate stochastic matching process, (ii) the market size (number of buyers) and the number of sellers operating in the market, and (iii) the mechanism of price determination. We assume throughout the paper that the pairs of matched buyers and sellers negotiate the terms of exchange (prices at which the transactions take place) through a noncooperative bargaining procedure.

Our first result characterizes the equilibria payoffs of the Rubinstein bargaining game with stochastic pairwise meetings that describes the bilateral negotiations ongoing in our decentralized market. This game has a unique Markov perfect equilibrium outcome for which we provide an explicit characterization. It also proves fruitful to analyze two-player negotiations with time-varying discount factors, and can thus be read as a self-contained contribution to the noncooperative bargaining theory. In the second part of the paper we explore market equilibria when decentralized price setting proceeds as described by our bargaining game. A characterization of market

¹For instance, using empirical data from the Marseille wholesale fish market, Weisbuch *et al.* (1998) and Herreiner (1997) isolate two distinct groups of buyers, loyalists and searchers. Indeed, these papers reveal empirically a lot of organization in terms of prices and buyers (...) (relationships) with sellers. In particular, one observes that the most frequent buyers (...) with very few exceptions visit only one seller, while less frequent buyers visit several sellers.

equilibria is provided. We then investigate the role of time (history) dependence of the matching process, the potential mutual benefits derived from the establishment of regular trading relationships, and the effect of introducing a bias in the selection of the first proposer within a matched pair for the bargaining round.

This paper owes a lot to the literature on decentralized bilateral trade surveyed by Osborne and Rubinstein (1990) and Jackson and Palfrey (1998), that provides a rich framework used here to analyze more complex markets. A common assumption to these models is random matching with constant probabilities either time independent –as in Gale (1986a,b), Rubinstein and Wolinsky (1985), Samuelson (1992), Wolinsky (1990) and Wooders (1998)– or time dependent –as in Binmore and Herrero (1988b), Gale (1987), McLennan and Sonnenschein (1991), Ponsatí (2000) and Rubinstein and Wolinsky (1990). Other contributions include Binmore and Herrero (1988a), Hendon *et al.* (1994), Shaked (1994) and Wolinsky (1987). This modeling constitutes an appropriate approach in large and anonymous markets. We depart from these papers in at least one respect: the probabilities of being matched anew need not be independent across agents and of past events. Indeed, our model allows for the existence of preferential trading links between economic agents by assigning different relative strength (frequency of activation) to agent-to-agent meetings. For the small markets considered (we take as primitives the finite sets of the two types of agents, buyers and sellers) we assume that the individuals know the full structure of trading relationships. The behavior of a given agent now not only depends on the current period but also on the names (labels) of her potential partners, on the role they are assigned within a matched pair during the bargaining phase, and on the overall market organization as captured by the stochastic matching technology.

More recently, Merlo and Wilson (1995, 1998) and Ereslan and Merlo (1999) consider a general class of bargaining games in which both the surplus to be allocated and the players moves evolve over time according to a stochastic process. These authors fully characterize the set of equilibrium payoffs, determine the conditions under which agreement is delayed, and discuss the corresponding advantage to proposing. Although similar in spirit, our model departs from these papers by focusing on a particular stochastic process and relating the sequential bargaining game obtained to a decentralized market mechanism.

The remainder of the paper is organized as follows. Section 2 describes the noncooperative bargaining game with stochastic pairwise meetings and the corresponding decentralized market. This game is solved in Section 3. Section 4 analyzes the market where transactions are concluded at pairwise meeting of agents. Section 5 applies these results, and the impact of the market organization and trading links on market variables is discussed. An appendix contains all the proofs.

2 The model

2.1 The economy

The agents in the model are of two different types. We denote by $N_k = \{1, \dots, n_k\}$ the finite set of agents of type k ($n_k \geq 1$), and by $\delta_{i_k} \in (0, 1)$ the discount factor of player $i_k \in N_k$, where $k = 1, 2$. Note that we do not assume so far that the agents have identical time preferences. When two agents of opposite types N_1 and N_2 are matched, they bargain over the partition of a unit surplus associated with the match. If a bilateral agreement is reached, the two bargaining partners leave the market with the agreed share of the unit surplus. The model will focus on steady state situations where this flow of departures is counterbalanced by an equal arrival flow of new agents of both types. One can think, for instance, that the two types of agents are sellers (with discount factor δ_s) and buyers (with discount factor δ_b) of a divisible good traded for some quantity of an indivisible good (money). If the agreed price after t periods is p the seller's and the buyer's utility are respectively $\delta_s^t p$ and $\delta_b^t (1 - p)$. If an agent never trades her utility is zero. The discount factors δ_s and δ_b apply to expected future benefits and represent time preferences. For the sellers, time preferences are associated to storing costs for storable goods, and to the proximity of the expiry date for perishable goods. For the buyers, time preferences measure impatience.

Time is discrete and the time periods in the market life of an agent are labeled with a nonnegative integer $t \in \mathbb{N}$.

2.2 The bargaining procedure

We assume that there is only one pair of matched agents of both types that bargain per period. For some $t \in \mathbb{N}$, let $i \in N_1$ and $j \in N_2$ be the two current matched agents. These two active agents bargain over the partition of a unit surplus. We assume without loss of generality that i is the proposer and j her respondent. The ordered pair $m^t = (i, j) = (\text{proposer}, \text{respondent})$ constitutes the state of the game at period t . The state indicates the labels of the matched agents and their respective roles (either proposer or respondent) within the matched pair during the bilateral bargaining contest. The proposer makes a two-player/one-cake proposal to the respondent. If the respondent accepts the offer, the unit surplus is split at the agreed-upon terms and the two agents leave the market with their shares. In the event of rejection, we move to the next period and the agents participate in the matching process.

2.3 The matching phase

In the matching phase, each agent in the market is matched, with positive probability, with an agent of the opposite type. Thus, at a given period,

an agent can (i) have no partner, (ii) have the same partner than in the previous period, or (iii) have a new partner. We allow for the probability of being matched anew to vary across agents and to depend on past events. More precisely, we assume that agents are matched according to a Markov process. A transition function $Q(m^{t+1} | m^t)$ gives then the probability that the new ordered pair of matched agents at $t + 1$ is m^{t+1} conditional on their being m^t at period t . The game starts in some state m^0 .²

Example 1 *The probability that i and j continue bargaining together at period $t + 1$ or, equivalently, the probability that i and j are matched anew conditional on i being the proposer and j her respondent at period t is: $Q((i, j) | (i, j)) + Q((j, i) | (i, j))$. Conditional on these two agents continue bargaining together at $t + 1$, agent i remains the proposer and j her respondent with probability $\frac{Q((i, j) | (i, j))}{Q((i, j) | (i, j)) + Q((j, i) | (i, j))}$ while with probability $\frac{Q((j, i) | (i, j))}{Q((i, j) | (i, j)) + Q((j, i) | (i, j))}$ the roles of the agents within the matched pair are reversed at $t + 1$ that is, j is the new proposer and i the new respondent.*

The state space \mathcal{M} consists of all the ordered pairs of agents of both types: $\mathcal{M} = \{(i, j) \in (N_1 \times N_2) \cup (N_2 \times N_1)\}$. This state space contains $2n_1n_2$ different elements. A probability measure on \mathcal{M} is a vector π in the $2n_1n_2$ -dimensional unit simplex $\Delta_{2n_1n_2}$. The transition function Q can then be represented by an $2n_1n_2 \times 2n_1n_2$ matrix $\Theta = [\theta_{hl,ij}]$, where $\theta_{hl,ij} = Q(m^{t+1} = (h, l) | m^t = (i, j))$. The elements of the Markov matrix Θ are nonnegative and each column sum is unity. If the pair of active agents are chosen at random at period t according to the probability distribution $\pi^t \in \Delta_{2n_1n_2}$, the probability distribution over the state (ordered pair of matched agents of both types) at period $t + 1$ is given by $\pi^{t+1} = \Theta \cdot \pi^t = \Theta^t \cdot \pi^0$.

Example 2 *In the standard bilateral bargaining game of alternating offers between two players labeled 1 and 2, both players alternate in making offers that is, $\theta_{12,21} = \theta_{21,12} = 1$. The corresponding transition matrix is thus*

$$\Theta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

2.4 Strategies and equilibrium

In this game, payoffs depend on the state (identity and role³ of the matched agents that bargain) and on the current actions. Moreover the states follow

²Alternatively, one can interpret the random matching technology as a Markov random field describing the conditional probability that any two agents communicate directly with each other and bargain over the terms for trade. To our knowledge, Fölmer (1974) is the first contribution that explicitly incorporates Markov fields in an economic context. Related papers are Kirman (1983) and Kirman *et al.* (1986) that analyze the welfare properties of noncompetitive allocations in an economy with stochastic communication represented by a random graph.

³Within a matched pair, either proposer or respondent.

a Markov process that is, the probability distribution on tomorrow's states is determined by today's states and actions. We concentrate on *Markov perfect equilibria*, namely those subgame perfect equilibria where strategies only depend on the identity and the role of the players that bargain (the state) that constitutes the only payoff-relevant information. We are thus assuming that each agent rule of behavior in front of a given bargaining partner is suited to this partner: a buyer (resp. a seller) may act differently depending on the identity of the seller (resp. buyer) she faces. For all $t \in \mathbb{N}$ and for all $m^t = (i, j) \in \mathcal{M}$, $(a_{ij}^t, 1 - a_{ij}^t)$, with $0 \leq a_{ij}^t \leq 1$, denotes the two-player/one-cake proposal made by the proposer i to her bargaining partner j at period t . If the respondent j accepts such a proposal she gets $1 - a_{ij}^t$ while the proposer i ends up with a_{ij}^t . From now on and for notational simplicity we omit the superscript t when there is no risk of confusion.

3 Bargaining with stochastic pairwise meetings

3.1 Solving the bargaining game

The following proposition characterizes the unique equilibrium payoff of the bargaining game with stochastic matching technology.⁴

Proposition 1 *The bargaining game described above has a unique Markov perfect equilibrium. For all $m = (i, j) \in \mathcal{M}$, where $i \in N_k$ and $j \in N_{3-k}$ for some $k = 1, 2$, the equilibrium shares are characterized by:*

$$1 - a_{ij} = \delta_j \sum_{l \in N_k} [a_{jl} \theta_{jl,ij} + (1 - a_{lj}) \theta_{lj,ij}] \quad (1)$$

In words, the proposer concedes to her respondent the expected share this player can obtain in the continuation game. At equilibrium, players are indifferent between their share today and their share tomorrow appropriately discounted.

Consider for instance the following variation of the standard two-player game of alternating offers between two players labeled 1 and 2. Suppose that at some round player 1 makes a sharing proposal to her bargaining partner player 2. If player 2 accepts this proposal the game ends. If on the contrary the proposal is rejected, we go to the next round where player 1 is still the proposer with probability p while it is player 2's turn to propose a sharing scheme with the complementary probability $1 - p$. If player 2 was the initial proposer, we assume that the identity of the proposer at the next round is determined similarly. Therefore, the transition matrix is given by $\theta_{21,12} =$

⁴It is straightforward to see that this proposition holds in a slightly more general setting where a partition of the population in two distinct types N_1 and N_2 is not required.

$\theta_{12,21} = 1 - p$ and $\theta_{12,12} = \theta_{21,21} = p$ that is, $\Theta = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}$. The case $p = 0$ corresponds to the standard two-player game of alternating offers and the case $p = 1$ coincides with a take-it-or-leave-it offer game. In the simple case where players have identical discount factors δ the equilibrium shares given by equations (6.1) are

$$a_{12}(p) = a_{21}(p) = \frac{1-\delta p}{1-\delta p + \delta(1-p)}.$$

In particular, $a_{ij}(p)$ increases with the probability p of player i remaining the proposer at the next round in case of rejection of the current sharing proposal. The minimum $a_{ij}(0) = \frac{1}{1+\delta}$ coincides with the Rubinstein outcome of the standard game, and the maximum $a_{ij}(1) = 1$ is the outcome of a take-it-or-leave-it game. At the limit $\delta \rightarrow 1$, the equilibrium shares coincide with the standard half-half partition, no matter the value of $p \neq 1$ that is, $\lim_{\delta \rightarrow 1} a_{ij}(p) = \frac{1}{2}$, $\forall p \in [0, 1)$.

3.2 Rubinstein bargaining with time-varying discount factors

A standard feature of noncooperative bargaining games is that players have time preferences, thereby discounting future periods by a constant discount factor $\delta \in (0, 1)$. Players with low δ have a strong preference for current payoffs with respect to future ones, and reciprocally. The discount factor δ thus measures players' impatience: the higher δ (resp. the lower δ), the more patient (resp. the more impatient) the player. Assuming that this discount factor is fixed through time implies that players maintain a constant impatience rate.

One could assume, though, that players change their tastes through time and, for instance, become more or less patient as time unravels and negotiations proceed.⁵ We shall prove that with an appropriate stochastic matching technology, noncooperative bargaining games with time-varying discount factors can be analyzed within the model of stochastic pairwise meetings studied so far. Indeed, as it will become clear, it suffices to treat the individuals at different time periods (with possibly different discount factors) as different people involved in a pairwise noncooperative bargaining game.

More precisely, let two players labeled 1 and 2 play a standard two-player game of alternating offers. For $k = 1, 2$, assume that player k 's discount

⁵Consider for instance a union bargaining with a firm. Evidence shows that a strike-bound firm may experience a decline in profitability after a certain point. Therefore, although the firm may maintain a tough position during the first days of negotiation (thus behaving as a patient player), as the strike gets longer, top executives of the firm certainly adopt a softer attitude (thus behaving as an impatient player). For a union-firm bargaining model with profitable opportunities decaying over time see Hart (1989).

factor δ_k can take a finite set of $n_k \geq 1$ different values, $\delta_k \in \{\delta_{k_1}, \dots, \delta_{k_{n_k}}\}$, and that this discount factor varies through time according to a Markov transition matrix $D^k = [D_{i,j}^k]$, where $D_{i,j}^k = \Pr\{\delta_k^{t+1} = \delta_{k_i} \mid \delta_k^t = \delta_{k_j}\}$, $\forall t \in \mathbb{N}$. It is easy to check that this game is identical to a bargaining game with stochastic matching technology as in the previous section, where the $2n_1n_2 \times 2n_1n_2$ Markov transition matrix $\Theta = [\theta_{hl,ij}]$ governing pairwise meetings is given by:

$$\theta_{hl,ij} = D_{i,i}^k D_{h,j}^{3-k}, \text{ if } i, l \in N_k \text{ and } j, h \in N_{3-k} \text{ for some } k = 1, 2$$

Applying Proposition 1 yields the following result.

Corollary 1 *Two-player bargaining games of alternating offers with time-varying discount factors have a unique Markov perfect equilibrium outcome.*

We now use this result to analyze the following example. Suppose that player 1 has a constant discount factor $\delta_1 \equiv \delta$ while the discount factor of player 2 can take two different values, $\delta_2 \in \{\delta_L, \delta_H\}$, where $\delta_L < \delta_H$. Player 2 is initially of the patient type, meaning that $\delta_2^{t=0} = \delta_H$, and she is bound to become more impatient as time unravels. To simplify matters, we suppose that player 2's discount factor varies through time according to the following transition matrix $D = \begin{bmatrix} 1 & d \\ 0 & 1-d \end{bmatrix}$, where $d = \Pr\{\delta_2^{t+1} = \delta_L \mid \delta_2^t = \delta_H\}$, $d \neq 0$, is the probability of player 2 becoming impatient tomorrow conditional on being patient today. According to this matrix, player 2 is almost surely impatient in the long run as $\Pr\{\delta_2^{t+1} = \delta_L \mid \delta_2^0 = \delta_H\} = 1 - (1-d)^t \rightarrow 1$ when $t \rightarrow \infty$.⁶

The bargaining game proceeds as follows. At $t = 0$, the first proposer player 1 submits a two-player/one-cake offer to player 2 (with discount factor $\delta_2^{t=0} = \delta_H$) that may accept it or reject it. In case of rejection we move to the next round where player 2 is the new proposer, with discount factor $\delta_2^{t=1} \in \{\delta_L, \delta_H\}$ determined by the transition matrix D that is, $\Pr\{\delta_2^{t=1} = \delta_L\} = d = 1 - \Pr\{\delta_2^{t=1} = \delta_H\}$. And so on.

Let $a(d) = \frac{1-\delta_H+d(2-d)\delta\delta_H a_{1L}}{1-(1-d)^2\delta\delta_H}$, where $a_{1L} = \frac{1-\delta_L}{1-\delta\delta_L}$ is the Rubinstein outcome of a standard two-player game of alternating offers between a player with discount factor δ and an impatient player with discount factor δ_L .

Lemma 1 *At round $t = 0$, player 1 proposes the share $1 - a(d)$ to player 2 that immediately accepts it, and player 1 ends up with $a(d)$.*

It is easy to check that $a(d)$ is an increasing function of the likelihood d with which a patient player becomes impatient. The minimum $a(0) =$

⁶The state "player 2 is impatient" is a transient state, and the state "player 2 is patient" is the only recurrent state.

$\frac{1-\delta_H}{1-\delta\delta_H} = a_{1H}$ corresponds to the Rubinstein outcome of a standard two-player game between a player with discount factor δ and a patient player with discount factor δ_H , and the maximum is $a(1) = 1 - \delta_H a_{L1}$.

4 The decentralized market

4.1 The market equilibrium

Suppose that the two types of agents are sellers and buyers of a divisible good traded for some quantity of an indivisible good (money) that is, $N_1 = S$ (sellers) and $N_2 = B$ (buyers). Clearly, the market conditions influence negotiations and hence the agreements with which these negotiations conclude. These market conditions include the chances that the negotiating parties have of meeting other partners if no agreement is reached, the corresponding cost of delayed agreement, the expected length required to achieve an alternative transaction, and the expected behavior of alternative partners. Assume for instance that buyers and sellers are matched according to the matching technology defined in section 2.3 and bargain over the terms of exchange (the price) according to the bargaining procedure defined in section 2.2. We define a *market equilibrium* of this economy to be a *Markov perfect equilibrium* of the noncooperative bargaining game with random matching. Proposition 1 allow us to fully characterize the market equilibrium:

Corollary 2 *There is a unique market equilibrium. Let $s \in S$ and $b \in B$ be, respectively, a seller and a buyer. In this equilibrium the seller s always proposes the price x_{sb} to the buyer b and accepts any price at least equal to y_{bs} from this buyer, and the buyer b always proposes the price y_{bs} to the seller s and accepts any price at most equal to x_{sb} , where the equilibrium prices satisfy, for all $(s, b) \in S \times B$:*

$$\begin{cases} 1 - x_{sb} = \delta_b \sum_{i \in S} [(1 - y_{bi}) \theta_{bi, sb} + (1 - x_{ib}) \theta_{ib, sb}] \\ y_{bs} = \delta_s \sum_{j \in B} (x_{sj} \theta_{sj, bs} + y_{js} \theta_{js, bs}) \end{cases} \quad (2)$$

Equations (2) fully characterize the market prices at which trade takes place between paired buyers and sellers. These prices need not be the same for all units that are sold. In fact, a seller (respectively, a buyer) acts differently depending on the identity of the buyer (respectively, the seller) she is matched with. Different buyers may then get their units at different prices. Indeed, the matching technology considered models a situation in which agents condition their behavior on the identities of their potential partners. In other words, *the trading process is not impersonal*. Rubinstein and Wolinsky (1990) obtain a similar result of non-anonymous price equilibrium. Trading in their model is frictionless (no cost to delay is introduced)

and the way these equilibrium prices are determined is left unmodelled. On the contrary, our paper considers costs to delay in trade, allows for a whole variety of trading relationships and provides a full characterization of the market equilibrium prices.

4.2 The agents values

Let $s \in S$ be a given seller.⁷ Seller s obtains a positive dividend every time she concludes a transaction with a buyer. The dividend she obtains with buyer b (the price at which the good is traded) can either be x_{sb} if the seller is the one that proposes the price (immediately accepted by the buyer), or y_{bs} if the buyer is the one that proposes the price (immediately accepted by the seller). The value V_s of seller s participating in the market thus depends on the market equilibrium prices and on the random matching technology determining the frequency with which a given seller is effectively matched with every possible buyer and, given that this seller is matched with a buyer, the probability with which the seller is either a proposer or a respondent within the matched pair while bargaining for the market price. This random matching technology is fully characterized by the Markov transition matrix Θ and the initial probability distribution π^0 over the pair of matched buyers and sellers at period 0. Indeed, at period $t \in \mathbb{N}$, the pair of active players are matched and their role (either proposer or respondent) within the matched pair are assigned with probability $\pi^t \in \Delta_{2BS}$, where $\pi^t = \Theta^t \cdot \pi^0$.

If the Markov transition process is such that the ergodic theorem applies,⁸ this process admits a unique ergodic set and a unique invariant (limiting-state) distribution π^* characterized by $\pi^* = \Theta \cdot \pi^*$. In that case, the probability distribution over the possible states of the system in the long-run is governed by π^* irrespective of the initial conditions. In other words, $\Theta^t \cdot \pi^0 \rightarrow \pi^*$ when $t \rightarrow \infty$ for all initial probability distributions $\pi^0 \in \Delta_{2BS}$, and π^* indicates the probabilities with which the possible trading links of the market are activated in the long-run assuming that buyers and sellers are matched according to the stochastic matrix Θ . Empirically, $\pi_{sb}^* + \pi_{bs}^*$ is linked to the frequency of transactions in a given market between seller s and buyer b .

In this paper we primarily focus on the steady state behavior of the economy, where the outflow of agents that conclude a transaction is exactly counterbalanced by an inflow of agents of the same type. Consistent with this approach, we concentrate (when possible) in the long-run behavior of the market. To this purpose, we define a *regular economy* to be an economy whose Markov process regulating the random matching technology satisfies the conditions of the ergodic theorem. Denote by $\pi^* \in \Delta_{2BS}$ the unique

⁷The case of a buyer $b \in B$ is similar.

⁸See for instance Stokey *et al.* (1989), Theorem 11.2.

invariant distribution of the stochastic matching process governing pairwise meetings in this economy.

Corollary 3 *For all $s \in S$ and $b \in B$, the agents values at the market equilibrium of a regular economy with invariant probability distribution π^* are:*

$$\begin{cases} V_s = \sum_{j \in B} (\pi_{sj}^* x_{sj} + \pi_{js}^* y_{js}) \\ V_b = \sum_{i \in S} [\pi_{ib}^* (1 - x_{ib}) + \pi_{bi}^* (1 - y_{bi})] \end{cases} \quad (3)$$

5 Applications

Abusing slightly notations denote by S (respectively, B) the cardinality of the set S of sellers (respectively, set B of buyers). We now examine some cases of particular interest and explicitly compute the corresponding equilibrium prices and agents values using equations (2) and (3).

5.1 Anonymous nite market

Assume first that $\theta_{hl,ij} = \frac{1}{2BS}$, $\forall (h,l), (i,j) \in \mathcal{M}$ and $\delta_i = \delta$, $\forall i \in B \cup S$. This case corresponds to an economy where agents are homogeneous in time preferences, all pairwise meetings between buyers and sellers are equally likely and, within a matched pair, the buyer and the seller are selected to be the proposer or the respondent with equal probability $\frac{1}{2}$. Denote by x the equilibrium price proposed by sellers to buyers and by y the equilibrium price proposed by buyers to sellers at the unique market equilibrium. These prices depend on the primitives of the model (B , S and δ) but are independent of agents labels and roles. The ergodic theorem clearly applies to this Markov transition matrix whose unique invariant distribution is $\pi_{ij}^* = \frac{1}{2BS}$, $\forall (i,j) \in \mathcal{M}$. An anonymous nite market is thus a regular economy in the sense defined in the previous section. Denote by V_s (respectively, V_b) the value of a seller (respectively, a buyer) in the market.

Proposition 2 *In an anonymous nite market the market equilibrium prices are $x = \frac{(2S-\delta)(B-\delta)}{S(B-\delta)+B(S-\delta)}$ and $y = \frac{\delta(B-\delta)}{S(B-\delta)+B(S-\delta)}$. The resulting agents values are $V_s = \frac{B-\delta}{S(B-\delta)+B(S-\delta)}$ and $V_b = \frac{S-\delta}{S(B-\delta)+B(S-\delta)}$.*

The common discount factor δ measures the cost to delay (through disagreement) and thus introduces a friction in the decentralized market. By letting δ converge to unity this friction (absent in the natural benchmark corresponding to the standard competitive market model) is removed. Denote by x^* and y^* the equilibrium prices at the limit $\delta \rightarrow 1$, and by V_s^* and V_b^* respectively the value of a seller and a buyer in the market at this same limit.

Corollary 4 *In a frictionless anonymous finite market, the equilibrium prices and agents' values are $x^* = 1 - V_b^* = \frac{(2S-1)(B-1)}{S(B-1)+B(S-1)}$ and $y^* = V_s^* = \frac{B-1}{S(B-1)+B(S-1)}$.*

We deduce from the previous expressions that $V_s^* \geq V_b^*$ if and only if $S \leq B$. Moreover, $V_b^* = 0$ (respectively, $V_s^* = 0$) only in the monopoly case where $S = 1$ (respectively, in the monopsony case where $B = 1$). Therefore, the members of the short side of the market have an advantage, but their advantage does not enable them to appropriate all the surplus. This result is in contrast with Corominas-Bosch (1999) that considers a market where sellers and buyers are connected through a network and make repeated alternating public offers for which she shows that the short side of the market gets all the surplus (provided it is well connected enough).⁹

We can also check that V_s^* increases with B and decreases with S while V_b^* increases with S and decreases with B .¹⁰ In words, the seller's value increases with the size of the market (at the expense of the buyer's value) and decreases with the number of competitors (at the benefit of the buyer's value). We now apply these results to some cases of particular interest:

Example 3 *Monopoly ($S = 1$): $x^* = y^* = V_s^* = 1$ and $V_b^* = 0$.*

Example 4 *Duopoly ($S = 2$): $x^* = \frac{3(B-1)}{3B-2}$, $y^* = V_s^* = \frac{B-1}{3B-2}$, and $V_b^* = \frac{1}{3B-2}$. Therefore, as $B \rightarrow \infty$ we have $x^* \rightarrow 1$, $y^* = V_s^* \rightarrow \frac{1}{3}$, and $V_b^* \rightarrow 0$.*

Example 5 *Identical sides ($B = S = n$): $V_b^* = V_s^* = y^* = 1 - x^* = \frac{1}{2n} \rightarrow 0$ as $n \rightarrow \infty$. We recover the standard half-half partition in the two-agent case ($n = 1$) and a take-it-or-leave-it offer in the infinite case ($n = \infty$).*

5.2 Anonymous large market

We now consider large markets where the primitives are not the finite set of the two types of agents, buyers and sellers, but matching probabilities. For concreteness, assume that in case of disagreement between two matched agents $s \in S$ and $b \in B$, (i) these two agents remain matched at the following period with probability $(1 - \alpha)(1 - \beta)$, (ii) $s \in S$ is matched anew with a different buyer while $b \in B$ remains unmatched with probability $\alpha(1 - \beta)$, or (iii) $b \in B$ is matched anew with a different seller while $s \in S$ remains

⁹See also Kranton and Minehart (1998) and Chatterjee and Dutta (1998) for related models.

¹⁰Indeed, let $F(z, t) = \frac{(2z-1)(t-1)}{z(t-1)+t(z-1)}$ defined on $D = (1, \infty) \times (1, \infty)$. Then, $\frac{\partial F(z, t)}{\partial z} = -\frac{t-1}{[z(t-1)+t(z-1)]^2} \leq 0$, and $\frac{\partial F(z, t)}{\partial t} = \frac{(2z-1)(z-1)}{[z(t-1)+t(z-1)]^2} \geq 0$, $\forall (z, t) \in D$. Similarly, let $G(z, t) = \frac{t-1}{z(t-1)+t(z-1)}$ defined on D . Then, $G(z, t)$ decreases with z and $\frac{\partial G(z, t)}{\partial t} = \frac{z-1}{[z(t-1)+t(z-1)]^2} \geq 0$, $\forall (z, t) \in D$.

unmatched with probability $(1 - \alpha)\beta$. Hence we assume that the two agents cannot both remain unmatched.¹¹

Proposition 3 *In a frictionless anonymous large market, the equilibrium prices are $x^* = \frac{\alpha(1+\beta)}{\alpha+\beta}$ and $y^* = \frac{\alpha(1-\beta)}{\alpha+\beta}$.*

When the probabilities of being matched with a different partner α and β are small –due for instance to high search costs or to very few market opportunities– we approximately get (at a first order approximation): $x^* \simeq y^* \simeq \frac{\alpha}{\alpha+\beta}$ that coincides with Rubinstein and Wolinsky (1985).

Remark 1 *Our model, though, differs from Rubinstein and Wolinsky (1985) in at least three respects. First, their paper assumes that two agents of different types i and j matched today cannot both remain unmatched at the next period, which is equivalent to requiring that $\theta_{hl,ij} = \theta_{lh,ij} = 0$ for all $h \in N_1, l \in N_2, h \neq i$ and $l \neq j$ in our model. Here, we allow for an additional probability $\sum_{l \in N_2 \setminus \{j\}} \sum_{h \in N_1 \setminus \{i\}} (\theta_{hl,ij} + \theta_{lh,ij}) \geq 0$ of agent i and j not being matched at period $t + 1$ conditional on their being matched at t . Second, their model assumes that, given a pair of matched agents, a random selection device selects one of them to be the proposer with probability $\frac{1}{2}$. Here, this is equivalent to assume that $\theta_{hl,ij} = \theta_{lh,ij}, \forall (h, l), (i, j) \in \mathcal{M}$. Finally, each agent of a given type is matched with an agent of the opposite type with the same probability in their paper. This is equivalent to impose that $\alpha_h = \sum_{l \in N_2} (\theta_{hl,ij} + \theta_{lh,ij})$ and $\beta_l = \sum_{h \in N_1} (\theta_{hl,ij} + \theta_{lh,ij})$ do not depend on the label of the agents ($h \in N_1$ and $l \in N_2$), which needs not be the case in our more general setting.*

5.3 Take-it-or-leave-it versus offer-counter-offer

In the previous markets both agents (sellers and buyers) have the same probability to be in a position to set prices. We now determine the effect of introducing a bias in the selection of the proposer within a matched pair. To this purpose, we suppose that sellers negotiate according to two different possible regimes: *offer-counter-offer* regime or *take-it-or-leave-it* regime. We denote by S_1 the set of sellers negotiating according to the *offer-counter-offer* regime. The remaining set of sellers $S_2 = S \setminus S_1$ is composed of those sellers negotiating according to the *take-it-or-leave-it* regime. By definition, $S_1 \cap S_2 = \emptyset$ and $S_1 \cup S_2 = S$: the negotiation regimes divide the sellers population into two mutually exclusive and collectively exhaustive subsets.

Let $s \in S_1$ and assume that the state of the system at period t is $m^t = (s, b)$ for some $b \in B$. At the current period, the seller s is matched with

¹¹Denote by M the number of matches per period. If the market size is relatively big with respect to M , approximately $\alpha = \frac{M}{B}$ and $\beta = \frac{M}{S}$, where α is the probability that a seller matched at the current period meets at the following period a new partner with whom she proceeds to the bargaining stage.

the buyer b , and within this matched pair and during the bargaining phase s is the proposer whilst b is her respondent. Player s makes a price proposal to player b at t . If the buyer accepts this offer, the good owned by the seller s is traded with the buyer b at the agreed-upon price, the match (s, b) is dissolved, we move to the next period $t + 1$ and a new ordered pair $m^{t+1} \in \mathcal{M}$ is formed. What if, on the contrary, b disagrees with the price proposed by s and rejects her offer? The seller $s \in S_1$ is of the *offer-counter-offer* type. Hence, if b rejects her offer at t , s passes to her current partner b the negotiation initiative. It is now b 's turn to make a (counter) price offer to s , as if they were playing a standard two-player sequential bargaining game of alternating offers. More precisely, we assume that $m^{t+1} = (b, s)$ with probability $1 - \zeta$ for some $0 < \zeta \ll 1$. The parameter ζ is a probability of breakdown of the standard bilateral bargaining due to the seller's unilateral departure from the current pair. Indeed, we assume that with probability ζ the seller does not wait for the buyer's counter offer and makes a price proposal to an outside buyer, all alternative matches being equally likely. More precisely,

Assumption 1 Let $s \in S_1$ and $b \in B$ such that $m^t = (s, b)$ for some $t \in \mathbb{N}$. If b rejects the price offer made by s at t , the matched pair at the next round is $m^{t+1} \in \mathcal{M}$ with $\Pr \{m^{t+1} = (b, s)\} = 1 - \zeta$ and $\Pr \{m^{t+1} = (s, b')\} = \frac{1}{B-1}\zeta, \forall b' \in B, b' \neq b$.

Let now $s \in S_2$ and assume that the state of the system at period t is $m^t = (s, b)$ for some $b \in B$. Here again, s makes a price proposal to b at t . We just have to specify what happens when b rejects this proposal. The seller $s \in S_2$ is of the *take-it-or-leave-it* type. Hence, if b rejects her offer at period t , s does not wait for a counter-offer by her current partner b at $t + 1$. In case of disagreement, s and b remain matched together at $t + 1$ with probability $1 - \xi$ for some $0 < \xi \ll 1$, and the negotiation proceeds as in the previous period where s makes another *take-it-or-leave-it* offer. With the complementary probability ξ the current match $m^t = (s, b)$ is dissolved as a result of the buyer's unilateral departure, this buyer is matched at the next round with a seller of the *offer-counter-offer* type and asks her for a price proposal. We assume for simplicity that all alternative matches are equally likely.

Assumption 2 Let $s \in S_2$ and $b \in B$ such that $m^t = (s, b)$ for some $t \in \mathbb{N}$. If b rejects the *take-it-or-leave-it* price offer made by s at t , the matched pair at the next round is $m^{t+1} \in \mathcal{M}$ with $\Pr \{m^{t+1} = (s, b)\} = 1 - \xi$ and $\Pr \{m^{t+1} = (s', b)\} = \frac{1}{S_2}\xi, \forall s' \in S_1$.

We assume that the negotiation initiative (the initiation of a price bargaining by submitting a first price offer) comes always from the seller side

of the market. This assumption does not exclude, though, the possibility that an interested buyer asks a particular seller for an offer. Buyers are all of the same type and only make price proposals when replicating by a counter-proposal to sellers of the *offer-counter-offer* type. Suppose that $m^{t-1} = (s, b)$ for some $b \in B$, $s \in S_1$ and $t \geq 1$. If b rejects the price offer made by s at $t-1$, with probability $1-\zeta$ the matched pair at t is $m^t = (b, s)$. Now it is b 's turn to make a (counter) price offer. If s rejects the price offer made by b at t , either the seller s replies to the buyer b at $t+1$ by a counter offer with probability $1-\zeta$, or the buyer quits the seller with the complementary probability ζ . Consequently, the current match $m^t = (s, b)$ is dissolved as a result of the buyer's unilateral departure, and this buyer is matched at the next round with a seller of the *take-it-or-leave-it* type and asks her for a price proposal. We assume for simplicity that all alternative matches are equally likely.

The parameters ζ and ξ correspond to a probability of breakdown of the regular bargaining relationships (either of the *offer-counter-offer* mode or of the *take-it-or-leave-it* mode). They capture a certain variability in the players' behavior. For instance, instead of replicating by a counter-offer to the price proposal of some seller $s \in S_1$, a buyer may switch over with probability ζ and ask some seller $s \in S_2$ for an offer.¹² Also, instead of waiting for another *take-it-or-leave-it* price offer by some seller $s \in S_2$, a buyer may initiate a negotiation with a seller of the opposite type with probability ξ . We assume for simplicity that $\xi = \zeta$ and denote by $\nu \equiv \xi = \zeta$ this common breakdown probability.

Denote by x_i the price proposal from sellers of type S_i , $i = 1, 2$ and by y the price proposal from buyers.

Proposition 4 *The market equilibrium prices are $x_1 = \frac{1}{1+\delta(1-\nu)}$, $x_2 = \frac{1-\delta^2(1-\nu)}{1-\delta^2(1-\nu)^2}$ and $y = \frac{\delta(1-\nu)}{1+\delta(1-\nu)}$. In the frictionless market ($\nu \rightarrow 0$ and $\delta \rightarrow 1$) we have $x_1^* = y^* = \frac{1}{2}$ and $x_2^* = 1$.*¹³

We now look for the invariant distribution of Θ . The Markov transition

¹²Similarly when some seller of the *offer-counter-offer* type has to make a counter-proposal.

¹³Note that $\lim_{\nu \rightarrow 0} \lim_{\delta \rightarrow 1} x_1 = \lim_{\delta \rightarrow 1} \lim_{\nu \rightarrow 0} x_1 = \frac{1}{2}$, and also $\lim_{\nu \rightarrow 0} \lim_{\delta \rightarrow 1} y = \lim_{\delta \rightarrow 1} \lim_{\nu \rightarrow 0} y = \frac{1}{2}$. On the contrary, $\lim_{\delta \rightarrow 1} \lim_{\nu \rightarrow 0} x_2 = 1$ whereas $\lim_{\nu \rightarrow 0} \lim_{\delta \rightarrow 1} x_2 = \frac{1}{2}$. The parameter ν corresponds to the fixed probability with which a negotiation terminates after a rejection –breakdown probability– whereas δ measures the cost to delay through disagreement. At the limit $\lim_{\nu \rightarrow 0} \lim_{\delta \rightarrow 1} x_2$, the time cost of bargaining rather than the fear of breakdown is the dominant consideration. The equilibrium agreement is thus driven by time impatience and coincides with the standard half-half partition. On the contrary, at the limit $\lim_{\delta \rightarrow 1} \lim_{\nu \rightarrow 0} x_2$ the breakdown effect overwhelms the cost to delay through disagreement, and the pairwise meetings mechanism influences the equilibrium outcome. We thus concentrate on this last case.

process defined above clearly satisfies the conditions of the ergodic theorem: this process admits a unique ergodic set and a unique invariant distribution π^* . The market considered here thus constitutes a regular economy in the sense defined in Section 4. The invariant distribution $\pi^* \in \Delta_{2BS}$ satisfies $\Theta \cdot \pi^* = \pi^*$. By symmetry, the components of π^* only take three different values denoted by π_1^* , π_2^* and π_b^* , where π_i^* is the probability that the match is (s, b) for some $s \in S_i$ (BS_i possible matches) for $i = 1, 2$ and π_b^* is the probability that the match is (b, s) , where necessarily $s \in S_2$ (BS_2 such matches). The normalization condition is then:

$$BS_1(\pi_1^* + \pi_b^*) + BS_2\pi_2^* = 1 \quad (4)$$

The equation for the invariant distribution $\Theta \cdot \pi^* = \pi^*$ also imposes:

$$\begin{cases} (1 - \zeta) \pi_1^* = \pi_b^* \\ (1 - \xi) \pi_2^* + BS_1 \frac{\zeta}{S_2} \pi_b^* = \pi_2^* \end{cases} \quad (5)$$

We obtain the invariant distribution by solving the system composed of (6.4) and (6.5).¹⁴ The invariant distribution is the limiting-state distribution of the stochastic matching technology. Specializing (6.3) the players market values in the long run are:

$$\begin{cases} V_1^* = B\pi_1^*x_1^* + B\pi_b^*y^* \\ V_2^* = B\pi_2^*x_2^* \\ V_b^* = S_1\pi_1^*(1 - x_1^*) + S_2\pi_2^*(1 - x_2^*) + S_1\pi_b^*(1 - y^*) \end{cases}$$

Corollary 5 *In the frictionless market, the agents' values are $V_1^* = \frac{1}{S_1(B+2)}$, $V_2^* = \frac{B}{S_2(B+2)}$ and $V_b^* = \frac{1}{B(B+2)}$.*

As expected, prices agreed-upon at equilibrium out of an *offer-counter-offer* bargaining regime coincide with the standard half-half partition ($x_1^* = y^* = \frac{1}{2}$). On the contrary, *take-it-or-leave-it* price-setting negotiations allow the sellers to extract all the trade surplus ($x_2^* = 1$). One can interpret the two negotiation regimes analyzed as different trading relationships. On the one hand, Rubinstein bargaining with alternating offers may represent a loyalist behavior in the market where a given buyer (nearly) always visits the same seller, and does not exert any threat of buying elsewhere (i.e. opting out) when negotiating over the terms for trade. On the other hand, buyers visiting several sellers and buying from any of them indistinctly – thus behaving as market searchers – are more likely to face *take-it-or-leave-it* price offers. According to Proposition 4 sellers charge a lower price to loyalist buyers ($x_1^* = y^* = \frac{1}{2}$) than to searchers ($x_2^* = 1$), thus providing buyers

¹⁴By letting $\xi = \zeta = \nu$, the last equations become $(1 - \nu) \pi_1^* = \pi_b^*$ and $(1 - \nu) \pi_2^* + BS_1 \frac{\nu}{S_2} \pi_b^* = \pi_2^*$.

with an incentive to establish stable trading relationships.¹⁵ Concerning the impact of market organization on agents' values, we deduce from Corollary 5 that $V_1^* \leq V_b^*$ if and only if $B \leq S_1$, meaning that the short side of the *loyal market* gets a bigger share of the trade surplus. Also, $V_1^* \leq V_2^*$ if and only if $S_2 \leq BS_1$. Notice that BS_1 is the maximum number of loyal trading links between loyalist buyers and sellers that can coexist in the market. It is thus a measure of the *loyal market size*. Therefore, the sellers operating in the smaller market (whose sizes are given respectively by BS_1 for the *loyal market*, and by S_2 for the *search market*) end up with a higher per-capita value.

References

- [1] Binmore, K.G. and C. Herrero (1988a): Matching and Bargaining in Dynamic Markets, *Review of Economic Studies* 55, 17-32
- [2] Binmore, K.G. and C. Herrero (1988b): Security Equilibrium, *Review of Economic Studies* 55, 33-48
- [3] Chatterjee, K. and B. Dutta (1998): Rubinstein Auctions: on Competition for Bargaining Partners, *Games and Economic Behavior* 23, 119-145
- [4] Corominas-Bosch, M. (1999): *On Two-Sided Network Markets*, Ph.D. thesis, Universitat Pompeu Fabra
- [5] Ereslan, H. and A. Merlo (1999): Majority Rule in a Stochastic Model of Bargaining, manuscript, University of Minnesota and New York University
- [6] Fölmer, H. (1974): Random Economies with Many Interacting Agents, *Journal of Mathematical Economics* 1, 51-62
- [7] Gale, D. (1986a): Bargaining and Competition. Part I: Characterization, *Econometrica* 54, 785-806
- [8] Gale, D. (1986b): Bargaining and Competition. Part II: Existence, *Econometrica* 54, 807-818
- [9] Gale, D. (1987): Limit Theorems for Markets with Sequential Bargaining, *Journal of Economic Theory* 43, 20-54
- [10] Hart, O. (1989): Bargaining and Strikes, *Quarterly Journal of Economics* 104, 25-44

¹⁵In fact, the establishment of regular trading relationships may be mutually profitable by allowing sellers to predict accurately the demand they face and determining their supply accordingly.

- [11] Hendon, E., Sloth, B. and T. Tranæs (1994): Decentralized Trade with Bargaining and Voluntary Matching, *Economic Design* 1, 55-78
- [12] Herreiner, D. (1997): *Market Organization*, Ph.D. thesis, University of Bonn
- [13] Jackson, M.O. and T. Palfrey (1998): Efficiency and Voluntary Implementation in Markets with Repeated Pairwise Bargaining, *Econometrica* 66, 1353-1388
- [14] Kirman, A. (1983): Communication in Markets: a Suggested Approach, *Economics Letters* 12, 101-109
- [15] Kirman, A., Oddou, C. and S. Weber (1986): Stochastic Communication and Coalition Formations, *Econometrica* 54, 129-138
- [16] Kranton, R. and D. Minehart (1998): A Theory of Buyer-Seller Networks, *American Economic Review* (forthcoming)
- [17] McLennan, A. and H. Sonnenschein (1991): Sequential Bargaining as a Noncooperative Foundation for Walrasian Equilibrium, *Econometrica* 59, 1395-1424
- [18] Merlo, A. and C. Wilson (1995): A Stochastic Model of Sequential Bargaining with Complete Information, *Econometrica* 63, 371-400
- [19] Merlo, A. and C. Wilson (1998): Efficient Delays in a Stochastic Model of Bargaining, *Economic Theory* 11, 39-55
- [20] Osborne, M.J. and A. Rubinstein (1990): *Bargaining and Markets*, Academic Press, San Diego, CA
- [21] Ponsatí, C. (2000): Search and Bargaining in Simple Markets, manuscript, Universitat Autònoma de Barcelona
- [22] Rubinstein, A. and A. Wolinsky (1985): Equilibrium in a Market with Sequential Bargaining, *Econometrica* 53, 1133-1150
- [23] Rubinstein, A. and A. Wolinsky (1990): Decentralized Trading, Strategic behavior and the Walrasian Outcome, *Review of Economic Studies* 57, 63-78
- [24] Samuelson, L. (1992): Disagreement in Markets with Matching and Bargaining, *Review of Economic Studies* 59, 177-187
- [25] Shaked, A. (1994): Opting Out: Bazaars versus Hi Tech. Markets, *Investigaciones Económicas* 18, 421-432
- [26] Stokey, N.L., Lucas, R.E. and E.C. Prescott (1989): *Recursive Methods in Economic Dynamics*, Harvard University Press

- [27] Weisbuch, G., Kirman, A. and D. Herreiner (1998): Market Organization and Trading Relationships , Working Paper GREQAM 98A32
- [28] Wolinsky, A. (1987): Matching, Search and Bargaining, *Review of Economic Studies* 42, 311-333
- [29] Wolinsky, A. (1990): Information Revelation in a Market With Pair-wise Meetings, *Econometrica* 58, 1-24
- [30] Wooders, J. (1998): Matching and Bargaining Models of Markets: Approximating Small Markets by Large Markets, *Economic Theory* 11, 215-224

6 Appendix: proofs

Proof of Proposition 1. It is easy to show that, at equilibrium, agents are indifferent between their share today and their share tomorrow appropriately discounted. Suppose without loss of generality that we are at round t and that the matched agents are $m^t = (i, j)$, $i \in N_1$ being the proposer and $j \in N_2$ the respondent. The share j gets as a respondent at round t if she accepts the two-player/one-cake proposal made by i is $1 - a_{ij}$. If she rejects such an offer, at the next round:

- agent j makes a proposal to agent $l \in N_1$ with probability $\theta_{jl,ij}$ and the corresponding expected payoff is $\sum_{l \in N_1} a_{jl} \theta_{jl,ij}$
- player j receives a proposal from agent $l \in N_1$ with probability $\theta_{lj,ij}$ and the corresponding expected payoff is $\sum_{l \in N_1} (1 - a_{lj}) \theta_{lj,ij}$.

Therefore, $\sum_{l \in N_1} [a_{jl} \theta_{jl,ij} + (1 - a_{lj}) \theta_{lj,ij}]$ is the expected payoff j gets at round $t + 1$ if she refuses the proposal made by i at round t . We now prove that the set of equations (1) has a unique solution. For all $(i, j) \in \mathcal{M}$ where $i \in N_k$ and $j \in N_{3-k}$ for some $k = 1, 2$ equations (1) are equivalent to

$$a_{ij} + \delta_j \sum_{l \in N_k} (a_{jl} \theta_{jl,ij} - a_{lj} \theta_{lj,ij}) = 1 - \delta_j \sum_{l \in N_k} \theta_{lj,ij}$$

We thus obtain a system of $2n_1 n_2$ equations with $2n_1 n_2$ unknowns that can be written in matrix form. For all pair $(i, j) \in \mathcal{M}$ of matched players of different types the diagonal term of the matrix is 1. Summing up the absolute values of the row terms of the matrix, except the 1 in the diagonal, we obtain:

$$\delta_j \sum_{l \in N_k} (\theta_{jl,ij} + \theta_{lj,ij}) \leq \delta_j \sum_{(h,l) \in \mathcal{M}} \theta_{hl,ij} = \delta_j < 1.$$

Therefore the matrix corresponding to the system of equation characterizing the proposals a_{ij} at equilibrium is dominant diagonal, thus invertible, implying uniqueness of the Markov subgame perfect equilibrium payoffs. \square

Proof of Lemma 1. It suffices to solve equations (1) with the matrix Θ given by $\theta_{H1,1H} = \theta_{1H,H1} = 1 - d$; $\theta_{H1,1L} = \theta_{1H,L1} = d$; $\theta_{L1,1L} = \theta_{1L,L1} = 1$ and $\theta_{hl,ij} = 0$ otherwise. \square

Proof of Proposition 2. Equations (2) become:

$$\begin{cases} 1 - x = \frac{\delta}{2BS} S(1 - y + 1 - x) \\ y = \frac{\delta}{2BS} B(x + y) \end{cases}$$

from which we obtain x and y . Rewriting equations (3) as

$$\begin{cases} V_s = \frac{B}{2BS} (x + y) \\ V_b = \frac{S}{2BS} (1 - x + 1 - y) \end{cases}$$

and using the fact that $x + y = \frac{2S(B-\delta)}{S(B-\delta)+B(S-\delta)}$ and $1 - x + 1 - y = \frac{2B(S-\delta)}{S(B-\delta)+B(S-\delta)}$ we obtain the desired result. \square

Proof of Proposition 3. Equations (2) become:

$$\begin{cases} 1 - x = \frac{\delta}{2} [(1 - \alpha)(1 - \beta) + \beta(1 - \alpha)] (1 - y + 1 - x) \\ y = \frac{\delta}{2} [(1 - \alpha)(1 - \beta) + \alpha(1 - \beta)] (x + y) \end{cases}$$

Solving this system of equations yields the desired result. \square

Proof of Proposition 4. The equilibrium equations (2) become

$$\begin{cases} 1 - x_1 = \delta(1 - \zeta)(1 - y) \\ 1 - x_2 = \delta(1 - \zeta)(1 - x_2) + \delta\xi(1 - x_1) \\ y = \delta(1 - \zeta)x_1 \end{cases}$$

Replacing the parameters ζ and ξ by ν , and solving these equations yields the desired result. \square

Proof of Corollary 5. Solving (4) and (5) where ξ and ζ are replaced with ν , and letting $\nu \rightarrow 0$ and $\delta \rightarrow 1$ we get $\pi_1^* = \pi_b^* = \frac{1}{BS_1(B+2)}$, $\pi_2^* = \frac{1}{S_2(B+2)}$. The desired result is then obtained by simple computation using the equilibrium prices of Proposition 4. \square