Chicken & Egg: 
Competition among Intermediation Service Providers

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This version, June 12, 2002

\(^1\)This paper has benefitted from the comments and criticisms of participants at the 2000 EEA Meeting in Bolzano, at conferences in Heidelberg and in Toulouse, and at various seminars in Vienna, Rotterdam, Lausanne and Paris. We are in particular indebted to P.Aghion, C.Shapiro and J.Tirole. Remaining errors are of course ours.

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Abstract

We analyze a model of imperfect price competition between intermediation service providers. We insist on the following features that are relevant for informational intermediation via the Internet: the presence of indirect network externalities, the possibility of using the non-exclusive services of several intermediaries at the same time, and the widespread practice of price discrimination based on users’ identity and on usage. Efficient market structures emerge in equilibrium, as well as some specific form of inefficient structures. Intermediaries have incentives to propose non-exclusive services, as this moderates competition and allows them to exert market power. The pricing and business strategies followed by intermediation services providers to protect their position or gain new market shares are analyzed in detail.
"Ultimately we’re an information broker. On the left side we have lots of products; on the right side we have lots of customers. We’re in the middle making the connections. The consequence is that we have two sets of customers: consumers looking for books and publishers looking for consumers. Readers find books or books find readers."

Jeff Bezos, Amazon.com
Leadership Online, Harvard Business School.

1 Introduction

While in the traditional brick and mortar economy, intermediaries often buy and resell goods, the development of the new technologies of information and communication have brought informational intermediation to the forefront of the “new economy”. Informational intermediation consists of services such as search, certification, advertising, price discovery, as opposed to storage, exposition or delivery. In these activities, users have larger expected gains, the larger the number of users on the other side of the market, a property referred to as indirect network externalities. This is the case for instance for individuals visiting a matchmaking (e.g. dating) service, for sellers of goods and services participating in a marketplace, as well as for buyers because a large number of sellers gives them access to more diversity. Indirect network externalities give rise to a “chicken-and-egg” problem: to attract buyers, an intermediary should have a large base of registered sellers, but these will be willing to register only if they expect many buyers to show up.

This paper proposes an analysis of the intermediation market that accounts for such specific aspects of informational intermediation as network externalities, non-exclusivity of services and price-discrimination. The paper makes two contributions. First, it determines the equilibrium market structures that are likely to emerge and characterizes their efficiency properties. Second, it provides a precise description and analysis of the pricing strategies that allow intermediation service providers to protect their business or to gain new business.

More precisely, we investigate an imperfect-competition, Bertrand game between two

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2See Shapiro-Varian (1998) for a general presentation of the economics of information.
3In the traditional buy and resell activity, this externality translates into the potential rationing of demand (see Yannelle (1989)).
matchmakers in the presence of indirect network externalities. Matchmakers rely on two pricing instruments: registration fees, which are user-specific and paid ex-ante, and a transaction fee, paid ex-post when a transaction takes place between two matched parties. We analyze both the case of exclusive and non-exclusive services, depending on whether users can have their request processed by only one intermediary or by several at the same time; in the latter case, users are said to engage in “multi-homing”.

Because of network effects and imperfect matching technologies, an efficient allocation may involve only one intermediary serving all users or, with non-exclusive technologies and low costs, both intermediaries serving all users, a situation we call “global multi-homing”. In our model, there always exist efficient equilibria. With non-exclusive services, however, there may also exist an inefficient equilibrium that involves multi-homing on one side of the market and single-homing on the other side.

We also characterize the relevant pricing strategies and the maximal profits that can be sustained in equilibrium. Due to indirect network effects, the key pricing strategies are of a “divide-and-conquer” nature, subsidizing the participation of one side (divide) and recovering the loss on the other side (conquer). Exclusivity then implies highly contestable market structures, where all potential profits are eroded in order to protect a monopoly position. With non-exclusive services, it is easier to “divide” but more difficult to “conquer”; intermediaries are then able to avoid fierce price competition and make positive intermediation profits in equilibrium. Moreover, the most profitable market equilibrium may precisely be the inefficient one.

Transaction fees appear to be a powerful weapon for intermediation service providers to gain market shares. To highlight this aspect, we compare our conclusions with the ones obtained when intermediaries are unable to monitor the transactions and thus to impose transaction fees. By and large, ruling out transaction fees raises intermediation profits. In the cases in which global multi-homing is efficient, however, transaction fees enable intermediaries to profitably differentiate, one offering low registration but high transaction fees, the other

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4See Innes and Sexton (1993) for an application to monopoly pricing, Segal (2001) for an application to mechanism design, and Jullien (2001) for an application to network competition.

5A companion proceeding paper (Caillaud-Jullien, 2001a) sketches some preliminary results in this case, derived from a similar model with simplifying assumptions (see below).
adopting the mirror pricing policy.

The paper is in a large part motivated by the development of Internet-related intermediation, which best fits our assumptions.\textsuperscript{6} Many B2B websites provide a bundle of services among which matching is critical. The website esteel.com, for example, records types and characteristics of orders, and connects buyers and sellers who want to trade some given quality of steel with some well-specified properties. Most portals, as well as information-oriented or trade-oriented B2C websites provides information and matching services, such as search facilities or certification.\textsuperscript{7} In these activities, the existence of indirect network effects is widespread and well documented.\textsuperscript{8}

Informational intermediation and matchmaking on the Internet are often non-exclusive. A websurfer looking for some specific good or service will usually visit and register with several intermediation service providers to increase his chances of finding a match. Similarly, firms offering various services register with different intermediaries in order to benefit from their different users bases.\textsuperscript{9} Exclusivity may sometimes be imposed by intermediaries so as to ensure that their efforts in processing the users’ demands end up with a transaction, or because registration involves the specific building of a profile which the intermediary may consider as proprietary. Understanding electronic intermediation markets therefore requires a careful analysis of the role of exclusivity.

Finally, a wide array of pricing strategies is observed in intermediation services on the Internet. The use of flat rates is quite common in e-commerce, but different users and different usages are treated differently. Access to general purpose portals is free for websurfers, while announcers pay on the basis of click-through, or of priority orders, e.g. access to top-screen banners. Auction websites charge fees that are proportional to the transaction price or even

\textsuperscript{6}Our analysis has some links with the literature on competing stock exchanges (Admati and Pfleiderer (1988), Pagano (1989)). In this literature, the indirect network effects translate into a positive feedback effect between volume and liquidity, but there is no price-discrimination.

\textsuperscript{7}See the study by the University of Austin, at www.internetindicators.com, for a decomposition and evaluation of different types of activities related to Internet, the survey of The Economist on e-commerce for a general presentation, and Kaplan and Sawhney (2000) for a discussion of auction sites and the various types of aggregation.

\textsuperscript{8}There are also pecuniary externalities between participants on the same side of the market which can be negative (see Baye-Morgan (2001)) but also positive, for instance for demand aggregators such as mobshop.com.

\textsuperscript{9}Non-exclusivity is not specific to Internet-related activities, but rather to the low cost of service per customer. In real estate or retail distribution for instance, the service may or may not be exclusive depending on the contractual agreement.
piecewise linear,\textsuperscript{10} but sellers also have to pay registration fees that depend on their reserve prices.

So, while the insights of our analysis are of wider applicability, they are of particular relevance for e-commerce.

The paper is organized as follows. Section 2 spells out the details of our model of intermediation, the equilibrium concept and the benchmark case of exclusive services. Section 3 analyzes users’ behavior when multi-homing is possible. Section 4 discusses the results. In section 5, we briefly present the results for the case where transaction fees are not available. We conclude in section 6. Proofs are relegated in the appendix.

2 A basic model with exclusive services

2.1 The framework

Consider a simple pairwise matching model with two homogenous populations, labelled $i = 1$ and 2, each consisting of a continuum of mass 1 of ex-ante identical agents. For a given agent, there exists a unique matching partner on the other side of the market with whom trade is valuable; the total gross gain from trade between matching partners is normalized to 1. Matched partners follow an efficient bargaining process to determine the transaction price, which yields a linear sharing of the total net trade surplus, with a share $u_i$ for the type-$i$ agent and a better bargaining position for $2$-agents: $u_2 \geq \frac{1}{2} \geq u_1$ and $u_1 + u_2 = 1$.\textsuperscript{11}

A given $j$-agent has zero probability of finding his matching partner by just picking randomly within the $i$-population. But he can turn to an intermediary, endowed with an information technology that can perform matchmaking services. This matchmaker builds a data base with the characteristics of the agents who register with it. For each potential matching pair, the information technology identifies the match with probability $\lambda \leq 1$, provided both agents are registered in the data base; the search fails otherwise. Hence, if $n_i$ randomly-drawn agents of type $i$ register with a matchmaker, a $j$-agent finds his matching partner with probability $\lambda n_i \in [0,1]$ through this intermediary. $\lambda$ characterizes the quality

\textsuperscript{10}The fees on final value at aucland.com are 4%, while at eBay.com they amount for 5,25%, 2.75% or 1.5% of the transaction price depending on the level of this price.

\textsuperscript{11}The net trade surplus between matching partners equals the gross trade surplus minus transaction fees that may be charged by intermediaries (see below).
of the matching process, the likelihood that there are no mistakes or errors in registration and data processing. It is related to the intermediary’s technology, not to the users’ characteristics; in particular, two processes performed by two matchmakers would succeed or fail independently.

Two matchmakers, \( k \in \{I, E\} \), compete using the same technology. Each matchmaker has a cost \( c_i \) of providing services to (a mass of) one \( i \)-agent.\(^{12}\) We assume that intermediation is efficient: \( \lambda > c \equiv c_1 + c_2 \).

Intermediaries can observe and verify the types of registered users and whether trade takes place, but not the transaction price; so, they can price-discriminate using two pricing instruments. First, matchmaker \( k \) can charge each \( i \)-user an up-front connection or registration fee \( p_k^i \). We do not restrict registration prices to be non-negative. A negative price can be the consequence of gifts given to joining members, or the result of the addition of free services to the basic free-of-charge matching service.

Second, matchmaker \( k \) can also charge a total transaction fee \( t_k^i \) conditional on the occurrence of trade. The net surplus to be shared among matched partners then becomes \( (1 - t_k^i) \geq 0 \).\(^{13}\) We impose that \( 0 \leq t_k^i \), since with negative transaction fees, even agents who are not matched would engage in trade. Our focusing only on the total transaction fee is a consequence of several assumptions: the value of trade between partners is constant and common knowledge, users engage in efficient bargaining, and only the occurrence of trade is observable. Models of efficient bargaining with transferable utility, e.g. a Nash bargaining solution with given weights or a bargaining price that equalizes users net utilities from bargaining, imply that users’ utilities only depend upon the sum of individual transaction fees, that is upon the total transaction fee.\(^{14}\) Note also that there is no trade distortion associated with the use of transaction fees: efficiency considerations therefore only relate to the

\(^{12}\)It includes the agent’s personal cost, the matchmaker’s cost of registration and information processing. In some cases, intermediaries finance themselves through advertising. In these cases, \( c_i \) should include the advertising revenue that a customer-\( i \) generates. This means that the cost \( c_i \) could be negative.

\(^{13}\)Our basic matching process and the possibility of imposing a transaction fee appear in Yavas (1994). This paper, however, does not allow for registration fees and focuses on the competition between a matchmaker and a search market with frictions.

\(^{14}\)Spelling out a bargaining model with heterogenous matching pairs and observable transaction prices would introduce more instruments for price discrimination. This would reinforce our conclusions with respect to the efficiency properties of equilibria. The level of sustainable profits would be different, but the impact of price discrimination would be qualitatively similar (see the discussion in subsection 3.4).
intermediation process, not to the possible trade distortion after a match. Finally, in many instances, transaction fees are difficult to implement because the agents may agree ex-post to bypass the matchmaker. We will present the analysis of such a situation in the last section.\(^{15}\)

In equilibrium, the agents’ expected surplus from trade must be non-negative. We thus restrict attention to prices \(P^k = (p^k_1, p^k_2, t^k)\), such that

\[
\lambda u_i(1 - t^k) - p^k_i \geq 0, \quad i = 1, 2.
\]

\(^{1}\) \(\frac{1}{2} \text{Timing and equilibrium} \)

We analyze a two-stage model. In a first stage, both matchmakers set prices \(P^k\) simultaneously and non-cooperatively. The resulting price system \(P = \{P^I, P^E\}\) is publicly observable. In a second stage, users simultaneously choose which matchmakers (if any) to register with.

Let us assume, for the rest of this section, that matchmakers offer exclusive services; that is, for technological or legal reasons, users can register with at most one intermediary.\(^{16}\) Let \(\mathcal{N} = \{n^I_i, n^E_i\}_{i=1,2}\) denote the distribution of agents across matchmakers, with \(n^k_i\) the number (proportion) of agents of type \(i\) who register with matchmaker \(k\). Let

\[
U_j(P, k, \mathcal{N}) = n^k_i \lambda u_j(1 - t^k) - p^k_j
\]

denote the net (indirect) expected utility of an \(i\)-agent registering with intermediary \(k\) for the prices \(P\) and the allocation \(\mathcal{N}\).\(^{17}\) Similarly, let

\[
\pi^k(P^k, \mathcal{N}) = \sum_{i=1,2} n^k_i(p^k_i - c_i) + \lambda n^k_1n^k_2t^k.
\]

denote matchmaker \(k\)’s profit from charging \(P^k\) given the distribution \(\mathcal{N}\).

With a continuum of users on each side of the market, the setting does not exactly correspond to a game. The definitions below are adapted from the standard concept of subgame-perfect equilibrium.

\(^{15}\)Our European Economic Review Proceeding (Caillaud-Julllien (2001a)) presents some results on exclusive services and ex-post monopoly for the case of perfect and costless matching technologies. But these assumptions deliver some non-robust conclusions. We focus here on non-exclusive services and will rely on this proceeding when relevant.

\(^{16}\)This assumption simplifies notation in the following definitions. In the next section, we indicate how to extend the definitions in the more general case.

\(^{17}\)By definition, \(U_i(P, \emptyset, \mathcal{N}) = 0\).
Definition 1: A distribution of users \( N \) is an equilibrium distribution for a price system \( P \) if for all \( k \in \{I, E, \emptyset\} \):

\[
n^k_i > 0 \implies U_i(P, k, N) = \max_{h \in \{I, E, \emptyset\}} U_i(P, h, N).
\]

A market allocation is a mapping \( N(\cdot) \) that associates to each feasible price system \( P \) an equilibrium distribution of users \( N(P) \).

In words, if some \( i \)-user registers with \( k \), then he must be as well off as if he registered instead with the other matchmaker or none. As a function of prices \( P \), \( n^k_i(P) \) determines \( j \)-users’ demand for matchmaker \( k \)'s services.

There can be multiple market allocations.\(^{18}\) Although most of our results do not rely on point predictions about the equilibrium outcome, we will use a mild refinement to focus on reasonable market allocations. This refinement amounts to ruling out increasing demand functions.

Definition 2: A market allocation \( N(\cdot) \) is monotone if \( \forall k, n^k_i(P^k, P^{-k}) \) is non-increasing in \( P^k \).

Monotonicity is not very restrictive. In particular, it imposes no restriction when, say, \( p^k_1 \) increases while \( p^k_2 \) decreases. Monotonicity is implied for instance by the selection criterion that requires that users coordinate on a Pareto undominated allocation (for users only).\(^{19}\)

Definition 3: An equilibrium is a pair \((P^*, N(\cdot))\), where (i) \( N(\cdot) \) is a monotone market allocation and (ii) \( P^* \) is a Nash equilibrium of the reduced-form pricing game induced by \( N(\cdot) \), with profits \( \pi^k(P, N(P)) \).

Intuitively, an equilibrium consists of a set of prices charged by matchmakers and of a description of how users choose among them for all possible prices. The allocation of

\(^{18}\)That network externalities are a source of multiplicity of equilibria is a well-known phenomenon, see e.g. Farell-Saloner (1985), Katz-Shapiro (1985, 1994).

\(^{19}\)The only caveat is that prices may be viewed as a signal of quality. In our model, the “quality” of the intermediation services depends on the mass of users registering and so, a low price could be perceived as a bad signal triggering a reduction in demand. But this effect is conceivable only if intermediaries have a better information on demand than consumers when they set prices, which is not the case in our model. We conjecture that a more detailed dynamic process would deliver the monotonicity restriction as a more natural property of equilibrium.
users corresponds to a system of demand functions for each matchmaker. Once demand is characterized, the first stage amounts to a classical price setting game.

It is convenient to interpret this equilibrium concept as a rational expectation equilibrium where, following the choice of a price system $P$, each infinitesimal user has expectations about how all other users will allocate among the different matchmakers; in equilibrium expectations are common and fulfilled. We shall use this interpretation repeatedly.

### 2.3 Competition for exclusive services

As is well-known, network externalities induce concentration. When users can register with at most one intermediary and $\lambda > c$, an efficient distribution of users requires all users to register with the same intermediary. We show below that, given the set of pricing instruments, all equilibria are efficient; that is, they all involve only one active matchmaker, say $I$, on the equilibrium path. Such equilibria are called *dominant-firm equilibria*.

A dominant-firm equilibrium price system $(P^I, P^E)$, if it exists, can always be sustained by a “bad-expectation” (or pessimistic) market allocation against $E$, that is, by a market allocation such that, after any price deviation by $E$, users coordinate on an equilibrium distribution with zero market share for $E$, whenever possible.\(^{20}\) So, a dominant-firm equilibrium must be such that no pricing strategy allows $E$ to earn a positive profit, when users have pessimistic beliefs against $E$.

Given $P = (p^I_1, p^I_2, t^I, p^E_1, p^E_2, t^E)$, there exists a bad-expectation distribution of users against $E$, with $n^E_i(P) = 0$ and $n^I_i(P) = 1$, as long as:

$$\lambda u_i(1 - t^I) - p^I_i \geq -p^E_i, \ i = 1, 2. \tag{2}$$

Under (2), users have no incentives to register with $E$ since they expect all others to register with $I$. To get a positive market share despite pessimistic beliefs, $E$ must adopt a *divide-&-conquer* strategy (hereafter DC-strategy). First, $E$ must subsidize one group, say divide $i$-users:

$$p^E_i < p^I_i - \lambda u_i(1 - t^I) \leq 0. \tag{3}$$

The distribution of users must then be such that: $n^E_i = 1$. Second, $E$ extracts part of the

\(^{20}\)More generally, on a distribution that yields non-positive profits for $E$ (see footnote 25).
ensuing externality benefits on the other group; it *conquers* \( j \)-users, with:

\[
p_j^E + \lambda u_j t^E < \lambda u_j + \inf \{ p_j^I, 0 \} ,
\]

since \( j \)-users rationally expect all \( i \)-users to register with \( E \). Note that the revenue from the transaction fee on \( i \)-users, \( \lambda u_j t^E \), does not appear, so that it is optimal for \( E \) to set the transaction fee at its maximal level \( t^E = 1 \).

To deny \( E \) an active participation in the market, \( I \)'s pricing strategy must thus be designed so that no such DC-strategy for \( E \) is profitable. The next proposition follows straightforwardly.\(^{21}\)

**Proposition 1**: With exclusive intermediation services, the only equilibria are dominant-firm equilibria, where one intermediary \( I \) captures all users, charges the maximal transaction fee (\( t^I = 1 \)), subsidizes registration and makes zero profit (\( p^I_1 + p^I_2 = c - \lambda )\).

The efficiency property of all equilibria is due to a tension between monotonicity and the nature of DC-strategies. With high transaction fees, intermediaries have an incentive to attract more customers by undercutting slightly the registration fees, so as to raise the number of transactions. On the other hand, low transaction fees imply relatively high registration fees, which raises the profitability of DC strategies.

The intuition for the zero-profit property runs as follows. In a dominant-firm equilibrium, the inactive matchmaker could deviate and offer to pay all users, through registration subsidies, slightly more than their expected surplus with the active matchmaker. This deviation attracts all users, independently of their beliefs, and generates maximal aggregate surplus \( \lambda - c \). Then, a maximal transaction fee enables the deviating matchmaker to capture this maximal aggregate surplus minus the users’ surplus in the candidate equilibrium. In equilibrium, such a deviation cannot be profitable. Hence consumers must receive the total surplus and the dominant firm cannot make a strictly positive profit. Finally, the transaction fee is maximal as it is the dominant firm’s best interest to design registration fees that are the most attractive for its customers even when they hold pessimistic beliefs against this matchmaker.\(^{21}\)

\(^{21}\)We refer the reader to Caillaud-Jullien [2001a, 2001b] for a detailed discussion of DC-strategies and dominant firm equilibria in the case of exclusive services. The present paper only takes this proposition as a benchmark result.
In a model with sequential entry, Proposition 1 characterizes the highest-profit, entry-deterrence equilibrium. In the absence of any fixed cost of entry, users’ beliefs constitute the key factor that determines entry barriers. The incumbent monopolizes the market but has to abandon all profits in order to deter entry: the market is highly “contestable”.

3 Multi-homing

As argued in the introduction, intermediation services, in particular Internet-based services, are usually not exclusive. Moreover, even when it applies, exclusivity often results from a choice by intermediation providers based on their evaluation of competition with non-exclusive services. This section therefore assumes that users can use the services of both matchmakers simultaneously; they can engage in “multi-homing”.

We assume that the matching processes performed by the two matchmakers are independent. So, when $j$-users engage in multi-homing, a $i$-user may have two motives to do so instead of registering with $I$ only. First, it increases the probability of a match by $(1 - \lambda)\lambda$, that is by the probability that $E$ performs the match while $I$ doesn’t; and second, in case of a double match, that is with probability $\lambda^2$, the $i$-user can save on transaction fees since he can conclude the transaction via the intermediary that imposes the lowest transaction fee and pay only $u_i \min\{t^I, t^E\}$. Note that the first effect corresponds to a net efficiency gain for the economy as a whole, while the second effect has no impact on efficiency.\(^{22}\)

As suggested above, there can be two types of efficient allocations, depending on whether, once all agents have register with one intermediary, it is efficient or not that they also register with the other. A market allocation is now defined as $N = \{n^I_i, n^E_i, n^M_i\}$, where $n^k_i$ is the mass of $i$-users registering with $k$ only (single-homing) and $n^M_i$ is the mass of users registering with both $I$ and $E$ (multi-homing). When $\lambda(1 - \lambda) < c$, efficiency requires single-homing ($n^I_i = 1$ for all $i$); but when $\lambda(1 - \lambda) > c$, global multi-homing ($n^M_i = 1$ for all $i$) is efficient.

We start by the critical analysis of $E$’s best response to prices $P^I$ under pessimistic beliefs. Then, we study the existence and the properties of equilibria, gathered in two classes.\(^{23}\)

\(^{22}\)If the probability of success was correlated across matchmakers, the benefit in terms of the total probability of a match would be smaller, and multi-homing would be less of an attractive option in terms of efficiency; on the other hand, double matches would be more frequent implying tougher price competition in transaction fees. The nature of the analysis, however, would be similar.

\(^{23}\)We adjust the equilibrium concept for the fact that multi-homing is possible and we maintain the
The first class consists of “pure equilibria”, where all agents of one type make the same deterministic choice: they all register with $I$ only, or with both $I$ and $E$. The second class consists of “mixed equilibria”, where some ex-ante identical agents end up making different choices ex post.\textsuperscript{24}

### 3.1 Best-response analysis

We first analyze $E$’s best response to $P_I$ under pessimistic beliefs.

Let $r^k_i \equiv p^k_i + \lambda u_i t^k$ denote the maximum revenue that can be extracted by $k$ on $i$-users. Expecting all other users to register with $I$, a given $i$-user prefers to register with $I$ instead of $E$ if (2) holds; moreover, he prefers to register with $I$ only, instead of registering with both $I$ and $E$, whenever $p^E_i \geq 0$ since multi-homing then only involves this additional registration charge. A profitable entry strategy for $E$ must be a DC-strategy, where a group of $i$-users enjoys registration subsidies: $p^E_i < 0$. The difference with the case of exclusivity is that here, any negative price $p^E_i < 0$ induces $i$-users to register with $E$ as a second home, to cash in the subsidy, while still maintaining their registration with $I$ if they expect $j$-users, for $j \neq i$, to register with $I$.

Even with $p^E_i < 0$, bad expectations may still prevent $E$ from making a positive profit.\textsuperscript{25} This occurs if $j$-users still register with $I$ and not with $E$, while $i$-users engage in multi-homing. Then, $E$ cannot earn revenues from $j$-users’ registrations and does not process any transaction. Given $(P_I, P^E)$, $n_j^I = n_j^M = 1$ is an equilibrium distribution of users if:

\begin{align}
    r_j^E &\geq r_j^I, \\
    r_j^E &\geq \lambda(1 - \lambda)u_j + \lambda^2 u_j \max\{t^I, t^E\}.
\end{align}

By (5), $j$-users prefer $I$ to $E$ since they are charged lower total expected fees; by (6), they do not themselves engage in multi-homing since the additional expected charge is larger than the sum of the benefits from multi-homing. Dividing $i$-users is almost costless, but $E$’s surplus

\textsuperscript{24}Our model is formally equivalent to a game with one agent of each type (and 2 intermediaries), choosing within a set of 4 pure strategies: register with $I$, register with $E$, register with both, or register with none. The labels pure and mixed correspond to pure strategy equilibria and mixed strategy equilibria in such a game.

\textsuperscript{25}According to the broad concept of bad-expectation market allocation mentioned in footnote 20, we pick a distribution that yields the lowest profits for $E$. 

11
from conquering $j$-users is limited by

$$r_j^E < \max \{ r_j^I; \lambda(1 - \lambda)u_j + \lambda^2u_j \max\{t^I, t^E\} \}. \quad (7)$$

$p_i^E < 0$ and (7) induce all users to register with $E$. Whether or not they also register with $I$ determines the profitability of $E$’s pricing strategy. Hence, three possible DC-strategies for $E$:

- **$E$ as a second-source**: $E$ charges $t^E \geq t^I$, users engage in multi-homing and conclude the transaction via $I$, in case of a double match. $E$ only processes the transaction when the match has failed at $I$.\(^{26}\)

- **$E$ as a first-source**: $E$ charges $t^E < t^I$, users engage in multi-homing and $E$ processes the transaction whenever it performs the match.

- **$E$ as a sole-source**: all matches take place through $E$, since at least one population of users does not register with $I$.

Note first that the profit as a second source is bounded from above by $\lambda(1 - \lambda) - c$, the total additional surplus generated by multi-homing. When multi-homing is not efficient, a second-source strategy cannot be profitable. When multi-homing is efficient, slightly negative registration fees and a maximal transaction fee allows $E$ to earn a profit $\pi^{SS}$ (almost) equal to this upper bound.

In the alternative strategies, $E$ processes all transactions after a successful match. Intuitively, being a first-source should be chosen whenever possible, as this is less demanding than acting as a sole source. Under $p_i^E < 0$ and (7), a first-source strategy is feasible if and only if there exists $t^E < t^I$ such that all users engage in multi-homing. Then, multi-homing is a market allocation if no user of type $h$ prefers registering with $E$ only: for all $h$,

$$r_h^I \leq \lambda(1 - \lambda)u_h + \lambda^2u_h \max\{t^I, t^E\}. \quad (7)$$

Let us define below a measure $z^I$ of the minimal surplus for any user of using $I$ as a second source: formally,

$$z^I = \min_h \{ \frac{\lambda(1 - \lambda)u_h + \lambda^2u_h t^I - r_h^I}{\lambda^2u_h} \}. \quad (7)$$

\(^{26}\)If $t^I = t^E$, we say that both intermediaries are a second-source, as they would be treated in the same way by customers considering multi-homing.
Users engage in multi-homing if and only if \( \max\{t^I, t^E\} \geq t^I - z^I \). \( E \) being a first-source requires that \( t^E \leq t^I \) and consequently is possible only if \( z^I \geq 0 \). If \( z^I < 0 \), the only alternative to being a second source for \( E \) is to act as a sole-source; then, \( E \) can charge a transaction fee \( t^E \) as high as \( t^I - z^I \).

The next proposition characterizes \( E \)'s best response among all three DC-strategies.

**Proposition 2**: Under pessimistic beliefs, \( E \)'s best response to prices \( P^I \) (if \( E \) sells) is one out of two strategies:

- If \( z^I \geq 0 \), \( E \) adopts a first-source strategy with \( t^E \rightarrow t^I \) or a second-source strategy;
- If \( z^I < 0 \), \( E \) adopts a sole-source strategy with \( t^E \rightarrow t^I - z^I \) or a second-source strategy;

The profit as a sole or first-source is \( \pi^F = \lambda(1 - \lambda)u_2 + \lambda(u_1 + \lambda u_2)t^E - c \).

Note that \( \pi^F > \pi^{SS} \) if and only if \( t^E > \frac{(1 - \lambda)u_1}{u_1 + \lambda u_2} \). Thus the best-response is determined by the highest transaction fee among \( t^I, t^I - z^I \) and \( \frac{(1 - \lambda)u_1}{u_1 + \lambda u_2} \), if \( t^I \) or \( t^I - z^I \) is the highest, \( t^E \) is set equal to this level, and in the last case, \( t^E \) is maximal. \( E \) can improve on being a second source if \( I \)'s transaction fee is high. Whether it will do so as a sole source or a first source depends on the level of \( I \)'s registration fees (through the term \( z^I \)).

### 3.2 Pure equilibria

Pure equilibria are such that users of the same population all make the same choice. They correspond either to equilibria that involve global multi-homing (\( n^M_1 = n^M_2 = 1 \)) or to dominant-firm equilibria (\( n^I_1 = n^I_2 = 1 \)). If an efficient equilibrium exists, it must necessarily be a pure equilibrium. Conversely, the intuition provided in the previous subsection suggests that if there exists a pure equilibrium, it must necessarily be efficient.

**Proposition 3**: The market allocation of a pure equilibrium is efficient.

**Proof.** If \( \lambda(1 - \lambda) > c \), a dominant-firm equilibrium cannot exist since the inactive firm can profitably use a second-source strategy and make a profit (almost) equal to \( \lambda(1 - \lambda) - c > 0 \) with a small registration subsidy to all users and a maximal transaction fee. If \( \lambda(1 - \lambda) < c \), a
global multi-homing equilibrium cannot exist since at least one firm would be second-source and would make losses.

For a given set of parameters, there can only be one type of pure equilibrium. As we shall see below, this uniqueness and efficiency property is a consequence of the possibility of charging transaction fees. In the rest of this subsection, we prove that efficient equilibria do actually exist and that they may involve positive profits for the active firms.

We first focus on global multi-homing equilibria when \(\lambda(1 - \lambda) > c\). A firm can secure a profit at least equal to \(\lambda(1-\lambda)-c\) by relying only on its transaction fee (with small registration subsidies). Existence should therefore not be an issue. This also suggests that equilibrium profits should be equal to the marginal contribution of each firm to total surplus, that is to \(\lambda(1 - \lambda) - c\). This intuition turns out to be wrong; it would be valid only if in equilibrium \(t_I = t^E (= 0 \text{ by monotonicity w.r.t. transaction fees})\). But if \(t_I < t^E\), registering with \(I\) in addition to \(E\) allows one user to reduce his transaction payment by an expected amount \(\lambda^2(t^E - t_I)\) compared to the option of single-homing with \(E\). This means that \(I\) contributes to the users’ surplus by more than \(\lambda(1 - \lambda) - c\). This translates into higher equilibrium profits.

**Proposition 4:** A global multi-homing equilibrium exists if and only if \(\lambda(1 - \lambda) > c\); the highest-profit equilibrium is characterized by \(t_I < t^E\) and profits \(\pi_I\) and \(\pi^E\) such that:

\[
\pi_I = \lambda(1 - \lambda) + \frac{\lambda^2(1 - \lambda)u_1}{\lambda u_2 + u_1} - c > \pi^E = \lambda(1 - \lambda) - c.
\]

The maximal-profit, global multi-homing equilibrium is not symmetric: matchmakers play different roles. Firm \(I\) sets a low transaction fee and acts as a first source of intermediation, that is as the provider through which transactions are implemented whenever possible, while \(E\) sets a high transaction fee and acts as a second source, concluding transactions between trading partners who have not been matched elsewhere. Overall \(E\) is cheaper in terms of registration fees for both categories of users, but once registered with \(E\), all users are still willing to register with \(I\) because this allows them to save on the transaction fee if they are matched. The equilibrium configuration exhibits endogenous differentiation between the matchmakers.

\(^{27}\)We will not consider the limit case \(\lambda(1 - \lambda) = c\).
Assuming \( \lambda(1 - \lambda) < c \), let us now study whether dominant-firm equilibria exist, with \( I \) as the dominant firm and \( E \) the potential entrant. The next proposition proves existence and characterizes the level of profit that can be sustained in a dominant-firm equilibrium.

**Proposition 5**: A dominant-firm equilibrium exists if and only if \( \lambda(1 - \lambda) \leq c \). The highest equilibrium profit \( \pi^{DI} \) is such that:

\[
\pi^{DI} = \frac{(\lambda - c)}{u_1 + \lambda u_2}(1 - \lambda)u_1 \leq c.
\]

Any profit that can be attained in a dominant-firm equilibrium can be supported by strategies with a zero transaction fee, \( t^I = 0 \): matchmaker \( I \) does not have to impose a transaction fee to make a profit, registration fees are sufficient. This contrasts with the results under exclusivity. Indeed, under exclusivity, the dominant firm protects its market share by making it costly for \( E \) to divide. With multi-homing, \( E \) can easily divide through small registration subsidies; so, \( I \) must reduce the benefits for \( E \) of conquering. This is best achieved by setting a low transaction fee which ensures that \( E \)'s services are used only as a second source in case of multi-homing.

With \( t^I = 0 \), entry with multi-homing cannot be profitable for \( E \) (because \( \lambda(1 - \lambda) \leq c \)). The dominant firm only has to prevent entry of \( E \) as a sole or first source. This is easier than in the case of exclusivity: \( I \) just has to set its prices so that global multi-homing prevails whenever entry occurs. The highest attainable profit is then strictly positive.

### 3.3 Mixed equilibria

With non-exclusive services, mixed equilibria can emerge where users of the same type make different choices. These equilibria must however involve “some multi-homing”, in a sense made precise by the next proposition.

**Proposition 6**: When intermediation services are not exclusive, there does not exist equilibria with two active firms \( (n^I > 0 \text{ and } n^E > 0) \) and no multi-homing \( (n^M_1 = n^M_2 = 0) \) if \( c \neq \frac{\lambda}{2} \) (that is, generically).

On the equilibrium path, the distribution of users must involve multi-homing by at least one group, say \( i \)-users.\(^{28}\) Users of the other group (\( j \)-users) are single-homing users who

\(^{28}\)More precisely, this group of \( i \)-users engage in some multi-homing; in limit cases, equilibria can be such that \( i \)-users are indifferent between registering with \( I \) only, or with \( E \) only, or with both. We omit the analysis.
register with only the least costly intermediary (by monotonicity). In equilibrium, matchmakers charge single-homing users identical total prices: 
\[ p^I_j + \lambda u_j t^I = p^E_j + \lambda u_j t^E \]
Moreover, monotonicity implies that reducing \( p^k_j \) so as to attract more single-homing users and to generate more transactions is not profitable, which amounts to:
\[ p^k_j + \lambda t^k \leq c_j \] 
(8)
Hence matchmakers make losses (or zero profit) on single-homers. But, for a given quality of the matching process, the additional benefit of registering with an additional matchmaker must be smaller, for \( j \)-users, than the corresponding additional price; otherwise, \( j \)-users would rather engage in multi-homing. It follows that if costs are trivial, no market-sharing equilibrium (that is, with \( n^M_i = 1, n^I_j > 0 \) and \( n^E_j > 0 \)) exists.

**Proposition 7**: For a fixed \( \lambda < 1 \), a market-sharing equilibrium does not exist when the costs are close to zero.

Note however that, when the matching process is almost perfect, the additional benefit of multi-homing is small and, for given costs, a market-sharing equilibrium may exist, as shown in Proposition 8 below.

Monotonicity has no bite with respect to registration fees charged on multi-homing users. Therefore, a high registration fee \( p^k_i \) for multi-homing users and potentially high profits can be supported in equilibrium. The following proposition takes into account all other possible deviations.

**Proposition 8**: Fix all parameters except \( \lambda \) and assume that \( \frac{c_i}{u_i} \leq \frac{c_j}{u_j} \). There exists a market sharing equilibrium for \( \lambda \) close to 1 if and only if \( 1 - c > \frac{c_i}{u_i} \). Under this sufficient condition, the maximal aggregate profit is attained in a symmetric market-sharing equilibrium with \( n^M_i = 1, n^I_j = n^E_j = \frac{1}{2}, t^I = t^E = 0 \), and it is approximately equal to:
\[ \inf \left\{ \frac{u_i}{u_j} c_j - c_i, \frac{u_i}{1 + u_i} \left( 1 - \frac{c_i}{u_i} - c \right) \right\} \]

In equilibrium, matchmakers’ equilibrium profits and users’ equilibrium surplus only depend upon the total prices charged on users. Positive transaction fees however leave room of these cases.
for potentially profitable deviations, e.g. first-source deviations for users who want to save on transaction fees. Therefore, in equilibrium, matchmakers extract the multi-homing users’ surplus through registration fees, and the transaction fees can be set equal to zero.

To provide a better intuition for Proposition 8, set \( \lambda = 1 \) and focus on symmetric equilibria with zero transaction fees. Monotonicity implies: \( p_j \leq c_j \). The multi-homing users’ matching surplus is equal to \( u_i \), which matchmakers could jointly extract with \( p_i = \frac{u_i}{2} \). With this price structure, second-source deviations fail since all matches are performed by both matchmakers and trade is concluded at the lowest transaction fee. There is no scope for first-source deviations either. Setting \( p_i^E = p_j^E \leq 0 \), however, enables a deviating firm to attract both sides of the market and, setting \( t^E < \max\{\frac{p_i}{u_i}, \frac{p_j}{u_j}\} \), \( E \) becomes a sole source; for, in this case, even if all users anticipate that their matching partner registers with \( I \), at least one group has an incentive to register only with \( E \), where their expected surplus is larger. This sole-source strategy is profitable against \( p_i = \frac{u_i}{2} \) and \( p_j = c_j \). To prevent such a deviation in equilibrium, the matchmakers must leave more surplus to the group of users with the largest price-to-utility ratio and therefore, depending on the parameters, either extract only part of the multi-homing users with \( p_i < \frac{u_i}{2} \), or price access below marginal cost for single-homing users (\( p_j < c_j \)).

Note that for \( \lambda = 1 \), the maximal profit in a dominant-firm equilibrium is 0. So, when the value of the intermediation services is large, an intermediary prefers to share the market in a market-sharing equilibrium rather than being either the dominant firm or the entrant in a dominant-firm equilibrium.

4 General discussion and extensions

This section discusses the implications of the previous results.

**Efficiency.** When matchmakers provide exclusive intermediation services, competition yields an equilibrium with an efficient market structure that involves monopolization. When all services are non-exclusive, an efficient equilibrium always exists; but there may also exist inefficient equilibria where matchmakers induce multi-homing by some users.\(^{29}\)

**Intermediation profits.** Under exclusive services, the market is highly contestable with low

\(^{29}\)In this form, these conclusions apply to the case of \( K > 2 \) matchmakers (proof available upon request).
(vanishing) profits. Non-exclusivity, however, induces less severe a degree of competition and allows positive profits in any type of equilibrium.\(^{30}\) When multi-homing is efficient, each matchmaker appropriates at least the marginal social benefit of allowing multiple registration. When single-homing is efficient, the dominant matchmaker’s profit is bounded from above by the total marginal cost of intermediation. Inefficient equilibria yield larger profits than those in a dominant-firm equilibrium when the matching process is very efficient.

*Consumers welfare.* The consumers’ welfare under exclusivity equals \(\lambda - c\). It can easily be seen that the consumers’ welfare is higher under exclusive services than in any equilibrium with non-exclusive services.\(^{31}\) Thus, from the total consumers’ welfare perspective, exclusivity is the best alternative even though it results in lower efficiency.

*Exclusivity choice and entry.* Suppose we consider a preliminary stage where matchmakers could freely and non-cooperatively choose whether to let users who register with them also register with their opponent. Exclusivity can be imposed unilaterally. In our model, exclusivity exacerbates competition between intermediation service providers and forces profits down to zero, while non-exclusivity allows a whole range of strictly profitable equilibria. So, in equilibrium, matchmakers would choose to allow for multiple registration.

When firms can choose to be exclusive, a more interesting question is: to which extent established firms will use exclusivity to deter entry? To discuss this issue, let us consider a situation with two periods. In the first period, firm \(I\) with quality \(\lambda^I\) enters and commits through irreversible technological choices to be exclusive or not. In the second period, a potential entrant appears, with quality \(\lambda^E\) drawn randomly from a common knowledge distribution. The entrant decides to enter or not, and whether to be exclusive or not in case of entry. Then firms make pricing decisions, given their exclusivity choices and their respective quality parameters. For simplicity, let us assume both firms incur a cost \(c\). The questions are then: will \(I\) choose to be exclusive? when does \(E\) enter and how?

For the sake of brevity of the discussion, we make restrictive assumptions that could be relaxed: we assume that only pure equilibria can emerge and that, in a global multi-homing equilibrium, \(E\) is second-source (as a new comer). \(E\) enters only if its profit is positive.

\(^{30}\)These conclusions also extend to the case of \(K\) identical intermediaries.

\(^{31}\)In global multi-homing equilibria, the total profit is at least \(2\lambda(1 - \lambda) - 2c\), so that the total consumers’ welfare is at most \(\lambda^2\), which is smaller than \(\lambda - c\) under the assumption that: \(\lambda(1 - \lambda) > c\).
• Under exclusivity, firm $E$ will be active if and only if $\lambda^E > \lambda^I$, capturing the whole market with equilibrium profit $\lambda^E - \lambda^I$ (assuming that $I$ doesn’t play a weakly dominated strategy). In particular, if $I$ chooses to be exclusive, entry occurs only when $\lambda^E > \lambda^I$, but $I$ loses the market in this case.

• Suppose now that $I$ chooses to be non-exclusive. This reduces barriers to entry and entails a cost for $I$, as entry now may occur for $\lambda^E < \lambda^I$. But at the same time, when $\lambda^E > \lambda^I$, $I$ may remain active if $E$ chooses non-exclusivity and if a global multi-homing equilibrium prevails.

Let $\Pi_{E}^{ss} = \lambda^I(1 - \lambda^E) - c$ and $\Pi_{E}^{ss} = \lambda^E(1 - \lambda^I) - c$ denote the second-source profits of $I$ and $E$ respectively. For $\lambda^E > \lambda^I$, $E$ prefers non-exclusivity with a global multi-homing equilibrium than exclusivity whenever $\Pi_{E}^{ss} > \lambda^E - \lambda^I$, which is equivalent to $\Pi_{I}^{ss} > 0$. Moreover when $\Pi_{I}^{ss} > 0$ and $\Pi_{E}^{ss} > 0$, the unique equilibrium under non-exclusivity precisely involves global multi-homing. This implies that, when $\lambda^E > \lambda^I$, a global multi-homing equilibrium with non-exclusive services emerges if $\Pi_{I}^{ss} > 0$, while $E$ enters and becomes the only active matchmaker otherwise.

Assume first that $c > \lambda^I(1 - \lambda^I)$. Then $\Pi_{E}^{ss} > 0$ is incompatible with $\lambda^E > \lambda^I$ and so $I$ loses the market whenever $\lambda^E > \lambda^I$. So, $I$ would rather concentrate on the case where $\lambda^E < \lambda^I$ and choose to be exclusive.

Now assume $\lambda^I(1 - \lambda^I) > c$. Then $E$ will enter as a second source when $\inf\{\Pi_{I}^{ss}, \Pi_{E}^{ss}\} > 0$, which reduces to $\lambda^E \in \left(\frac{c}{1 - \lambda^I}, \frac{\lambda^I - c}{\lambda^I}\right)$, and as a sole source if $\lambda^E > \frac{\lambda^I - c}{\lambda^I}$. Choosing non-

\[ p^I_{1} + p^E_{2} = c - \lambda^I \]
and the entrant’s profit is $\lambda^E - \lambda^I$.

The analysis of pure equilibria under exclusivity with different quality parameters is omitted as it follows similar steps as the one in the previous section.

Whether $E$ chooses to be exclusive or non-exclusive to enter as a sole source may depend on the equilibrium selection under non-exclusivity.

$E$ cannot enter in a global multi-homing equilibrium if $\lambda^E < \lambda^I$ since $\Pi_{E}^{ss} < 0$. But there is the possibility that under non-exclusivity $E$ becomes a sole source if $\lambda^E$ is smaller but close to $\lambda^I$. This follows from the fact that profits are positive in proposition 5.

If $\lambda^E < \lambda^I$, $\Pi_{E}^{ss} > 0$ so that $E$ cannot be a sole source, implying that $E$ enters if $\Pi_{E}^{ss} = \inf\{\Pi_{I}^{ss}, \Pi_{E}^{ss}\} > 0$; if $\lambda^E > \lambda^I$, $E$ chooses global multi-homing whenever $\Pi_{I}^{ss} = \inf\{\Pi_{I}^{ss}, \Pi_{E}^{ss}\} > 0$.

\[ p^I_{1} + p^E_{2} = c - \lambda^I \]
exclusivity over exclusivity yields for $I$ a net minimal gain equal to:

$$\Pr \left\{ \lambda^I < \lambda^E < \frac{\lambda^I - c}{\lambda^I} \right\} E \left\{ \Pi^{ss}_I | \lambda^I < \lambda^E < \frac{\lambda^I - c}{\lambda^I} \right\} - \Pr \left\{ \frac{c}{1 - \lambda^I} < \lambda^E \leq \lambda^I \right\} E \left\{ \Pi^M - \Pi^{ss}_I | \frac{c}{1 - \lambda^I} < \lambda^E \leq \lambda^I \right\}. $$

where $\Pi^M$ denotes the monopoly profit, equal to $\lambda^I - c$. Rearranging, the condition becomes:

$$\Pr \left\{ \lambda^E > \lambda^I | \frac{c}{1 - \lambda^I} < \lambda^E < \frac{\lambda^I - c}{\lambda^I} \right\} > E \left\{ \frac{\lambda^I \lambda^E}{\lambda^I - c} | \frac{c}{1 - \lambda^I} < \lambda^E < \frac{\lambda^I - c}{\lambda^I} \right\}. $$

The choice of non-exclusivity over exclusivity only depends upon the distribution of $\lambda^E$ conditional on the equilibrium under non-exclusivity being global multihoming. $I$ will choose to be non-exclusive if it is more likely to face a more efficient entrant in this range than a less or equally efficient one. For example, when $c = 0$, the condition becomes $\Pr \left\{ \lambda^E > \lambda^I \right\} > E \left\{ \lambda^E \right\}$, so that $I$ chooses to be non-exclusive if $\lambda^I$ is below some threshold.

To sum up, in our simple model of entry, the first mover will choose to enter with exclusive services when the quality of its matching technology is high enough; this enables him to deter entry most of the time and to monopolize the market, but it implies a risk of being driven out of the market if a very efficient entrant appears and captures all the market. When the quality of $I$’s matching is low, however, $I$ will propose non-exclusive services; entry will take place quite often but when the entrant’s quality is not too high, the first mover will still be active on the market. Of course, high quality entrants will still drive $I$ out by acting as sole sources.

The strategic use of transaction fees. The impact of transaction fees is quite contrasted between the situations with and without exclusivity. Under exclusive services, matchmakers use transaction fees as an additional instrument to extract profit and overcome consumers’ coordination failures. With non-exclusive services, transaction fees can still be used to capture efficiency gains generated by an aggressive registration policy but a crucial point is the possibility to propose a smaller transaction fee than the opponent’s (in first-source strategies). So, in dominant-firm equilibria or in market-sharing equilibria, matchmakers are forced to set zero transaction fees to limit the possibility of profitable deviations. In global multihoming equilibria, however, matchmakers endogenously differentiate, relying on all pricing
instruments: one sets a low transaction fee and acts as a first source, the second charges a high transaction fee and captures the benefit of acting as a second source.

5 Competition without transaction fees

When transactions do not give rise to physical or monetary exchanges, such as for pure informational intermediation or pure matching, or when they are difficult or costly to monitor, the possibility of using transaction fees is not a reasonable assumption. In this section, we investigate how our findings are modified under the restrictive assumption that transaction fees cannot be used. Since we have sketched some of this analysis elsewhere, we only provide the main results and intuition here.\footnote{All the proofs are in our working paper [2001b], where transaction fees are restricted to $t \in [0, T]$ for a given $T$.}

When transaction fees are not feasible, a deviating intermediary has fewer instruments to generate efficiency gains and to capture them. Therefore, larger profit levels can be sustained in equilibrium and other types of equilibrium may emerge. With exclusive services and no transaction fees, Proposition 1 is modified as follows:

1. There may exist inefficient equilibria, where both matchmakers are active and the market is segmented; these equilibria are symmetric and equilibrium profits are null.\footnote{However, the market allocation would be unstable (at fixed prices).}

2. There exist dominant-firm equilibria with positive maximal profits for the dominant given by: $\lambda \inf \{u_1, u_2 - u_1\}$.

With non-exclusive services, the analysis of best responses is somewhat simpler, since only sole-source strategies matter. Moreover, there is no scope for endogenous differentiation. From the proof of Proposition 4, a firm cannot obtain more than $\lambda(1 - \lambda) - c$ in a global multi-homing equilibrium. Whenever this is positive, prices such that $p^E_i = \lambda(1 - \lambda)u_i$ are indeed equilibrium prices. For any other $P^E$, users register with $I$ (except when monotonicity has some bite). Thus $E$ cannot obtain more than the additional surplus $\lambda(1 - \lambda) - c$. Proposition 4 becomes:
Proposition 9: Suppose transaction fees are not feasible, a global multi-homing equilibrium exists if and only if multi-homing is efficient. The equilibrium with maximal profits is symmetric; profits are equal to $\lambda(1 - \lambda) - c$.

As for dominant-firm equilibria, we saw in the discussion of Proposition 5 that transaction fees are not needed for the dominant matchmaker; but they constitute an instrument for entry. So, when transaction fees are not available, the entrant has fewer instruments. Either it subsidizes one group of users and undercuts the registration fee for the other group, so as to become a sole source. Its deviation profit is then equal to: $\max\{p'_1, p'_2\} - c$, and if the dominant matchmaker sets prices equal to the total marginal cost $c$, no such deviation is profitable. Or, it acts as a second source, subsidizes one group of users and charges the other group (say, group $h$) at most the expected benefit from multi-homing, $\lambda(1 - \lambda)u_h$. When $c \geq \lambda(1 - \lambda)u_2$, this strategy is not profitable either. This leads us to modify Proposition 5 as follows:

Proposition 10: Suppose transaction fees are not feasible, a dominant-firm equilibrium exists if and only if $c \geq \lambda(1 - \lambda)u_2$. The highest attainable profit for the dominant firm is equal to $c$.

Note that the argument in Proposition 3 cannot be replicated in the absence of transaction fees. Indeed, the previous result shows that pure equilibria are not necessarily efficient since, for $u_2 \leq \frac{c}{\lambda(1 - \lambda)} \leq 1$, a dominant-firm equilibrium exists although multi-homing is efficient.

Finally, we sketch the analysis of market-sharing equilibria for the case where $\lambda = 1$. In this case, the same steps as in Proposition 8 show that the highest-profit market-sharing equilibrium is symmetric and involves $p_j \leq c_j$, $p_i \leq \frac{u_i}{2}$, $n_i^M = 1$ and $n_j^E = \frac{1}{2}$. Now, consider indeed the candidate equilibrium: $p_i = \frac{u_i}{2}$ and $p_j = c_j$. A deviating matchmaker might consider undercutting $p_i$; but the same distribution of users can prevail, making this deviation unprofitable. By $p_j \leq c_j$, undercutting $p_j$ cannot be profitable either. Other deviations involve a subsidy to one group of users, say $h$-users, and for the other group of users (7) becomes: $p_{E-h}^h < p_{-h}$. These deviations are therefore undercutting deviations themselves, hence non-profitable. Maximal profits of $\frac{u_i}{2} - c_i$ can then be sustained provided $u_i \geq 2c_i$. 

22
Proposition 11: Suppose transaction fees are not feasible, a market-sharing equilibrium exists for $\lambda = 1$ if and only if $\inf \{ \frac{c_1}{u_1}, \frac{c_2}{u_2} \} \leq \frac{1}{2}$. The highest attainable profit in a market-sharing equilibrium is equal to $\max_h \{ \frac{u_h}{c_h} \}$. Consequently, the first conclusion drawn for exclusive services also applies for non-exclusive services. Namely, when transaction fees are not available, inefficient market configurations can emerge in equilibrium: dominant-firm equilibria may be supported even though they are inefficient, and market-sharing equilibria exist for a wider range of parameters, when $\lambda$ is close to 1. The second conclusion does not extend, though. Equilibrium profits in a dominant-firm equilibrium or a market-sharing equilibria are indeed larger when transaction fees are not feasible, but equilibrium profits in global multi-homing equilibria are smaller. The intuition has already been alluded to in the previous section. In dominant-firm equilibria or market-sharing equilibria, transaction fees are only an additional instrument for deviations; ruling them out can only improve equilibrium profits. In global multi-homing equilibria, they play a central role in extracting users’ surplus and ruling them out puts limits on attainable profits.

6 Conclusion

This paper has proposed a framework to analyze imperfect competition between matchmakers with indirect network externalities, with a particular emphasis on relevant features of the intermediation activity on the Internet. Intermediation services usually are not exclusive and users often heavily rely on the services of several intermediation providers. Users visit many intermediation websites, firms advertise on many marketplaces,...

As should be expected, multiple equilibria exist. Under the assumption that any generated matching surplus is efficiently shared, we prove that, depending upon the imperfection and cost of the matching technology, the efficient market structure may be monopolistic or duopolistic, and that an equilibrium with the efficient market structure always exists. But inefficient equilibria also exist, especially when the matching technology is effective or the ability to rely on transaction fees is limited. The intermediation market is moreover partially contestable: depending upon the pricing instruments and the exclusivity of services, concentrated market structures may go along with limited or zero intermediation profits. In-
termediation providers still have an incentive to open up the intermediation market so as to allow users to turn to several intermediaries simultaneously: this moderates price competition and reinforces market power and intermediation profits.

We have also characterized relevant business strategies on the intermediation market. These are divide-and-conquer strategies, where one side of the market is subsidized and profits are made on the other side of the market. The possibility of such business strategies have strong consequences in terms of market equilibrium and market structures that are likely to emerge. Moreover, the use of transaction fees is shown to be central in these pricing and business strategies.

Intermediation markets, and particularly Internet-based markets, have therefore some strong specificities. The design of competition policy rules with respect to such markets should then take these characteristics into account. Concentration may not necessarily carry strong inefficiencies; in fact, the opposite may be true. Intermediation profits may be larger in market-sharing configurations and the users’ surplus may have better protection in concentrated markets where one large intermediary dominates provided that there is enough contestability.

These first conclusions must obviously be challenged by further research. In particular, the potential impact of intermediation pricing on the efficiency of trade between users must be investigated, using a model where the bargaining over the matching surplus may be affected by the matchmakers’ business strategies. Rochet-Tirole (2001) is one attempt in this direction.
References


Appendix

**Proof of Proposition 1.** The proof of the existence of dominant-firm equilibria and the characterization of pricing are similar to Caillaud-Jullien [2001a], hence omitted.

Suppose there exists an equilibrium with prices \( P = (P^I, P^E) \) and an inefficient distribution of users (two active firms). Let \( s_i = \lambda u_i(1 - t^k)n^k_j - p^k_i \) denote the ex ante surplus of \( i \)-users in this equilibrium. The profits are:

\[
\Pi^k = \lambda n^k_i n^k_2 - \sum_i (c_i + s_i)n^k_i \geq 0.
\]

Firm \( k \) could undercut slightly and serve the whole market (because \( n^k_i > 0 \)) with profit approximately equal to:

\[
\lambda - c - s_1 - s_2 - \sum_i \lambda u_i(1 - t^k)n^{-k}_j \leq \Pi^k.
\]

It is shown in our working paper (2001b) that these 4 inequalities are only consistent with
\( n^k_i = \frac{1}{2}, t^I = t^E = 0, p^E_i = p^E = p_1 \) and \( p_1 + p_2 = c \) (zero profit).

But with the DC strategy described in (3) and (4), a firm could obtain \( p_i + \inf\{p_j, 0\} + \lambda u_j - c \). Using \( p_i - c = -p_j \) and \( p_j \leq \frac{\lambda u_j}{2} \) (\( s_j \geq 0 \)), we see that the deviation profit is strictly positive, a contradiction with the zero profit result.

**Proof of Proposition 2.** We follow the steps of analysis provided in the text.

First, \( E \) can always choose to act as second-source with profit \( \pi^{SS} \) close to \( \lambda (1 - \lambda) - c \) by setting \( t^E = 1 \) and \( p^E_i = p^E = p_1 \) slightly negative. For this choice of prices, multi-homing is indeed an equilibrium distribution since users obtain \( \lambda u_i - r^I_i \geq 0 \) if they all register with \( I \) and \( E \), while they have only 0 if they register with \( E \) only. Then, \( E \) is indeed a second source.

Suppose that \( 0 \leq z^I \). In this case, there exists a market allocation where all users register with \( I \) for all \( P^E \). So, \( E \)'s alternative to second-sourcing is to set prices such that, for some \( i, t^E < t^I, p^E_i < 0 \) and

\[
r^E_j \leq \max\{r^I_j, \lambda u_j [1 - \lambda + \lambda t^I]\} = \lambda u_j [1 - \lambda + \lambda t^I],
\]

and act as a first source. \( E \)'s profits are then given by:

\[
p^E_i + p^E_j + \lambda t^E - c < \lambda t^E u_i + \lambda u_j [1 - \lambda + \lambda t^I] - c.
\]
Setting optimally $t^E$ as close as possible to $t^I$, with $p^E_i$ and $r^E_j$ as large as possible, yields maximal profits for $i = 1$ and $j = 2$ almost equal to:

$$
\pi^E = \lambda u_1 t^I + \lambda u_2 \left[1 - \lambda + \lambda t^I \right] - c.
$$

Suppose now that $z^I < 0$. Then $I$ cannot be a second source. $E$ may choose to act as a sole source. This occurs if $p^E_i < 0,$

$$
r^E_j < \max \{r^E_j, \lambda u_j \left[1 - \lambda + \lambda \max \{t^I, t^E\} \right] \}
$$

and one group of users does not register with $I$, that is:

$$
\begin{align*}
r^I_i &> \lambda u_i \left[1 - \lambda + \lambda \max \{t^I, t^E\} \right] \\
or \quad r^J_j &> \lambda u_j \left[1 - \lambda + \lambda \max \{t^I, t^E\} \right].
\end{align*}
$$

This last condition reduces to $t^E \leq t^I - z^I$ (conditions are simpler than (7) because $p^E_i < 0$ and $r^E_j < r^I_j$). $E$’s profits are given by:

$$
\pi^E = \lambda (1 - \lambda) u_i + \lambda^2 u_i t^E
$$

and one group of users does not register with $I$, that is:

$$
\begin{align*}
r^I_i &\geq \lambda (1 - \lambda) u_i + \lambda^2 u_i t^E \geq r^I_i \quad \text{for all } i, \\
or \quad r^J_j &\geq \lambda (1 - \lambda) u_i + \lambda^2 u_i t^E \geq r^J_j \quad \text{for all } i.
\end{align*}
$$

Proof of Proposition 4. Consider an asymmetric multi-homing equilibrium with $t^I < t^E$. Given prices satisfying $\lambda u_i \geq p^k_i + \lambda u_i t^k$, a global multi-homing equilibrium distribution requires:

$$
\lambda (1 - \lambda) u_i + \lambda^2 u_i t^E \geq r^I_i \quad \text{for all } i, \quad \text{(9)}
$$

$$
\lambda (1 - \lambda) u_i \geq r^E_i - \lambda^2 u_i t^E \quad \text{for all } i. \quad \text{(10)}
$$

We also know that $\pi^E = \lambda (1 - \lambda) - c$, which is only possible if $p^E_i + \lambda (1 - \lambda) u_i t^E = \lambda (1 - \lambda) u_i$, for $i = 1, 2$ (and thus $z^E = 0$).

First, it cannot be profitable for $I$ to undercut prices. $E$ could however undercut $I$ with a slightly lower transaction fee (still preserving multi-homing by monotonicity), thereby becoming a first source instead of a second source. Such a strategy changes the revenues
raised by transaction fees from $\lambda(1 - \lambda)t^E$ to $\lambda t^I$; it is not profitable if $t^I$ is small enough, that is:

$$t^I \leq (1 - \lambda)t^E. \quad (11)$$

For the other deviations, we apply Proposition 2. For $I$, using $z^E = 0$, we obtain:

$$\pi^I \geq \max \{ \lambda(1 - \lambda)u_2 + \lambda(u_1 + \lambda u_2)t^E, \lambda(1 - \lambda) \} - c.$$

For $E$, using $t^I - z^I = \max_i \left\{ \frac{r^I_i - \lambda(1 - \lambda)u_i}{\lambda^2 u_i} \right\}$, the conditions that deviations as a first source and as a sole source are not profitable reduce to:

$$t^I \leq \frac{(1 - \lambda)u_1}{u_1 + \lambda u_2} \quad (12)$$

$$p^I_i + \lambda u_i t^I \leq \left[ \frac{\lambda + u_1}{\lambda u_2 + u_1} \right] \lambda(1 - \lambda)u_i. \quad (13)$$

Now set any $t^E$ such that $\frac{u_1}{u_1 + \lambda u_2}(1 - \lambda)[u_1 + \lambda + \lambda u_2] \geq t^E \geq \frac{u_1}{u_1 + \lambda u_2}(1 - \lambda), \ t^I = 0$ and $p^I_i = \left[ \frac{\lambda + u_1}{\lambda u_2 + u_1} \right] \lambda(1 - \lambda)u_i$; this yields an equilibrium with maximal profits equal to:

$$\pi^I = \lambda(1 - \lambda)\left( \frac{\lambda + u_1}{\lambda u_2 + u_1} \right) - c = \lambda(1 - \lambda) + \frac{\lambda^2(1 - \lambda)u_1}{\lambda u_2 + u_1} - c.$$

**Proof of Proposition 5.** Note first that it is not necessary to look at undercutting strategies with the monotonicity restriction since $I$ could not possibly gain by undercutting while the users distribution following $E$’s undercutting is not restricted at all since $E$ has no market share in a dominant-firm equilibrium. Therefore, we only need to guarantee that $E$’s best response profits in Proposition 2 are non-positive.

The conditions that deviations as a first source and as a sole source are not profitable for $E$ reduce to:

$$\lambda(1 - \lambda)u_2 + \lambda(u_1 + \lambda u_2)t^I \leq c, \quad (14)$$

$$\lambda(1 - \lambda)u_2 + \lambda(u_1 + \lambda u_2)\max_i \left\{ \frac{r^I_i - \lambda(1 - \lambda)u_i}{\lambda^2 u_i} \right\} \leq c. \quad (15)$$

The last inequality can be written as:

$$r^I_i \leq \left( \frac{c + (1 - \lambda)u_1}{u_1 + \lambda u_2} \right) \lambda u_i.$$

Setting maximal $r^I_i$ in these constraints along with $t^I = 0$, yields maximal profit equal to:

$$\left( \frac{c + (1 - \lambda)u_1}{u_1 + \lambda u_2} \right) \lambda - c = \frac{(\lambda - c)}{u_1 + \lambda u_2}(1 - \lambda)u_1.$$
This profit is smaller than \( c \) under the assumption that \( c \geq \lambda(1 - \lambda) \).

\( E \)'s pricing strategy can then be given by \( P^E = P^I \). Note, first, that \( p^E_i = p^I_i > 0 \) so that \( E \) is indeed not active in equilibrium. Then, assume that users hold pessimistic beliefs against \( I \) when \( I \) attempts to deviate by increasing one price; \( I \) has no profitable deviation.

**Proof of Proposition 6.** Suppose there exists an equilibrium such that the equilibrium distribution of users satisfies: \( 0 < n_i^I = 1 - n_i^E < 1 \). The necessary conditions are:

\[
0 \leq \lambda n_i^I u_i(1 - t^I) - p_i^I = \lambda n_j^E u_i(1 - t^E) - p_i^E
\]

\[
\leq \lambda n_j^I u_i(1 - t^I) + \lambda n_j^E u_i(1 - t^E) - p_i^I - p_i^E.
\]

This implies that: \( \lambda n_j^I u_i(1 - t^k) = p_j^k \) for all \( i = 1, 2, j \neq i \) and \( k = I, E \). Intermediaries profits then become:

\[
\pi^k = \lambda n_i^1 n_2^k - c_1 n_1^k - c_2 n_2^k \geq 0.
\]

If a firm slightly reduces one of its prices, it captures the whole market in a monotonic market allocation. A necessary equilibrium condition is then:

\[
\lambda n_i^1 n_2^k - c_1 n_1^k - c_2 n_2^k \geq p_i^k + p_2^k + \lambda t^k - c
\]

\[
\implies \lambda\left(n_2^k u_1 + n_1^k u_2\right) (1 - t^k) + \lambda t^k - c.
\]

Using the fact that \( n_i^{1-k} = 1 - n_i^k \) and the non-negativity of profits, it follows:

\[
\lambda n_i^1 n_2^k \geq c_1 n_1^k + c_2 n_2^k \geq \lambda\left(n_2^{1-k} u_1 + n_1^{1-k} u_2\right) (1 - t^{1-k}) + \lambda t^{1-k} - \lambda n_1^{1-k} n_2^{1-k}
\]

(16)

for \( k = I, E \). Summing these double inequalities for \( k = I \) and \( E \) yields:

\[
2\lambda(n_i^1 n_2^E + n_i^E n_2^E) \geq \lambda + \lambda t^I (1 - n_i^1 u_1 - n_2^E u_2) + \lambda t^E (1 - n_i^E u_1 - n_2^E u_2) \geq \lambda.
\]

Since \( n_i^E = 1 - n_i^I \), the inequality between the extreme left-hand side and the extreme right-hand side is possible only for \( n_i^E = \frac{1}{2} \) and \( t^k = 0 \) for \( i = 1, 2 \) and \( k = I, E \). The double inequality (16) then yields \( \lambda = 2c \). ■

**Proof of Proposition 7 and 8.** Consider a candidate equilibrium \( (p_i^k, p_j^k, t^k) \), \( 0 < n_i^I = 1 - n_i^E < 1 \) and \( n_i^M = 1 \). On the equilibrium path, the market allocation satisfies:

\[
0 \leq \lambda u_i n_j^I (1 - t^k) - p_i^k
\]

\[
\lambda u_j (1 - \lambda + \lambda \max\{t^I, t^E\}) \leq p_j^I + \lambda u_j t^I = p_j^E + \lambda u_j t^E \leq \lambda u_j.
\]
The matchmakers’ profits are given by
\[
\pi^k = p_i^k - c_i + n_j^k(p_j^k + \lambda t^k - c_j).
\]

Price equilibrium conditions consists of (8), for undercutting deviations, and of the conditions given in Proposition 2, that is:
\[
\begin{align*}
\pi^k &\geq \lambda(1 - \lambda)u_2 + \lambda(u_1 + \lambda u_2)t^{-k} - c \\
\pi^k &\geq -(1 - \lambda)u_1 + (u_1 + \lambda u_2)\max_h\left\{\frac{p_{h^k} - \lambda u_h t^{-k}}{\lambda u_h}\right\} - c \\
\pi^k &\geq \lambda(1 - \lambda) - c
\end{align*}
\]

Consider a reduction in transaction fees \(t^k\) while maintaining constant \(p_i^k + \lambda u_i n_j^k t^k\) and \(p_j^k + \lambda u_j t^k\). This preserves profits and utility levels while relaxing all equilibrium conditions. It follows that we can look for an equilibrium with \(t^l = t^E = 0, p_i^l = p_j^E = p_j \leq c_j\). Then, taking the average of the conditions over the two firms, one can conclude that, if there exists a market-sharing equilibrium with \((p_i^k, p_j^k, n_j^k)\), then there also exists a symmetric market-sharing with \(p_i = \frac{p_i^l + p_j^E}{2}, n_j^k = \frac{1}{2}\), the same \(p_j\) and where total profits of intermediation are preserved. Hence, we can narrow our analysis to the search for the highest-profit, symmetric equilibrium, with zero transaction fees.

Matchmakers’ profits is given by: \(\pi = p_i - c_i + \frac{1}{2}(p_j - c_j)\). So, in the plane \((p_i, p_j)\), the set of equilibrium conditions for a symmetric, zero-transaction fee equilibrium consists of the intersection a rectangle, given by:
\[
p_i \leq \frac{\lambda u_i}{2}, \quad \lambda(1 - \lambda)u_j \leq p_j \leq \inf\{\lambda u_j, c_j\},
\]

of a cone, given by:
\[
2(\lambda u_j + (1 - \lambda)u_1)p_i \leq \lambda u_i [p_j + c_j + 2(1 - \lambda)u_1],
\]
\[
[\lambda u_i + \lambda + 2(1 - \lambda)u_1]p_j \leq 2\lambda u_j \left[p_i + (1 - \lambda)u_1 + \frac{1}{2}c_j\right],
\]

and of a last condition on profit:
\[
\pi = p_i - c_i + \frac{1}{2}(p_j - c_j) \geq \max\{\lambda(1 - \lambda) - c, 0\}.
\]

The rectangle is not empty only if: \(c_j \geq \lambda(1 - \lambda)u_j\). Hence Proposition 7: for a given \(\lambda < 1\), costs must be large enough for a market-sharing equilibrium to exist.
Straightforward computation shows that the intersection of the rectangle and the cone is non-empty if and only if:
\[ c_j + \lambda^2 u_i \geq \lambda(1 - \lambda)(1 - 2u_1), \]
and within this intersection, maximal profits are obtained for:
\[ p_i = \inf \left\{ \frac{\lambda u_i}{2}, \left( \frac{c_j + (1 - \lambda)u_1}{\lambda u_j + (1 - \lambda)u_1} \right) \lambda u_i \right\} \]
\[ p_j = \inf \left\{ \left( \frac{\lambda u_i + 2(1 - \lambda)u_1 + c_j}{\lambda u_i + 2(1 - \lambda)u_1 + \lambda} \right) \lambda u_j, c_j \right\}. \]

When \( \lambda \) goes to 1, (22) holds, profit-maximizing prices converge to:
\[ p_i \longrightarrow \inf \left\{ \frac{u_i}{2}, u_i, \frac{c_j}{u_j} \right\} \]
\[ p_j \longrightarrow \inf \left\{ u_j \left( \frac{u_i + c_j}{u_i + 1} \right), c_j, c_j \right\}, \]
so that profit converges to:
\[ \pi \longrightarrow \inf \left\{ \frac{u_i}{u_j} c_j - c_i, \frac{u_i}{1 + u_i} \left( 1 - \frac{c_i}{u_i} - c \right) \right\}. \]
Finally, (21) reduces to the non-negativity of profit, which is equivalent to:
\[ c + \frac{c_i}{u_i} \leq 1 \quad \text{and} \quad \frac{c_j}{u_j} \geq \frac{c_i}{u_i} \]
(23)
The conditions thus define \( i \) as the type with the smaller cost to utility ratio. The condition \( c_i < \frac{u_i}{2} \) then necessarily holds (as \( c < \lambda = 1 \)). Thus (23) is the only condition for the existence of the equilibrium in the limit case where \( \lambda \) goes to 1. \( \blacksquare \)