

Job Matching, Social Network and Word-of-Mouth Communication*

Antoni Calvó-Armengol[†]

Universitat Pompeu Fabra and CERAS

Yves Zenou[‡]

University of Southampton, GAINS and CEPR

July 13, 2000

Abstract: In our model, workers are embedded within a network of social relationships and can communicate through word-of-mouth. They can find a job either through formal agencies or through informal networks of contacts (word-of-mouth communication). From this micro scenario, we derive an aggregate matching function that has the standard properties but fails to be homogenous of degree one. The latter is due to negative externalities generated by indirect neighbors (neighbors of neighbors) that slow down word-of-mouth information transmission, especially in dense networks. We then show that there exists a unique labor market equilibrium and that, because of these negative externalities, the equilibrium unemployment rate increases with the network size in dense networks.

Keywords: microfoundation of the matching function, job search, network size.

JEL Classification: D83, J64

*Part of this work was written while the first author was visiting the University of Southampton whose hospitality is gratefully acknowledged. We also like to thank the participants of the Education and Employment Economics Group Annual Conference and in particular Adriana Kugler and Alan Manning.

[†]Department of Economics, Universitat Pompeu Fabra, Ramon Trias Fargas 25-27, 08005 Barcelona, Spain. Email: antoni.calvo@econ.upf.es

[‡]Department of Economics, University of Southampton, Southampton SO17 1BJ, UK. Email: yz@soton.ac.uk

1 Introduction

Individuals seeking for jobs read newspapers, go to employment agencies, browse in the web and mobilize their local networks of friends and relatives. Although underestimated by the bulk of the search and matching literature, personal contacts often play a prominent role in matching job-seekers with vacancies. Empirical evidence suggests indeed that about half of all jobs are filled through contacts.¹ Networks of personal contacts mediate employment opportunities which flow through word-of-mouth and, in many cases, constitute a valid alternative source of employment information to more formal methods.

The aim of this paper is twofold. First, we provide an explicit micro scenario in which finding a job depends both on formal and informal methods. According to this scenario, workers are linked to each other by a social network, the members of this network can communicate through word-of-mouth and agents partly rely on friends to gather information about employment opportunities. Second, we establish a relationship between the network of personal contacts and the information transmission protocol and the aggregate job matching process. Our analysis of a labor market based on search and an explicit social network structure sheds light on the social dimension of job matching.

In our model, individuals are not isolated one with respect to the other. Rather, they are embedded within a network of social relationships. We represent this social network by an undirected graph where nodes stand for the agents and a link between two nodes means that the corresponding agents can communicate directly. For most of the analysis, we focus on symmetric social networks where all agents have the same number of direct acquaintances. We refer to this number as the network size. Given a network of contacts, information about employment opportunities can be transmitted

¹Sociologists and labor economists have produced a broad empirical literature on labor market networks. In fact, the pervasiveness of social networks and their relative effectiveness varies with the social group considered. For instance, Holzer (1988) shows that among 16-23 years old workers who reported job acceptance, 66% used informal search channels (30% direct application without referral and 36% friends/relatives), while only 11% use state agencies and 10% newspapers. See also Corcoran *et al.* (1980) and Granovetter (1995). More recently, Topa (1999) argues that the observed spatial distribution of unemployment in Chicago is consistent with a model of local interactions and information spillovers, and may thus be generated by agent's reliance in informal methods of job search such as networks of personal contacts.

between any two direct neighbors through word-of-mouth communication. More precisely, when a job is available in the economy, workers can match with such a vacancy using either formal or informal methods. When an unemployed worker hears directly from a vacancy, we assume that s/he takes the job, and this is considered as a formal method (since the social network plays no role). If on the contrary the worker hearing directly from a vacancy is currently employed, we assume that s/he transmits this information to her/his direct unemployed neighbors. Unemployed workers getting a job with the help of their local social network –as described above– rely on informal methods of job search.

We first show that the relationship between network structure (namely size) and job-finding is not as straightforward as it is commonly viewed. Indeed, in the standard social network literature (especially in sociology), more contacts are thought to be an advantage since they are more network members who can potentially broker job vacancies and job seekers. We show that this result crucially depends on the size of the network. Indeed, in a symmetric social network, each individual worker can receive information from her/his *direct* neighbors. However, each of her/his neighbors also has a direct set of acquaintances –*indirect* neighbors from the viewpoint of the first worker– that may benefit from this information. In our model direct neighbors are beneficial whereas indirect neighbors are detrimental. More direct contacts provide job seekers with a higher probability of receiving information about job openings and the unemployed prefer a large set of direct acquaintances to broaden their potential employment channels. But the better a worker is connected, the higher the number of unemployed direct neighbors that can potentially benefit from the information s/he holds about available jobs. As a result, the unemployed prefer a small set of indirect acquaintances to release the constraints of information sharing with a potentially bigger set of information recipients. In other words, *indirect* neighbors generate a negative externality over their direct set of acquaintances. We show that rising the network size has a positive impact on the individual probability to find a job through friends in sparse social networks. On the contrary, increasing the network size in dense networks slows down word-of-mouth information transmission

We then obtain a well-defined aggregate matching function which gives the number of job matches per unit of time. This endogenous matching function is derived from an explicit micro scenario where the structure of personal contacts and the job information transmission process is spelled

out in detail. The corresponding reduced form is expressed in terms of the unemployed worker and vacant firm pools, and the social network underlying players talks. Contrarily to previous contributions also providing for micro foundations for matching functions, the expression obtained here is neither an exponential nor a min one. This matching function is increasing and strictly concave in both the unemployment and the vacancy rates. Moreover, the (extension of the standard) matching function we provide clearly relates job matching to individual social embeddedness and captures complex spillovers within social networks of interrelated personal contacts. In particular, we find a non-monotonic relationship between network size and the rate at which matches occur. With this matching function in hand, we can fully characterize the labor market equilibrium whose existence and uniqueness is established. We show that the resulting equilibrium unemployment rate decreases with the network size in sparse networks while it increases when the pattern of links is dense.

There have been several attempts to find a micro foundation of the standard macroeconomic matching function. The most popular reduced form is the exponential matching function that was first employed by Butters (1977) to model contacts between buyers and sellers in commodity markets.² More recently, Lagos (2000) has proposed an alternative micro approach by deriving an aggregate matching function which takes the form of a min function. Our micro foundation of the matching function based on word-of-mouth communication gives insights on the relationship between job search, job matching and social network. In fact, there have been few theoretical attempts to model this link. Notable exceptions include Diamond (1981), Montgomery (1991, 1992), Mortensen and Vishwanath (1994) and Kugler (2000) that contribute to the theoretical literature on equilibrium wage determination in search markets. However, in all these approaches, the modelling of the social network is quite shallow. To our knowledge, the first paper to explic-

²This matching function owes its origin to the well-known and extensively analysed urn-ball model in probability theory. According to this model, the labor market is visualized as ‘urns’ (vacancies) to be filled by ‘balls’ (workers). Because of a coordination failure inherent to any random placing of the balls in the urns, matching is not perfect and one can interpret the resulting mismatches in terms of labor market frictions. In most cases, the system steady state can be approximated by an exponential-type matching function as the population becomes large. See for instance Hall (1979), Pissarides (1979), Peters (1991), Blanchard and Diamond (1994), Burdett, Shi and Wright (2000), Smith and Zenou (2000).

itly model the structure of social contacts by an undirected network in a labor market context is Boorman (1975).³ Following this early contribution, Calvó-Armengol (2000) develops a model specifying at the individual level both the decision to establish or to maintain social ties with other agents, and the process by which information about jobs is obtained and transmitted. The analysis focuses on the impact that an endogenous determination of job contact networks has on the effectiveness of information transmission and on the aggregate unemployment level. On the contrary, the present paper builds an aggregate matching function stemming from an explicit network structure, and determines the impact a partial reliance on social networks as a method of job search has on labor market outcomes.

The remaining of the paper is as follows. The next section describes the social network and the information transmission protocol within this network. Section 3 derives the aggregate matching function and examines its main properties. The characterization, the existence and the uniqueness of the labor market equilibrium is established in section 4. Section 5 concludes and all the proofs are presented in Appendix.

2 Social Network and Word-of-Mouth Communication

Social networks are links and associations between people of a common ilk. These can be friends, acquaintances and colleagues. Networks are evident between family members, but are also established between friends and neighborhood residents. In this section, we model the social network between people by means of graph theory.

2.1 The social network

We consider a finite population of workers $N = \{1, \dots, n\}$. In our model, individuals are not isolated one with respect to the other. Rather, they are embedded within a network of social relationships. More precisely, each

³A recent and growing literature stresses the role of networks in explaining a wide range of economic phenomena among which labor markets are just an example. See for instance Jackson and Wolinsky (1996), Bala and Goyal (2000) and the references therein. For a previous model of word-of-mouth communication see for instance Ellison and Fudenberg (1995).

worker i is in *direct* contact with a group of workers (her/his set of friends or relatives) and we assume that each pair of directly connected workers connected can communicate with each other through word-of-mouth. A direct link between two individuals i and j is denoted by ij . The collection of all existing links constitutes the prevailing social network of personal relationships denoted by g . Such a social network is modelled as an undirected graph in which binary relationships are symmetric that is, whenever i is connected to j according to g ($ij \in g$), then j is also connected to i according to g ($ji \in g$).

Given a social network g , we denote by $N_i(g)$ the set of all direct neighbors of worker i . Formally, $N_i(g) = \{j \in N \setminus \{i\} : ij \in g\}$. We also denote by $n_i(g)$ the cardinal of the set $N_i(g)$ that is, the number of direct neighbors of i with whom s/he can directly communicate. For example, Figure 1a corresponds to a star-shaped graph in which worker 1 can communicate with every other individual in the economy whereas workers 2 to $n = 6$ can directly communicate only with worker 1. Figure 1b illustrates the case of the complete graph where every worker can directly communicate with everybody.

An interesting case to be considered is when all workers have the same number of direct neighbors that is, $n_i(g) = s$ for all $i \in N$. Such a graph is called a symmetric graph and s is the size of the corresponding social network. The complete graph described in Figure 1b is a particular case of a symmetric network where $s = n - 1 = 5$.

[Insert Figures 1a and 1b here]

2.2 Word-of-mouth information transmission

The labor market environment is as follows. Time is discrete and continues forever. At any point in time, each of the n workers is either employed or unemployed. At period t , the unemployment pool is denoted by U_t and the corresponding unemployment rate by $u_t = U_t/n$. There are also V_t vacancies to be filled and each worker directly hears of a vacancy with probability $v_t = V_t/n$. We refer to v_t as the job arrival rate or the vacancy rate. At each period, currently employed workers lose their jobs with some probability δ . This process is taken to depend only on the general state of the economy and hence is treated as exogenous to the labor market. The timing of the model is as follows (see Figure 2). At the end of period t , the unemployment and employment rates are respectively equal to u_t and $1 - u_t$. At the beginning of

period $t + 1$, there is a technological shock and employed workers lose their jobs with the breakdown probability δ . The resulting employment rate is $(1 - \delta)(1 - u_t)$. Then, V_{t+1} vacancies are posted and jobs are filled according to the procedure described below. At the end of period $t + 1$, the unemployment and employment rates are respectively equal to u_{t+1} and $1 - u_{t+1}$. And so on. From now on, and for notational simplicity, we omit the subscript t when no confusion is possible.

At each period, and once the technological shock has occurred, any worker (employed or unemployed) hears directly of a vacant job with probability $v = V/n$. There are two cases to be considered. First, the directly informed worker is unemployed. Then, s/he takes this job immediately. This means that this worker has found the job through a formal employment agency and, consequently, does not rely on her/his social network to be reemployed. Second, the directly informed worker is employed. Obviously, this worker does not need this job and transmits this information to one of her/his direct unemployed neighbors, if any. We assume that unemployed workers are treated on an equal footing, which means that all unemployed direct neighbors have the same probability to be informed. Observe that, according to this information transmission protocol, job information can only flow through word-of-mouth from an employed to an unemployed that is, between workers with different employment status. Indeed, vacancies are assumed to be posted for one period which coincides with the time required to transmit information to direct neighbors. Therefore, if the informed worker is both employed and does not have any unemployed worker in her/his direct vicinity, the job slot is lost. Similarly, if an unemployed worker hears of two (or more) vacancies through word-of-mouth from two (or more) direct employed neighbors, we assume that s/he selects one job randomly, the other job(s) being lost. Finally, one (or more) job(s) is (are) also lost when an unemployed worker hears of jobs both directly and through friends.

In our model, workers partly rely on friends to gather information about potential jobs. Denote by $\theta \equiv (1 - \delta)(1 - u)$ the individual probability of remaining employed after the technological shock and before vacancies are posted for the current period. Conditional on being unemployed and not hearing directly of a vacancy, the individual probability of finding a job through contacts for worker i depends on the prevailing social network g and

is given by:

$$P_i(g, u, v) = 1 - \prod_{j \in N_i(g)} \left[1 - v\theta \frac{1 - \theta^{n_j(g)}}{(1 - \theta) n_j(g)} \right] \quad (1)$$

The following comments are in order. First, $\prod_{j \in N_i(g)} \left[1 - v\theta \frac{1 - \theta^{n_j(g)}}{(1 - \theta) n_j(g)} \right]$ denotes the individual probability of worker i not hearing of a vacancy through word-of-mouth communication. Second, fix a worker $j \in N_i(g)$ in the direct neighborhood of worker i . Then, $1 - v\theta \frac{1 - \theta^{n_j(g)}}{(1 - \theta) n_j(g)}$ represents the probability of the employed worker j not transmitting information to worker i about the job s/he has heard about. Thus, the complementary probability $v\theta \frac{1 - \theta^{n_j(g)}}{(1 - \theta) n_j(g)}$ corresponds to j knowing of a job (probability v), not needing it (probability θ) and transmitting this job information to i (probability $\frac{1 - \theta^{n_j(g)}}{(1 - \theta) n_j(g)}$). The probability of i being the unemployed worker selected among all the unemployed neighbors of j to be told about the existing vacancy can be decomposed as follows:

$$\frac{1 - \theta^{n_j(g)}}{(1 - \theta) n_j(g)} = \theta^{n_j(g)-1} + \sum_{k=1}^{n_j(g)-1} \binom{n_j(g)-1}{k} \frac{\theta^{n_j(g)-k-1} (1 - \theta)^k}{k + 1}$$

According to this expression, worker i is the recipient of the job information held by her/his employed neighbor j if either s/he the only unemployed neighbor of j (probability $\theta^{n_j(g)-1}$) or s/he is the one selected among the $k + 1$ unemployed friends of j (probability $\frac{\theta^{n_j(g)-k-1} (1 - \theta)^k}{k + 1}$).

In Figures 1a and 1b, we have calculated this probability $P_i(g, u, v)$ for a star-shaped graph and a complete graph. From Figure 1a, it is clear that individual 1 has the highest probability to find a job through word-of-mouth since s/he is connected to everybody whereas all the others have the same probability since they are only connected to individual 1 ($P_1 > P_2 = P_3 = P_4 = P_5 = P_6$). In Figure 1b, all individuals have the same number of direct neighbors (symmetric graph), which implies that they all have the same probability to find a job through contacts ($P_1 = P_2 = P_3 = P_4 = P_5 = P_6$). Observe however that, in both cases, all individuals have the same probability v to find a job through formal methods since this job-finding process does not depend on the social network.

From now on, we focus on symmetric social networks where all workers have the same number of neighbors equal to s . We refer to s as the network

size. In a symmetric network of size s , the individual probability of hearing of a job through word-of-mouth is then:

$$P(s, u, v) = 1 - \left[1 - v \frac{\theta(1 - \theta^s)}{(1 - \theta)s} \right]^s \quad (2)$$

As stated above, Figure 1b depicts a particular example of a symmetric social network when $s = n - 1 = 5$.

Proposition 1 *The properties of $P(s, u, v)$ are the following:*

- (i) $P(\cdot, u, v)$ is strictly concave in s , increasing between 0 and \bar{s} and decreasing between \bar{s} and $n - 1$, where \bar{s} is the unique global maximum of $P(\cdot, u, v)$;
- (ii) $P(s, \cdot, v)$ is decreasing in u . Moreover, there exists some $\tilde{\delta} \in [0, 1)$ such that $P(s, \cdot, v)$ is strictly convex in u when $\delta \geq \tilde{\delta}$;
- (iii) $P(s, u, \cdot)$ is increasing and strictly concave in v .

The following comments are in order. First, fix u and v . The individual probability $P(\cdot, u, v)$ to find a job through word-of-mouth within the network of social contacts exhibits diminishing returns to network size s . In other words, the marginal impact of adding a new connection to everybody decreases with the total number of pairwise links in the society. Moreover, $P(\cdot, u, v)$ increases with s in sparse networks ($s < \bar{s}$) while it decreases with s in densely connected labor market networks ($s > \bar{s}$). To understand this result, observe that increasing the network size has both a (positive) direct and (negative) indirect effect. On one hand, rising the network size expands the available *direct* connections to every worker. Workers are better connected and, consequently, the potential job information they can benefit from increases. Indeed, the probability that at least one direct contact is informed about a job opening is $1 - (1 - v)^s \nearrow 1$ as $s \rightarrow +\infty$. On the other hand, rising the network size also increases the potential number of unemployed workers directly connected to an employed and informed worker. The information held by every employed worker is now shared by a larger group of unemployed workers. The individual probability of being randomly selected by an employed direct friend as the information recipient is $\frac{1 - \theta^s}{(1 - \theta)s} \searrow 0$ as $s \rightarrow +\infty$.

Therefore, every unemployed worker suffers from the information sharing constraints exerted by the unemployed *indirectly* connected to her/him. Stated differently, expanding one's neighborhood has a negative impact on the current direct friends as it reduces their (individual) probability to gather job opportunities through social contacts. Workers relative locations thus create a *negative network externality* for their direct vicinity. This indirect negative effect prevails in networks of large size: $[1 - (1 - v)^s] \left[\frac{1 - \theta^s}{(1 - \theta)^s} \right] \rightarrow 0$ as $s \rightarrow +\infty$. Increasing the network size of dense networks ($s > \bar{s}$) slows down word-of-mouth information transmission.⁴ This result contradicts the common view that more contacts are an advantage since, apparently, more network members can potentially broker job vacancies and job seekers.

Second, when the unemployment rate u increases, two effects are in order: (i) the likelihood that a worker, who is directly informed of a vacancy through formal channels (arrival rate v), is unemployed increases, and also (ii) the number of unemployed directly connected to every informed and employed worker rises. This implies that u and $P(s, \cdot, v)$ are negatively correlated. To understand the positive impact of the vacancy rate v on the individual probability of finding a job through friends $P(s, u, \cdot)$ a similar intuition applies.

3 The matching function

As stated above, unemployed workers find jobs from two different channels. Either they find their job directly through *formal* methods –such as advertisement or employment agencies– with probability v , or they gather information about jobs through *informal* methods –in our case, the network of social contacts– with probability $P(s, u, v)$. In this context, the job acquisition rate or individual hiring probability of an unemployed worker is:

$$h(s, u, v) = v + (1 - v)P(s, u, v) \quad (3)$$

At each period of time, there are $nu = U$ unemployed workers that find a job according to $h(s, u, v)$. Since this probability is independent across different individuals, the number of job matches taking place per unit of time is just $nuh(s, u, v)$. Therefore, the matching function for our labor market where workers partly rely on personal contacts to find a job is given

⁴The threshold value \bar{s} is uniquely determined by $\frac{\partial P(s, u, v)}{\partial s} = 0$.

by:^{5,6}

$$m(s, u, v) = u[v + (1 - v)P(s, u, v)] \quad (4)$$

We can thus express the aggregate rate at which job matches occur as a function of the unemployed worker and vacant firm pools, and the social network underlying players talks. This endogenous matching function is derived from an explicit micro scenario where the structure of personal contacts and the job information transmission process is spelled out in detail. Contrary to previous contributions also providing micro foundations for matching functions, the well-defined reduced function obtained here is neither an exponential nor a min one. Moreover, the central role of the network of contacts in matching job-seekers with vacancies is made explicit, and the link between $m(s, u, v)$ and the network size s is precisely the key element of our model.

Proposition 2 *The properties of the matching function $m(s, u, v)$ are the following:*

- (i) $m(\cdot, u, v)$ is strictly concave in s , increasing between 0 and \bar{s} and decreasing between \bar{s} and $n - 1$, where \bar{s} is the unique global maximum of $P(\cdot, u, v)$;
- (ii) $m(s, \cdot, v)$ is increasing and strictly concave in u on $[0, \bar{u}]$ for some $0 < \bar{u} \leq 1$;
- (iii) $m(s, u, \cdot)$ is increasing and strictly concave in v .

We have the following comments. First, even though our matching function is quite different to the ones found in the literature, it has the same natural properties: it is increasing and strictly concave in both u and v .⁷

⁵To be more precise this matching function corresponds to the rate at which job matches occur per unit of time. It suffices therefore to multiply $m(s, u, v)$ by n to get the number of matches per unit of time.

⁶It is easy to verify that the matching function for a general social network g , not necessarily symmetric, is equal to:

$$m(g, u, v) = nu \left[v + (1 - v) \frac{1}{n} \sum_{i \in N} P_i(g, u, v) \right]$$

where $P_i(g)$ is given by (1).

⁷For u , this is true only on a restricted domain, i.e. on $[0, \bar{u}]$, where \bar{u} is quite large.

However, it is easily verified that it is not homogeneous of degree one. This last property is in contrast with the assumption of a constant return to scale aggregate matching function, common in a broad range of the search-theoretic literature.⁸ There is nonetheless a huge body of controversial empirical work to assess whether this feature is encountered in real-life labor markets, and the absence of constant returns to scale certainly does not undermine the validity of our reduced form matching function.⁹ Second, there is a non-monotonic relationship between the job matching rate and the network size. In fact, because word-of-mouth communication plays a crucial role in our model, the workers and their direct set of acquaintances impose an important externality to one another. This externality between workers and personal contacts implies that network size is relevant in determining the rate at which unemployed find jobs. The non-monotonic relationship is just a direct consequence of the ambiguous impact network size has on the individual probability $P(s, u, v)$ to find a job through friends. Recall that network size has a positive impact on $P(s, u, v)$ in sparse networks, whereas it has a negative impact on $P(s, u, v)$ in dense ones. Finally, we can deduce from (4) the following simple expression for the individual probability $f(s, u, v)$ for firms to fill a vacancy:

$$f(s, u, v) = u \left[1 - \left(1 - \frac{1}{v} \right) P(s, u, v) \right] \quad (5)$$

Clearly, the properties of both the job-hiring rate $h(s, u, v)$ and the job-filling rate $f(s, u, v)$ as functions of the network size s are immediately deduced from that of $P(s, u, v)$ namely, strictly concave in s , increasing between 0 and \bar{s} and decreasing between \bar{s} and $n - 1$. Moreover, the job-hiring rate $h(s, u, v)$ is decreasing in u and increasing in v whereas the job-filling rate $f(s, u, v)$ is increasing in u and decreasing in v .¹⁰ In other words, given a vacancy rate v (and a network size s), when the number of unemployed increases, it is more difficult to find a job but easier to fill a vacancy. Similarly, given an unemployment rate u (and a network size s), it becomes easier to find a job but more difficult to fill a vacancy as the number of vacancies increases.¹¹

⁸See for example the recent survey of Mortensen and Pissarides (1999).

⁹See for instance Coles and Smith (1996), Petrongolo and Pissarides (2000) and the references therein.

¹⁰See Lemmata 1 and 2 in appendix.

¹¹See Pissarides (2000) for a thorough account and description of such trading external-

4 The labor market equilibrium

4.1 Characterization of the equilibrium

Firms and workers are all (*ex ante*) identical. A firm is a unit of production that can either be filled by a worker whose production is y units of output or be unfilled and thus unproductive. We denote by γ the search cost for the firm per unit of time, by w the wage paid by the firms when a match is realized and by r the discount factor. We assume that the wage is exogenous. At every period, matches between workers and firms depend upon the current network of social contacts of size s and the current state of the economy given by the unemployment rate u and the vacancy rate v . We focus on the steady state equilibrium.

Definition 1 *Given a network size s and the associated matching technology $m(s, \cdot, \cdot)$, a (steady-state) labor market equilibrium $(u^*(s), v^*(s))$ is determined by a free-entry condition for firms and a steady-state condition on unemployment flows.*

Let us start with the free-entry condition and the resulting labor demand. At period t , the intertemporal profit of a filled job and of a vacancy are denoted respectively by $I_{F,t}$ and $I_{V,t}$. Recall that the job-filling rate f is defined by (5). Since time is discrete, we have:

$$\begin{aligned} I_{F,t} &= y - w + \frac{1}{1+r} [(1-\delta)I_{F,t+1} + \delta I_{V,t+1}] \\ I_{V,t} &= -\gamma + \frac{1}{1+r} [(1-f)I_{V,t+1} + f I_{F,t+1}] \end{aligned}$$

In the steady state, both $I_{F,t} = I_{F,t+1} = I_F$ and $I_{V,t} = I_{V,t+1} = I_V$. Following Pissarides (2000), we assume that firms post vacancies up to a point where $I_V = 0$. We deduce from this free entry condition the following relation between u and v :

$$\frac{m(s, u, v)}{v} = \gamma \frac{r + \delta}{y - w} \quad (6)$$

In other words, the value of a job is equal to the expected search cost, i.e. the cost per unit of time multiplied by the average duration of search for

ities. Note also that $1/h$ and $1/f$ can be interpreted as the mean duration respectively of unemployment and of vacancies.

the firm. This equation can be mapped in the plane (u, v) and is referred to as the labor demand curve. We then close the model by the following steady-state condition on flows:

$$m(s, u, v) = \delta(1 - u) \quad (7)$$

As above, this equation can be mapped in the plane (u, v) and is referred to as the Beveridge curve. The two equations (6) and (7) with two unknowns u and v fully characterize the labor market equilibrium $(u^*(s), v^*(s))$ as a function of the network size s .

Proposition 3 *Suppose that $\gamma(r + \delta)/(y - w) > \delta/(1 + \delta)$. Then, for all network size s , there exists a labor market equilibrium $(u^*(s), v^*(s))$. If $\gamma(r + \delta)/(y - w)$ is small enough, this equilibrium is unique.*

Observe that the condition on the parameters $\gamma(r + \delta)/(y - w) > \delta/(1 + \delta)$ that guarantees the existence of the equilibrium is very likely to be satisfied. Indeed, we deduce from (6) that $\gamma(r + \delta)/(y - w)$ is equal to the job-filling rate $f(s, u, v)$. A sufficient condition for $f(s, u, v) > \delta/(1 + \delta)$ to hold is $f(s, u, v) > \delta$ that is, the job-filling rate be higher than the job-destruction rate, which is obviously true in most labor markets.

4.2 Social network and unemployment

We now investigate the different properties of the labor market equilibrium and focus on the relationship between the equilibrium unemployment rate $u^*(s)$ and the size of the social network s . We assume from now on that the conditions for uniqueness are met.

Proposition 4 *The equilibrium unemployment rate $u^*(s)$ decreases with s when $s < \bar{s}$, while it increases when $s \geq \bar{s}$.*

Our matching function depends explicitly on the structure of personal contacts and the labor market equilibrium captures the influence of the frictions due to workers social embeddedness on market outcomes. In particular, we know from propositions 1 and 2 that in a sparse network ($s < \bar{s}$), both the individual probability $P(\cdot, u, v)$ to find a job through word-of-mouth and the matching function increase with the network size s . We deduce from the free entry condition (6) that, holding the arrival rate v fixed, unemployment

decreases. The Beveridge curve (7) then implies that unemployment must also decrease to equalize flows out with flows in. Since the two effects have the same sign, $u^*(s)$ decreases with s . When the social network of contacts is dense ($s \geq \bar{s}$), the opposite result holds since negative network externalities prevail in networks of large size and both $P(\cdot, u, v)$ and $m(\cdot, u, v)$ decrease with s .

The impact of the network size s on the equilibrium vacancy rate $v^*(s)$ is ambiguous both when the network is sparse ($s < \bar{s}$) or dense ($s \geq \bar{s}$). Indeed, two opposite effects now in place. On one hand, increasing the size of a sparse network improves the transmission of information through word-of-mouth communication. As a result, matches are more frequent and we deduce from the free entry condition (6) that more vacancies are posted. In other words, $v^*(s)$ and s are positively correlated. On the other hand, rising the size of a sparse network by creating additional direct connections increases the number of matches between workers and firms. We then deduce from the Beveridge curve (7), that vacancies decrease in order to guarantee that the flows out of unemployment are still equal to the flows in unemployment. Therefore, $v^*(s)$ and s are negatively correlated. When the network is dense, this ambiguity remains and is sustained by the opposite intuition: $v^*(s)$ and s are both negatively and positively correlated due to (6) and (7) respectively.

5 Conclusion

In recent years, a growing literature consisting both on empirical work and theoretical contributions has stressed the prominence of social networks in explaining a wide range of economic phenomena. In particular, the prevalent social contacts strongly determine, or at least influence, economic success of individuals in a labor market context.

In this paper, we have analyzed the matching between unemployed workers and vacant jobs in a social network context. More precisely, each individual, who is embedded within a network of social relationships, can find a job either through formal agencies or through informal networks (word-of-mouth communication). From this micro scenario, we first derive an aggregate matching function that have the standard properties but fails to be homogenous of degree one. This is because there is non-monotonic relationship between the size of the social network and the probability to find a job: increasing the size of sparse networks is beneficial to workers whereas it is

detrimental in dense networks. Indeed, increasing the network size of dense networks slows down word-of-mouth information transmission and creates negative network externalities. We then close the model by introducing the behavior of firms and show that there exists a unique labor market equilibrium under mild conditions on the parameters of the economy. Finally, and because of the previous result, we show that the equilibrium unemployment rate decreases with the network size in sparse networks while it increases in dense networks.

In addition to our conclusions, we hope that the analysis conducted in this paper demonstrates that an accurate modelling of social networks provides fruitful theoretical insights, useful to motivate and to guide further empirical scrutiny of real-life labor markets.

References

- [1] Bala, V. and S. Goyal (2000), “A non-cooperative theory of network formation”, *Econometrica*, forthcoming.
- [2] Blanchard, O.J. and P. Diamond (1994), “Ranking, unemployment duration, and wages”, *Review of Economic Studies* 61, 417-434.
- [3] Boorman, S.A. (1975), “A combinatorial optimization model for transmission of job information through contact networks”, *Bell Journal of Economics* 6, 216-249.
- [4] Burdett, K., Shi, S. and R. Wright (2000), “Pricing with frictions”, Unpublished manuscript, University of Pennsylvania.
- [5] Butters, G. (1977), “Equilibrium distributions of sales and advertising prices”, *Review of Economic Studies* 44, 465-491.
- [6] Calvó-Armengol, A. (2000), “Job contact networks”, Unpublished manuscript, Universitat Pompeu Fabra and CERAS.
- [7] Coles, M.G. and E. Smith (1996), “Cross-section estimation of the matching function: evidence from England and Wales”, *Economica* 63, 589-597.
- [8] Corcoran, M., Datcher, L. and G.J. Duncan (1980), “Most workers find jobs through word of mouth”, *Monthly Labor Review* 103, 33-35

- [9] Diamond P. (1981), “Mobility costs, frictional unemployment, and efficiency”, *Journal of Political Economy* 89, 798-812.
- [10] Ellison, G. and D. Fudenberg (1995), “Word-of-mouth communication and social learning”, *Quarterly Journal of Economics* 110, 93-126.
- [11] Granovetter, M.S. (1995), *Getting a Job: A Study of Contacts and Careers*, Cambridge, MA: Harvard University Press.
- [12] Hall, R.E. (1979), “A theory of the natural unemployment rate and the duration of employment”, *Journal of Monetary Economics* 5, 153-169.
- [13] Holzer, H. (1988), “Search method used by unemployed youth”, *Journal of Labor Economics* 6, 1-20.
- [14] Jackson, M.O. and A. Wolinsky (1996), “A strategic model of social and economic networks”, *Journal of Economic Theory* 71, 44-74.
- [15] Kugler, A. (2000), “Employee referrals and the inter-industry wage structure”, Unpublished manuscript, Universitat Pompeu Fabra.
- [16] Lagos, R. (2000), “An alternative approach to search frictions”, *Journal of Political Economy*, forthcoming.
- [17] Montgomery, J.D. (1991), “Social networks and labor-market outcomes: toward and economic analysis”, *American Economic Review* 81, 1408-1418.
- [18] Montgomery, J.D. (1992), “Job search and network composition: Implications of the strength-of-weak-ties hypothesis”, *American Sociological Review* 57, 586-596.
- [19] Mortensen, D.T. and C.A. Pissarides (1999), ‘New developments in models of search in the labor market’, in Card, D. and O. Ashenfelter (eds.), *Handbook of Labor Economics*, Amsterdam: Elsevier Science, ch.39, 2567-2627.
- [20] Mortensen, D.T. and T. Vishwanath (1994), “Personal contacts and earnings: it is who you know!”, *Labour Economics* 1, 187-201.
- [21] Peters, M. (1991), “Ex ante price offers in matching games non-steady states”, *Econometrica* 59, 1425-1454.

- [22] Petrongolo, B. and C.A. Pissarides (2000), “Looking into the black box: a survey of the matching function”, CEPR Discussion Paper 2409, London.
- [23] Pissarides, C.A. (1979), “Job matchings with state employment agencies and random search”, *Economic Journal* 89, 818-833.
- [24] Pissarides, C.A. (2000), *Equilibrium Unemployment Theory*, 2nd edition, Cambridge: MIT Press.
- [25] Smith, T.E. and Y. Zenou (2000), “A discrete-time stochastic model of job matching”, CEPR Discussion Paper, London.
- [26] Topa, G. (1999), “Social interactions, local spillovers and unemployment”, Unpublished manuscript, New York University.

A Appendix

Proof of Proposition 1.

Let $q(s, \theta) \equiv \frac{\theta(1-\theta^s)}{s(1-\theta)}$. Then, $P(s, u, v) = 1 - Q(s, u, v)$ where $Q(s, u, v) = [1 - vq(s, \theta)]^s$. The properties of $P(\cdot)$ can thus be deduced from that of $Q(\cdot)$ established below:

(a) $Q(s, u, \cdot)$ is decreasing and strictly convex with respect to v . Indeed, differentiating once with respect to v gives: $\frac{\partial Q}{\partial v} = -sQ \frac{q}{1-vq} < 0$. Differentiating twice we get: $\frac{\partial^2 Q}{\partial v^2} = -s \frac{\partial Q}{\partial v} \frac{q}{1-vq} - sQ \frac{q^2}{(1-vq)^2}$. Replacing $\frac{\partial Q}{\partial v}$ by its expression above gives: $\frac{\partial^2 Q}{\partial v^2} = s(s-1)Q \frac{q^2}{(1-vq)^2} > 0$.

(b) $Q(s, \cdot, v)$ is increasing with respect to u . Moreover, there exists $\tilde{\delta} \in [0, 1)$ such that $Q(s, \cdot, v)$ is strictly concave with respect to u as long as $\delta \geq \tilde{\delta}$. Indeed, simplifying by $(1-\theta)$ gives $q(s, \theta) = \frac{1}{s}(\theta + \dots + \theta^s)$. Hence, $q(s, \cdot)$ is increasing with respect to θ , implying that $Q(s, u, v) = [1 - vq(s, 1-u)]^s$ is increasing with respect to u . From $\theta = (1-\delta)(1-u)$ we deduce that:

$$\frac{\partial^2 Q}{\partial u^2} = (1-\delta)^2 \times \frac{\partial^2}{\partial \theta^2} [1 - vq(s, \theta)]^s.$$

Differentiating twice gives:

$$\frac{\partial^2}{\partial \theta^2} [1 - vq]^s = -vs [1 - vq]^{s-2} \left[(1 - vq) \frac{\partial^2 q}{\partial \theta^2} - v(s-1) \left(\frac{\partial q}{\partial \theta} \right)^2 \right]$$

Hence, $\frac{\partial^2 Q}{\partial u^2} < 0$ is equivalent to $(1 - vq) \frac{\partial^2 q}{\partial \theta^2} - v(s-1) \left(\frac{\partial q}{\partial \theta} \right)^2 > 0$ where

$$\begin{cases} q(s, \theta) = \frac{1}{s}(\theta + \dots + \theta^s) \\ \frac{\partial q(s, \theta)}{\partial \theta} = \frac{1}{s}(1 + \dots + s\theta^{s-1}) \\ \frac{\partial^2 q(s, \theta)}{\partial \theta^2} = \frac{1}{s}(2 + \dots + s(s-1)\theta^{s-2}) \end{cases}$$

At $\theta = 0$ we have: $q(s, \theta) = 0$, $\frac{\partial q(s, \theta)}{\partial \theta} |_{\theta=0} = \frac{1}{s}$ and $\frac{\partial^2 q(s, \theta)}{\partial \theta^2} |_{\theta=0} = \frac{2}{s}$. Therefore, $\frac{\partial^2 Q(s, u, v)}{\partial u^2} |_{\theta=0} < 0$ is equivalent to $2s > v(s-1)$ which is true. Denote by $\tilde{\theta}$ the smallest positive root of the polynomial R in θ of degree $2(s-1)$ given by: $R(\theta) \equiv (1 - vq) \frac{\partial^2 q}{\partial \theta^2} - v(s-1) \left(\frac{\partial q}{\partial \theta} \right)^2$. If $R(\theta) > 0$ for all $\theta > 0$ we set $\tilde{\theta} = +\infty$ by definition. From $R(0) > 0$ and by continuity, we deduce that $R(\theta) > 0$ on $[0, \tilde{\theta})$. Let $\tilde{\delta} = 1 - \min\{\tilde{\theta}, 1\}$. Then, $R(\theta) > 0$ on $[0, 1 - \tilde{\delta})$ implying that $\frac{\partial^2 Q(s, u, v)}{\partial u^2} < 0$ for all $u \in$

$[0, 1]$ that is, $Q(s, \cdot, v)$ strictly concave with respect to u , as long as $\delta \geq \delta$.

- (c) $Q(\cdot, u, v)$ is strictly convex, decreasing in $[0, \bar{s}]$ and increasing on $[\bar{s}, +\infty)$. We prove it in four claims.¹²

Claim 1 The function $q(\cdot, \theta)$ is strictly convex on $[1, +\infty)$.

Proof. Fix $\theta \in (0, 1)$. We need to show that $\gamma(s) = \frac{1-\theta^s}{s}$ is strictly convex. Differentiating twice we get $\gamma''(s) = \frac{1}{s^3}\zeta(s)$ where $\zeta(s) = 2(1-\theta^s) + 2s\theta^s \ln \theta - s^2\theta^s (\ln \theta)^2$. We now prove that $\zeta(s) > 0, \forall s \in [1, +\infty)$. Differentiating once gives $\zeta'(s) = -s^2\theta^s (\ln \theta)^3 > 0$. Therefore, ζ increases with minimum $\zeta(1) = 2(1-\theta) + 2\theta \ln \theta - \theta (\ln \theta)^2$. We thus need to show $\kappa(\theta) \equiv \zeta(1)$ is positive, $\forall \theta \in (0, 1)$. Differentiating once gives: $\kappa'(\theta) = -(\ln \theta)^2$. Therefore κ decreases on $(0, 1)$ with infimum $\kappa(1) = 0$. *Q.E.D.*

Claim 2 The function $Q(\cdot, u, v)$ is strictly convex on $[1, +\infty)$.

Proof. First note that if f is a strictly convex and decreasing function ($f' < 0$ and $f'' > 0$) and g is strictly concave ($g'' < 0$), then $f \circ g$ is strictly convex. Indeed, $(f \circ g)'' = f''(g) \times [g']^2 + f'(g) \times g'' > 0$. We have $Q(s, u, v) = [1 - vq(s, \theta)]^s$. Also, for all $d \in (0, 1)$, the function $x \mapsto d^x$ is strictly convex and decreasing on $[1, +\infty)$, and $1 - q(\cdot, \theta)$ is strictly concave (from the previous claim) and takes values on $(0, 1)$. Therefore, $Q(\cdot, u, v)$ is strictly convex. *Q.E.D.*

Claim 3 $\frac{\partial Q(s, u, v)}{\partial s} \Big|_{s=1} < 0$.

Proof. From $Q(s, u, v) = [1 - vq(s, \theta)]^s$ we deduce that $\frac{\partial Q}{\partial s} = \frac{Q}{1-vq} \Phi(s)$ where $\Phi(s) = [1 - vq(s, \theta)] \ln(1 - vq(s, \theta)) - sv \frac{\partial q(s, \theta)}{\partial s}$. Therefore, $\frac{\partial Q(s, u, v)}{\partial s} \Big|_{s=1}$ is equivalent to $\Phi(1) < 0$. Direct computation gives $\frac{\partial q(s, \theta)}{\partial s} = -\frac{\theta}{1-\theta} \left[\frac{1-\theta^s}{s^2} + \frac{\theta^s}{s} \ln \theta \right]$. From $q(1, \theta) = \theta$ and $\frac{\partial q(s, \theta)}{\partial s} \Big|_{s=1} = -\theta \left[1 + \frac{\theta}{1-\theta} \ln \theta \right]$, and letting $\alpha \equiv v\theta$ we obtain $\Phi(1) = (1 - \alpha) \ln(1 - \alpha) + \alpha \left(1 + \frac{\theta}{1-\theta} \ln \theta \right)$. We denote by $\varphi_\theta(\alpha)$ this last expression, where $\alpha \in (0, \theta)$. Establishing that $\frac{\partial Q(s, u, v)}{\partial s} \Big|_{s=1} < 0$ is thus equivalent to proving that $\varphi_\theta(\alpha) < 0$ on $[0, \theta], \forall \theta \in (0, 1)$. Fix $\theta \in (0, 1)$. Differentiating once with respect to α we get: $\varphi'_\theta(\alpha) = -\ln(1 - \alpha) + \frac{\theta}{1-\theta} \ln \theta$

¹²This proof follows closely that of Lemma 2 in Calvó-Armengol (2000).

and $\varphi''_{\theta}(\alpha) = \frac{1}{1-\alpha} > 0$. Therefore, φ_{θ} is convex implying that φ'_{θ} increases on $[0, \theta]$ with maximum $\varphi'_{\theta}(\theta) = \frac{1}{1-\theta} [\theta \ln \theta - (1-\theta) \ln(1-\theta)]$. We now prove that $\varphi'_{\theta}(\theta) < 0$. It is straightforward to see that $x \mapsto x \ln x - (1-x) \ln(1-x)$ is worth 0 at $x = 0, \frac{1}{2}$ and 1, takes negative values on $(0, \frac{1}{2})$ and positive values on $(\frac{1}{2}, 1)$. We distinguish two cases:

- (i) $\theta \leq \frac{1}{2}$. Then $\theta \ln \theta - (1-\theta) \ln(1-\theta) \leq 0$ implying that $\varphi'_{\theta}(\theta) \leq 0$. Moreover, φ'_{θ} increases on $[0, \theta]$. Therefore, $\varphi'_{\theta}(\alpha) < 0, \forall \alpha \in [0, \theta]$ implying that φ_{θ} decreases on $[0, \theta]$. But $\varphi_{\theta}(0) = 0$. Hence, $\varphi_{\theta}(\alpha) < 0, \forall \alpha > 0$;
- (ii) $\theta > \frac{1}{2}$. Now $\theta \ln \theta - (1-\theta) \ln(1-\theta) > 0$ implying that $\varphi'_{\theta}(\theta) > 0$. We already know that $\varphi'_{\theta}(0) < 0$ and that φ'_{θ} increases. Therefore, there exists some $\alpha_{\theta}^* \in (0, \theta)$ such that φ_{θ} decreases on $[0, \alpha_{\theta}^*]$ and increases on $[\alpha_{\theta}^*, \theta]$. As $\varphi_{\theta}(0) = 0$ it thus suffices to show that $\varphi_{\theta}(\theta) < 0$ to conclude that $\varphi_{\theta}(\alpha) < 0, \forall \alpha > 0$. We have $\varphi_{\theta}(\theta) = (1-\theta) \ln(1-\theta) + \theta \left(1 + \frac{\theta}{1-\theta} \ln \theta\right)$. We know that $(1-\theta) \ln(1-\theta) < \theta \ln \theta$ when $\theta \in (\frac{1}{2}, 1)$. Therefore, $\varphi_{\theta}(\theta) < \frac{\theta}{1-\theta} [1 - \theta + \ln \theta]$. It is easy to check that $x \mapsto 1 - x + \ln x$ is negative on $(0, 1)$. Indeed, this function is worth 0 at $x = 1$ and increases on $(0, 1)$. Hence, $\varphi_{\theta}(\theta) < 0$ implying that $\varphi_{\theta}(\alpha) < 0, \forall \alpha \in [0, \theta]$. *Q.E.D.*

Claim 4 For high values of $s \in [1, +\infty)$, the function $Q(\cdot, u, v)$ increases towards its limit $\lim_{s \rightarrow +\infty} Q(s, u, v) = \exp\left(-\frac{v\theta}{1-\theta}\right)$.

Proof. We know that $\frac{\partial Q}{\partial s} = \frac{Q}{1-vq} \Phi(s)$ where:

$$\Phi(s) = [1 - vq(s, \theta)] \ln(1 - vq(s, \theta)) - sv \frac{\partial q(s, \theta)}{\partial s}$$

and $\frac{\partial q(s, \theta)}{\partial s} = -\frac{\theta}{1-\theta} \left[\frac{1-\theta^s}{s^2} + \frac{\theta^s}{s} \ln \theta \right]$. When $s \rightarrow +\infty$ we thus have $s \frac{\partial q(s, \theta)}{\partial s} \sim -\frac{\theta}{(1-\theta)s}$, $q(s, \theta) \sim \frac{\theta}{(1-\theta)s}$ and $\ln(1 - vq(s, \theta)) \sim -\frac{v\theta}{(1-\theta)s}$. Hence, $\Phi(s) \sim \left[\frac{v\theta}{(1-\theta)s} \right]^2$ meaning that $\frac{\partial Q}{\partial s} > 0$ for high values of s . Therefore, $Q(\cdot, u, v)$ increases towards its limit $\exp\left(-\frac{v\theta}{1-\theta}\right)$ when $s \rightarrow +\infty$. *Q.E.D.* ■

Proof of Proposition 2.

Recall that $m(s, u, v) = u[v + (1 - v)P(s, u, v)]$. Therefore,

- (a) the properties of the matching function $m(\cdot, u, v)$ with respect to s are deduced from that of $P(\cdot, u, v)$ given in Proposition 1(ii).
- (b) With some algebra and using Proposition 1 we get:

$$\begin{cases} \frac{\partial m(s, u, v)}{\partial v} = u[1 - P(s, u, v)] + u(1 - v) \frac{\partial P(s, u, v)}{\partial v} > 0 \\ \frac{\partial^2 m(s, u, v)}{\partial v^2} = -2u \frac{\partial P(s, u, v)}{\partial v} + u(1 - v) \frac{\partial^2 P(s, u, v)}{\partial v^2} < 0 \end{cases}$$

proving that $m(s, u, \cdot)$ is increasing and concave with respect to v .

- (c) With some algebra we get:

$$\begin{cases} \frac{\partial m(s, u, v)}{\partial u} = v + (1 - v) \frac{\partial}{\partial u} [uP(s, u, v)] \\ \frac{\partial^2 m(s, u, v)}{\partial u^2} = (1 - v) \frac{\partial^2}{\partial u^2} [uP(s, u, v)] \end{cases}$$

Simplifying by $(1 - v)$, we deduce from (2) that $P(s, u, v) = 1 - \left[1 - \frac{v}{s}(\theta + \theta^2 + \dots + \theta^s)\right]^s$, where $\theta = (1 - \delta)(1 - u)$. Fix v and s and let $R(u) \equiv uP(s, u, v)$. Clearly, $R(u)$ is a polynomial in u of degree $2s + 1$, with roots 0 and 1 (that is, $R(0) = R(1) = 0$) and strictly positive on $(0, 1)$ (that is, $R(u) > 0, \forall 0 < u < 1$). Therefore, $R'(u) = u \frac{\partial P(s, u, v)}{\partial u} + P(s, u, v)$ is a polynomial of degree $2s$ that has a unique root $\tilde{u} \in (0, 1)$ corresponding to the global maximum of R on $[0, 1]$. From $R'(u)$ continuous and $R'(0) = P(s, 0, v) > 0$ we deduce that $R'(u) > 0$ on $(0, \tilde{u})$ and that $R''(u)$ is negative locally around \tilde{u} that is, $R''(u) < 0$ on $(\tilde{u} - \varepsilon, \tilde{u} + \varepsilon)$ for some $\varepsilon > 0$. We also deduce from $R''(u) = u \frac{\partial^2 P(s, u, v)}{\partial u^2} + 2 \frac{\partial P(s, u, v)}{\partial u}$ and Proposition 1(ii) that $R''(0) = 2 \frac{\partial P(s, u, v)}{\partial u} \Big|_{u=0} < 0$. If $R''(u)$ were to change sign on $[0, \tilde{u}]$, by continuity of R'' and because both $R''(0) < 0$ and $R''(\tilde{u}) < 0$, it would imply that $R''(u)$ had two distinct roots on $(0, \tilde{u})$, which is impossible because successive derivatives of polynomials have nested roots, and $R'(u)$ has only one root on $[0, 1]$. Therefore, $R''(u) < 0$ on $[0, \tilde{u}]$. Let $\bar{u} = \arg \max \{u \in [0, 1] \mid R' > 0 \text{ and } R'' < 0 \text{ on } [0, u]\}$. Clearly, $0 < \tilde{u} \leq \bar{u} \leq 1$. \blacksquare

Lemma 1 *The hiring probability $h(s, u, v) = \frac{m(s, u, v)}{u}$ is decreasing and convex in u and increasing and concave in v . The properties of $h(\cdot, u, v)$ with respect to s are the same than that of $P(\cdot, u, v)$.*

Proof. Recall that $h(s, u, v) = v + (1 - v)P(s, u, v)$. With some algebra and using Proposition 1 we get:

$$\begin{cases} \frac{\partial h(s, u, v)}{\partial u} = (1 - v) \frac{\partial P(s, u, v)}{\partial u} < 0 \\ \frac{\partial^2 h(s, u, v)}{\partial u^2} = (1 - v) \frac{\partial^2 P(s, u, v)}{\partial u^2} > 0 \\ \frac{\partial h(s, u, v)}{\partial v} = 1 - P(s, u, v) + (1 - v) \frac{\partial P(s, u, v)}{\partial v} > 0 \\ \frac{\partial^2 h(s, u, v)}{\partial v^2} = -2 \frac{\partial P(s, u, v)}{\partial v} + (1 - v) \frac{\partial^2 P(s, u, v)}{\partial v^2} < 0 \end{cases}$$

which completes the proof. ■

Lemma 2 *The filling probability $f(s, u, v) = \frac{m(s, u, v)}{v}$ is increasing in u and decreasing in v . The properties of $h(\cdot, u, v)$ with respect to s are the same than that of $P(\cdot, u, v)$.*

Proof. Recall that $f(s, u, v) = u \left[1 - \left(1 - \frac{1}{v} \right) P(s, u, v) \right]$. With some algebra and using Proposition 1 we get:

$$\begin{cases} \frac{\partial f(s, u, v)}{\partial u} = \frac{f(s, u, v)}{u} - u \left(1 - \frac{1}{v} \right) \frac{\partial P(s, u, v)}{\partial u} > 0 \\ \frac{\partial f(s, u, v)}{\partial v} = -\frac{u}{v^2} P(s, u, v) - u \left(1 - \frac{1}{v} \right) \frac{\partial P(s, u, v)}{\partial v} < 0 \end{cases}$$

which completes the proof. ■

Proof of Proposition 3.

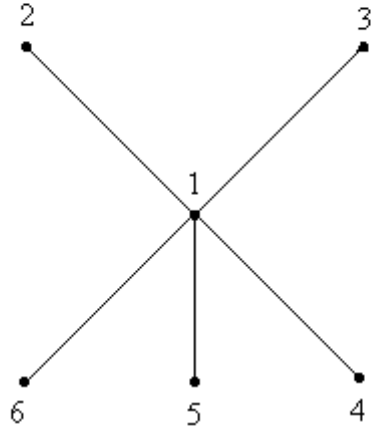
Fix the network size s . We first prove that along the Beveridge curve, u is decreasing in v . Indeed, let (u, v) and (u', v') both satisfying (7) with $v' > v$. By definition, $m(s, u, v) = \delta(1 - u)$ and $m(s, u', v') = \delta(1 - u')$. Suppose that $u' \geq u$. Then, $m(s, u', v') \leq m(s, u, v)$. But we deduce from Proposition 2 that $m(s, u, v) < m(s, u, v') \leq m(s, u', v')$ which yields to a contradiction. Therefore, $u' < u$. We now prove that along the curve in the plane (u, v) obtained from the free entry condition (6), u is increasing in v . Indeed, from the implicit function theorem we get: $\frac{dv}{du} = -\frac{\frac{\partial(m/v)}{\partial u}}{\frac{\partial(m/v)}{\partial v}} > 0$ according to Lemma 2. If a labor market equilibrium exists on $[0, \bar{u}] \times [0, 1] \subseteq [0, 1]^2$, it is thus unique. We now prove existence. At $v = 1$, $m(s, u, 1) = u$. We deduce from (7) that $\left(\frac{\delta}{1+\delta}, 1 \right)$ belongs to the Beveridge Curve and from (6) that $\left(\gamma \frac{r+\delta}{y-w}, 1 \right)$ satisfies the free entry condition (which requires that

$\gamma \frac{r+\delta}{y-w} \leq 1$). A necessary and sufficient condition for an equilibrium to exist is thus $\gamma \frac{r+\delta}{y-w} > \frac{\delta}{1+\delta}$. Clearly, when $\gamma \frac{r+\delta}{y-w} \leq \bar{u}$, the equilibrium is unique. ■

Proof of Proposition 4.

Suppose first that $s < \bar{s}$. Let (u, v) on the Beveridge Curve, thus satisfying (7), and let s' such that $s < s' < \bar{s}$. We know from Proposition 2 that $m(s, u, \cdot)$ increases with v and that $m(s', u, v) > m(s, u, v)$. Therefore, if we keep u constant while increasing the network size from s to s' , the vacancy rate adjusts by decreasing. As a result, the Beveridge Curve (that decreases on the plane (u, v)) shifts downwards. Let now (u, v) satisfy (6). We know from Lemma 2 that $f(s, u, v) = \frac{m(s, u, v)}{v}$ is an decreasing function of v and that $f(s', u, v) > f(s, u, v)$. Therefore, the vacancy rate adjusts by increasing and the curve associated to the free entry condition shifts upwards on the plane (u, v) . One can check geometrically that $u^*(s') < u^*(s)$. Suppose now that $s \geq \bar{s}$ and let $s' > s$. Following a similar reasoning it is straightforward to see that the Beveridge Curve now shifts upwards while the free entry condition curve shifts downwards, implying that $u^*(s') > u^*(s)$. ■

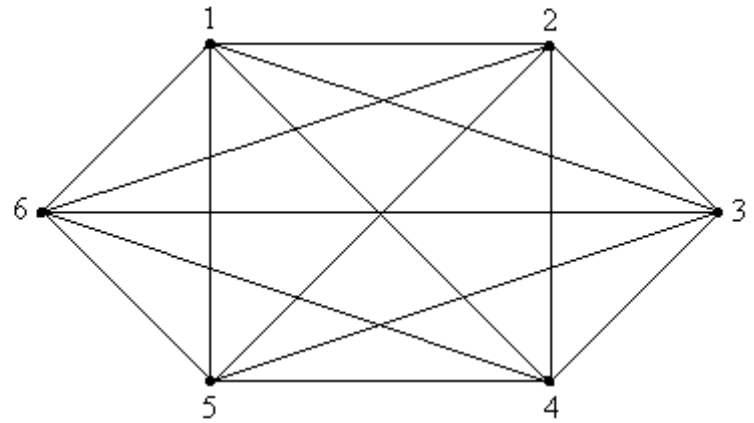
Figure 1a: Star-shaped graph
centered on worker 1 ($n = 6$)



$$P_1 = 1 - [1 - v\theta]^5 >$$

$$v\theta \frac{1 - \theta^5}{5(1 - \theta)} = P_2 = P_3 = P_4 = P_5 = P_6$$

Figure 1b: Complete graph ($n = 6$)



$$P_1 = P_2 = P_3 = P_4 = P_5 = P_6$$

$$= 1 - \left[1 - v\theta \frac{1 - \theta^5}{5(1 - \theta)} \right]^5$$