On Bargaining Partner Selection When Communication is Restricted*

Antoni Calvó-Armengol†

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Abstract

This paper analyzes the optimal selection of a bargaining partner when communication among players is restricted. We define a two-stage selection and bargaining game. In the first stage, players independently choose a partner. In the second stage, one player chosen randomly has an idea to be implemented jointly with an associate as a new firm and initiates a round of negotiations with her selected partner. With homogeneous time preferences, *ex post* payoffs do not depend on players relative locations: the unique equilbrium agreed-upon payoffs coincide with the standard half-half partition. With heterogeneous time preferences, the unique subgame perfect equilibrium bargaining outcomes depend both on players locations and discount factors. Still, selecting the most impatient neighbor is an equilibrium strategy for a geometric class of communication networks that includes the star among many others.

Keywords: noncooperative bargaining, bargaining partner, communication network, time preferences.

Journal of Economic Literature Classification Numbers: C72, C78, D20

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†Department of Economics, Universitat Pompeu Fabra, Ramon Trias Fargas 25-27, 08005 Barcelona, Spain, and CERAS-ENPC, 28 rue des Saints-Pères, 75007 Paris, France. http://www.econ.upf.es/~calvoa. Email: antoni.calvo@econ.upf.es

1. Introduction

Social and economic networks play a prominent role in explaining a wide range of economic phenomena. Examples that have been studied include social learning and coordination, information transmission in labor markets, the diffusion of new technologies, educational attainment and stigma for crime among others.[1] The aim of this paper is to determine how individual bargaining behavior is affected by the existence of social networks that restrict feasible pairwise meetings. More precisely, we analyze the optimal selection of a bargaining partner when communication among players is restricted, and players are in contact with only a limited subset of other players.

In our paper, the nature of the communication structure supporting agent-to-agent meetings or talks is made explicit and modeled as a graph. The nodes represent the players and a link between two nodes means that the corresponding players can communicate directly. This communication network introduces natural restrictions over bilateral meetings and only players that are in direct contact with each other can negotiate together. The relative location of players in the graph (more or less connected) may then have an impact on bargaining agreements, and co-bargainer selection becomes a strategic issue. Moreover, we already know that more impatient players are more eager to accept tougher proposals than patient ones, and reciprocally. The optimal selection of a bargaining partner is thus presumably governed by the following two players attributes: relative positions in the network of communication and relative rates of impatience.

We directly tackle the question of co-bargainer selection with communication restrictions with the following two-stage game. In the rst stage, players independently choose a partner among their (limited) set of direct acquaintances. In the second stage, one player randomly selected has an idea to be implemented jointly with an associate as a new rm, and initiates a round of negotiations with her selected partner. In case of rejection, the partner submits a proposal to her selected partner that may coincide with the initial promoter (standard two-player alternating offers game) or not. This bargaining procedure is a simple adaptation of the Rubinstein (1982) noncooperative bargaining game to the case of a nite population of players connected through a graph. Its three main features are the following. First, players are only allowed to make proposals to their direct neighbors. Second, bargaining is sequential and any player can only bargaining with one partner at a time. In particular, simultaneous offers to two different neighbors are not possible. Finally, the pair of neighbors that bargain at every round corresponds to the partners strategically selected during the rst stage of the game. In particular, we take the extreme view that, in case of rejection, the

[1] See for instance Jackson and Wolinsky (1996) and the references therein.

initial promoter is doomed to inactivity. At the end of each section, we discuss the role of this *a priori* over-simplifying assumption and check for the robustness of our results. We claim that the (apparently) simple setting analyzed in this paper provides valuable insights to understand the impact the structure of interaction (the communication network) has on individual strategic behavior within a bargaining context.

The results we obtain are the following. First, we show that the second stage of the selection and bargaining game −the bargaining phase− has a unique perfect equilibrium outcome. Uniqueness here is established for a bargaining game with endogenously determined outside options, where the outcome of opting out is not exogenously given but corresponds to bargaining agreements, themselves in uenced by endogenous outside options of the same type. When all players have the same discount factors, these unique equilibrium agreed-upon payoffs coincide with the standard half-half partition, no matter the respective positions of the players in the network of communications. With homogeneous time preferences, switching over to an alternative partner is payoff-equivalent to playing a standard alternating offers game. As a result, all players are identical from the bargaining game viewpoint and all directly connected players are worth the same as bargaining partners, irrespective of their location. As *ex post* payoffs are immune to players locations, no player has a strategic advantage to discriminate among her personal contacts according to their relative social embeddedness. We then introduce *ex ante* payoffs that strongly depend on players positions and thus capture to some extent the asymmetries induced by the geometry of communication links.

On the contrary, when players have different discount factors, the subgame perfect equilibrium bargaining payoffs depend strongly both on players relative positions in the communication graph and on their time preferences. With heterogeneous time preferences, location matters for the optimal selection of a bargaining partner. We prove that selecting the most impatient neighbor is an equilibrium strategy for a rich class of communication networks that we call strati ed graphs. Strati ed graphs satisfy the general requirement that any two players having an acquaintance in common also share their relatively more impatient neighbor. Star-shaped communication structures −where one central player is connected to peripheral isolated players− is a trivial example of such graphs. The rule-of-thumb that consists on selecting the most impatient contact, though, is not always an equilibrium strategy. We provide some (counter) examples illustrating this point.

The remainder of the paper is organized as follows. Section 2 presents the communication network, describes the $n-$player bargaining game and states the main result namely, the noncooperative bargaining game with communication restrictions has a unique perfect equilibrium outcome. Section 3 deals with the special case of homogeneous time preferences while section 4 is devoted to the more general case where players may have different discount factor. Section 5 concludes and an appendix contains all the proofs.

2. Noncooperative bargaining in communication networks

2.1. The network of communications

In many social and economic situations, agents are in direct contact with only a limited subset of other agents. When this is the case, bilateral volunteer meetings are limited to those pair of agents that can communicate with each other directly (or that know each other directly). We represent these restrictions on the feasible pairwise meetings by a graph where the nodes are identified with the players and a link between two nodes means that the corresponding players can bargain bilaterally. The communication network connecting players thus determines the individual bargaining possibilities.

Let $N = \{1, \ldots, n\}$ be the finite set of players ($n \geq 2$) and denote by $\delta_i \in (0, 1)$ the discount factor of player $i \in N$. We only consider undirected graphs. The complete graph denoted by g^N is thus the set of all subsets of N of size two, and the set of all possible graphs on N is $\mathcal{G} \equiv \left\{g \mid g \subseteq g^N\right\}$. Let $g \in \mathcal{G}$. A link in g between two players i and j, denoted by $ij \in g$, means that these players can communicate directly or, equivalently, that they are direct neighbors.[2] We introduce the following definition:

Definition 1. *For all $g \in \mathcal{G}$ and for all $i \in N$, i s neighborhood denoted by $N_i(g)$ is the set of players in N directly connected to i when the communication network is g that is, $N_i(g) = \{j \in N \setminus \{i\} \mid ij \in g\}$.*

Note that by definition $i \notin N_i(g)$. Also, $i \in N_j(g)$ is equivalent to $j \in N_i(g)$. In words, the neighborhood relationship is non reflexive and symmetric. We denote by $n_i(g)$ the cardinality of $N_i(g)$.[3] To avoid trivialities we restrict to $\mathcal{G}^* = \{g \in \mathcal{G} \mid n_i(g) \geq 1, \forall i \in N\}$ that is, to those networks where all players have at least one neighbor.

Example 1. *Suppose that a seller labeled s owns an indivisible object to be sold to one of m potential buyers labeled $b_1, \ldots, b_m$. The corresponding communication structure can be represented by a star-shaped graph centered on s where the peripheral players are the buyers (see Figure 2.1).*[4]

[2] Throughout the paper, we use the terms *graph*, *network* or *communication structure* without

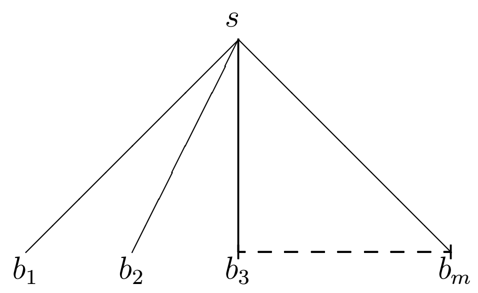

Figure 2.1: a star-shaped network centered on s.

3. The selection and bargaining game

3.1. Description of the game

An informal version of the game we consider is the following. Suppose that some player (the promoter) has an idea about a new and pro table economic activity to exploit jointly with an associate. The set of potential partners the promoter has at her disposal, presumably limited, is constituted by the players in the population she is in direct contact with. The promoter selects one player among her acquaintances, proposes this player to become her associate, and engages a bilateral negotiation to x their respective shares of the outcome delivered by the new business activity. If the two players agree on how to share the bene ts the negotiation ends, the rm is created, and its pro ts are distributed according to the agreed shares. Of course, these shares depend on the players preferences and on the negotiation rules. Moreover, as players may hold asymmetric positions in the communication network that determines the bargaining possibilities, the issue of optimal bargaining partner selection has to be contemplated.

We model this bargaining game as a two-stage game. In the rst stage, players choose independently to whom they wish to submit bargaining proposals for a potential collaboration in a joint project. In the second stage, one player randomly chosen has an idea about a business opportunity to exploit jointly with an associate and initiates a round of negotiations with her selected partner. In case of rejection, the potential associate submits a proposal to her selected partner that may coincide with the initial promoter or not. We assume for simplicity that

distinction.

[3] In graph theory, $n_i(g)$ is the degree of node i that is, the number of nodes adjacent to i or, equivalently, the number of links that are incident with i.

[4] Related models where the good is sold through a bargaining procedure are discussed by Hendon and Tranæs (1991) where the buyers have different valuations for the good, by Jéhiel and Moldovanu (1995 a,b) that investigate the effect of externalities between buyers, and by Calvó-Armengol (1999).

partners bargain over a cake of size one. We rst introduce a de nition:

De nition 2. *For all $g \in \mathcal{G}^*$, $\tau_g : N \longrightarrow N$ where $\tau_g(i) \subseteq N_i(g)$, $\forall i \in N$ is the selection correspondence on g that gives for any player i the subset $\tau_g(i) \subseteq N_i(g)$ of neighbors player i chooses to bargain with.*

Given a connected undirected bilateral graph g on N, the two-stage selection and bargaining game we analyze can now be described as follows:

> *Stage One.* For all $i \in N$, player i determines the subset $\tau_g(i)$ of neighbors in $N_i(g)$ she chooses to bargain with.
>
> *Stage Two.* Noncooperative bilateral negotiation with random selection of the promoter $i \in N$, two-player/one-cake bargaining offer at the rst round by the initial proposer i to her respondents in $\tau_g(i)$. In case of refusal, sequential activation of pairs of bargaining partners from neighborhood to neighborhood according to τ_g.

Two comments are in order. First, according to this bargaining procedure, a player can only make offers to her neighbors and can never make simultaneous offers to two different neighbors. Second, the bargaining partner is always selected from the set of direct neighbors and, in case of rejection, the respondent becomes the new proposer at the next round. In fact, for the bilateral negotiation to carry on correctly between a promoter and her selected partner, it is reasonable to assume that both the owner of the idea (the promoter) and her bargaining partner (the potential associate) have a good knowledge of the valued properties of the new business opportunity. Implicitly we are requiring that the idea is transferable and that the potential associate becomes completely aware of the promoter s project during the bargaining contest. Therefore, if no agreement is reached between the initial promoter and her selected partner, these two agents share a common idea about a new business opportunity. If both players can contact different potential associates, they may then act as two different promoters for the creation of two different rms. Here we take the extreme view that the initial promoter is spoliated of her idea in case of rejection, and condemned to a passive behavior from then on. We discuss at the end of each section the implications of such a drastic assumption according to which the initial proposer loses all claim to the surplus once, and if, the rst partner exits the bargaining table.

We assume from now on that for all $i \in N$, $\tau_g(i)$ is a singleton.[5] Any selection function τ_g de nes a sequence of proposers and respondents for the bargaining

[5] We can assume for instance that the following tie-breaking rule applies: when some player $i \in N$ is indifferent between neighbors $j_1, \ldots j_m \in N_i(g)$ (concerning the outcome she can obtain when bargaining with any of them separately), the respondent with the smallest index is selected that is, $\tau_g(i) = \min\{j_1, \ldots j_m\}$.

game on the graph g the following way. Denote by p_t the label of the proposer at some period $t \in \mathbb{N}$. Suppose that player p_t makes an offer to her bargaining partner (neighbor) $\tau_g(p_t)$. If the offer is accepted the game ends. Otherwise the rejector $\tau_g(p_t)$ becomes the new proposer at the next round $t+1$ that is, $p_{t+1} = \tau_g(p_t)$. And so on. The game proceeds until, and if, a bilateral agreement between two neighboring players is reached. The noncooperative bargaining game on the communication network actually played is fully described by the identity of the initiator $i = p_0$ of the game and the selection function τ_g. We denote by $\langle \tau_g, i \rangle$ such a game.

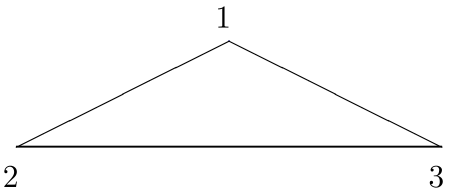

Figure 3.1: three players on a circle.

Example 2. *$N = \{1, 2, 3\}$ and $g = g^N$ (see Figure 3.1). The possible bargaining games $\tau_{g^N}, 1\rangle$ initiated by player 1 (equivalently, the possible selection functions) are:*

$\tau_{g^N}(1)$	$\tau_{g^N}(2)$	$\tau_{g^N}(3)$	*sequence*
2	1	–	$1 \to 2 \to 1 \to 2 \to 1 \to 2 \to \ldots$
2	3	1	$1 \to 2 \to 3 \to 1 \to 2 \to 3 \to \ldots$
2	3	2	$1 \to 2 \to 3 \to 2 \to 3 \to 2 \to \ldots$
3	–	1	$1 \to 3 \to 1 \to 3 \to 1 \to 3 \to \ldots$
3	1	2	$1 \to 3 \to 2 \to 1 \to 3 \to 2 \to \ldots$
3	3	2	$1 \to 3 \to 2 \to 3 \to 2 \to 3 \to \ldots$

3.2. Analysis of the game

The main result of this section is the following. Let $g \in \mathcal{G}^*$.

Proposition 1. *For all promoter $i \in N$ and for all selection function τ_g, the game $\langle \tau_g, i \rangle$ has a unique perfect equilibrium outcome.*

Recall that in our game, players that bargain have at their disposal outside opportunities de ned as the endogenously expected pro ts from negotiations with alternative bargaining partners. Despite the presence of such endogenous outside options corresponding to bargaining agreements −themselves in uenced by endogenous outside options of the same type−, uniqueness of the subgame perfect equilibrium outcome holds.[6] Moreover, as in Rubinstein (1982), in a subgame perfect equilibrium players agree to a proposal if it offers at least as much as what they can expect to get in the future (either with the current partner or with some other co-bargainer) appropriately discounted. Note also that we do not need to assume that players have a common discount factor to obtain uniqueness.

In our framework, players are connected through an arbitrary communication structure. They may thus differ by their relative positions in the network. In that sense, the communication network is a potential source of heterogeneity within the nite population. Individual preferences constitute a second source for population heterogeneity. Indeed, players may differ in their relative rates of impatience and the population can be either homogeneous or heterogeneous in time preferences. We analyze these two cases separately. We rst consider the case where all players have the same discount factor. The unique individual attribute differentiating players is then their (eventual) asymmetric location in the graph. We then allow for different discount factors. The optimal selection of a bargaining partner may now be in uenced both by the relative rates of impatience and by the relative positions of players in the communication network.

4. Co-bargainer selection with homogeneous time preferences

4.1. Solving the base model

Let $g \in \mathcal{G}^*$. The main result of this section is:

Corollary 1. *For all promoter $i \in N$ and for all selection function τ_g, the unique perfect equilibrium outcome of the game $\langle \tau_g, i \rangle$ coincides with the standard agreement $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$. In particular, the optimally selected partner $\tau_g(i)$ can be indistinctly any player in $N_i(g)$.*

In words, all bargaining sequences are payoff-equivalent and, whoever the promoter, all her neighbors are worth the same *ex post* as bargaining partners. This

[6] The proof relies on a generalization of Shaked and Sutton (1984) s argument where the maximum and the minimum of the subgame perfect equilibrium outcomes coincide. In our context, though, the stationarity of the game underlying Shaked and Sutton method of analysis takes a particular form and is induced by the communication network underlying players talks. See the appendix for more details.

result is counter-intuitive as it states that the different relative positions of players in a network of communications do not confer different payoffs. Whatever the location of a player (either isolated or well-connected) and whatever the bargaining partner selected in her neighborhood (either isolated or well-connected) the rst player always proposes in equilibrium the standard partition $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ that her partner accepts immediately. Letting $\delta \rightarrow 1$, thus eliminating the frictions in the negotiations due to discounting, this subgame perfect equilibrium outcome becomes $\left(\frac{1}{2}, \frac{1}{2}\right)$. Hence, whenever two neighbors agree on a joint collaboration, the derived bene ts are distributed according to the standard half-half splitting rule irrespective of their eventual asymmetric locations in the graph.

According to the previous result, no promoter $i \in N$ has a strategic advantage to discriminate among her neighbors: *ex post*, the equilibrium agreed-upon partition is half-half with all of them. The intuition for this result is the following. We know that outside options are irrelevant to the nal deal unless a bargainer can gain more by getting elsewhere [the Outside Option Principle, see Binmore *et al.* (1989)]. Suppose here that a (very) well-connected player receives a splitting proposal from one of her neighbors. In case of disagreement, the rejector has at her disposal a large set of acquaintances from which she can pick an alternative partner −differing from the neighbor that has just submitted the proposal− for the negotiations to carry on. In that sense, having many neighbors is equivalent, at least in spirit, to having the possibility to opt out after a rejection.[7] Players may hold asymmetric positions in the network of communications and thus the possibility to opt out may vary. But all players have the same discount factor and time preferences are the leading force governing players behavior during negotiations. By opting out no player can expect to obtain a higher payoff than by replicating with a counter-offer to the current bargaining partner. Switching over does not lead to a gain leverage and such outside options do not constitute a credible threat to ongoing negotiations. Consequently, neighborhood sizes have no effect on the equilibrium partition and all players (all locations) are identical from the bargaining game viewpoint.

We can thus assume that player i treats all her neighbors on an equal footing and selects her bargaining partner within her neighborhood with uniform probability. Formally, for all $j \in N_i(g)$ let $\Pr\{\tau_g(i) = j\} = \frac{1}{n_i(g)}$. Assume further that all players in the population have the same probability $\frac{1}{n}$ to become a promoter

[7] Strictly speaking, outside options are commonly introduced in bargaining games as exogenous xed (or sometimes random) amounts [see for instance Binmore *et al.* (1989) and Ponsatí and Sákovics (1998, 1999)]. In our model, the outcome of opting out is not exogenously given but determined as a negotiation outcome with an outside partner. For other examples of outside options varying on account of players actions see for instance Compte and Jéhiel (1999a, b).

(equal flow of opportunities). Given a network of communications we can easily compute, for any player i in the population, her expected payoff $E\pi_i(g)$ in a game where all players have an equal opportunity to become a promoter of business projects and all potential associates are treated equally:

$$E\pi_i(g) = \frac{1}{2n}\left[1+ \sum_{j\in N(g,i)} \frac{1}{n(g,j)}\right], \forall i \in N$$

The vector $\{E\pi_i(g)\}_{i\in N}$ give the *ex ante* distribution of payoffs at the limit $\delta \to 1$. It is obviously related to the particular random device that determines the identity i of the promoter and selects her bargaining partner $\tau_g(i)$ among the available set $N_i(g)$ of potential associates.[8] We deduce from this expression that even though *ex post* payoffs are immune to players relative locations, *ex ante* payoffs as defined here strongly depend on the frequency of activation of pairs of neighbors and thus on players positions. To some extent, they capture the asymmetries induced by the geometry of links. In particular, one can check that the *ex ante* payoff $E\pi_i(g)$ of player i increases with the size of her neighborhood and decreases with the size of her neighbors neighborhood.

4.2. Bargaining duplication, firm multiplication and competition

In the bargaining procedure analyzed so far, when a promoter submits a proposal to a potential associate, the offer can be accepted or rejected. In case of disagreement we move to the next round and the rejector becomes the new proposer. From this round on, the initial promoter does not play any active role unless she is submitted a proposal at some stage by some direct active neighbor (for instance, the initial rejector may respond to the promoter s proposal, once rejected, by a counter-offer). We are thus implicitly assuming that when the potential associate rejects an offer, the promoter is spoliated of his idea about a new and profitable economic activity and doomed to inactivity. Although it is reasonable to suppose that during the initial negotiation round the potential associate becomes

[8] More generally, assume that for all $i \in N$ and for all $j \in N_i(g)$, $\Pr\{\tau_g(i) = j\} = p_{ij}$, where $0 \leq p_{ij} \leq 1$ and $\sum_{j\in N(g,i)} p_{ij} = 1$. Assume further that i is the promoter with probability $0 \leq q_i \leq 1$, where $\sum_{i\in N} q_i = 1$, and that these probabilities are independent. The resulting expected payoffs are $E\pi_i(g) = \frac{1}{2}\left[q_i + \sum_{j\in N(g,i)} q_j p_{ji}\right], \forall i \in N$. We refer the reader to Calvó-Armengol (2000) for a more detailed analysis and discussion. In particular, $E\pi_i(g)$ is player i s expected payoff at the unique stationary subgame perfect equilibrium outcome of the bargaining game where players meet according to the probabilities $\left\{q_i, (p_{ij})_{j\in N}\right\}_{i\in N}$.

completely aware of the characteristics of the rm s project (thus allowing her to act separately as an active promoter at the next stage), it is not sensible to assume that the initial promoter is simultaneously deprived of her own idea and condemned to a passive behavior. In fact, in real-life negotiations the promoter plays an active role permanently.

To get rid of this over-simplifying assumption of idea spoliation by the bargaining partner, we consider the following variation of the bargaining procedure. At some period, a promoter $i \in N$ contacts a potential associate $j \in N_i(g)$, exposes her intention to collaborate jointly in the creation of a new rm and makes a pro t-splitting proposal that j may accept or reject. Suppose that j rejects the offer. At the next period, j can submit a counter-offer to i. Also, if both players are sufficiently well-connected[9] both the initial promoter i and the rejector j can act separately as two different promoters for the creation of two different rms. It suffices for them to select each a different bargaining partner and initiate with their respective co-bargainers two separate rounds of negotiation.

Multiple rms resulting from the various agreed pairwise collaborations may then be created due to this bargaining duplication. These rms derive from a common idea and thus certainly compete in the same market. The resulting *ex post* oligopoly competition yields then to a reduction of the per- rm pro t.[10] To simplify matters, we restrict to four players located on a line (see Figure 4.1). The promoter (player 1) selects a bargaining partner among her two potential candidates 2 and 3, where player 3 is isolated in the communication network whilst player 2 has an additional neighbor, player 4.

If the promoter bargains with her isolated neighbor player 3, pro ts are divided at equilibrium according to a half-half splitting rule. A unique rm is created yielding a monopoly pro t $\pi_M = 1$ and player 1 ends up with $\frac{\pi_M}{2}$. Suppose on the contrary that the promoter makes her rst offer to her well-connected neighbor player 2. If she rejects the offer, at the next round player 2 can either replicate to the promoter by a counter-offer or quit the current negotiations and promote separately the creation of a rm with player 4. In the rst case both player 1 and player 2 end up with $\frac{\pi_M}{2}$. In the second case, two distinct pairs of players $(1, 3)$ and $(2, 4)$ negotiate for the creation of two different rms. Within each pair, pro ts

[9] Formally, $\min \{n_i(g), n_j(g)\} \geq 2$ and $N_i(g) \backslash \{j\} \neq N_j(g) \backslash \{i\}$.

[10] If rms compete à la Bertrand, the expected per- rm pro t with more than one rm is zero. If rms compete à la Cournot, oligopoly pro ts are not null. In particular, if $k \geq 2$ Cournot oligopolists produce the output at a xed unit cost c in a homogeneous market with linear demand curve $p = A - bq$, the (Nash) pro t of a single rm is $\frac{(A-c)^2}{b(k+1)^2} < \frac{(A-c)^2}{4b}$, where $\frac{(A-c)^2}{4b}$ is the monopoly pro t. Denoting π_k the per- rm pro t with $k \geq 2$ oligopolies ($\pi_M = \pi_1 = 1$ being the monopoly pro t) we have $\pi_k = \frac{4}{(k+1)^2}$. In particular, the Cournot duopoly pro t $\pi_D = \pi_2$ is equal to $\pi_D = \frac{4}{9}$.

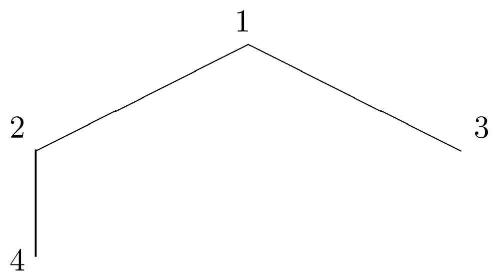

Figure 4.1: four players on a tree.

are divided evenly. But the resulting oligopoly competition results in duopoly pro ts $\pi_D < \pi_M$, and each player receives $\frac{\pi_D}{2}$. Therefore, duplicating the ongoing negotiations cannot result in a higher payoff than playing a standard two-player bargaining. Consequently, acting separately as an additional promoter is a non-credible threat and it does not affect the equilibrium strategies and outcomes. As before, payoffs are always distributed according to a half-half splitting rule and all neighbors are worth the same *ex post* as potential associates.

5. Co-bargainer selection with heterogeneous time preferences

5.1. Solving the base model

We now turn to the more general case where players may have differ discount factors. Consider rst the standard two-player game of alternating offers between players 1 and 2. At the unique perfect equilibrium of this game the rst player to move, e.g. player 1, proposes the partition $\left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \delta_2\frac{1-\delta_1}{1-\delta_1\delta_2}\right)$ that player 2 accepts. Player 1 thus gets a share $a_{12} = \frac{1-\delta_2}{1-\delta_1\delta_2}$ of the cake satisfying $\frac{\partial a_{12}}{\partial \delta_2} = -\frac{1-\delta_1}{(1-\delta_1\delta_2)^2} < 0$. Hence, the more one s rival is impatient (the smaller δ_2), the bigger the share obtained.

Suppose that all potential partners the promoter has at her disposal are isolated one with respect to each other, meaning that the promoter is the only person they are in direct contact with. The communication network is then a star-shaped graph as in Figure 2.1. Clearly, in this case, the promoter selects her relatively more impatient neighbor. The question that naturally arises is whether this result can be generalized to a nite population of players connected through any communication structure. More concretely, we now ask if selecting the most impatient neighbor is a subgame perfect equilibrium of the two-stage selection and bargaining game with a population heterogeneous in time preferences. We show with two (counter) examples that the general answer is no. In what follows $a_{ij} = \frac{1-\delta_j}{1-\delta_i\delta_j}$ for $i \neq j$ denotes player i s outcome of a standard two-player bargaining game between player i and player j where i is the rst proposer. We can check that $a_{ij} = 1 - \delta_j a_{ji}$.

Example 3. $N = \{1, 2, 3, 4\}$ *and* g *is a line (see Figure 4.1). Suppose that player 1 (the promoter) selects a bargaining partner among two potential candidates 2 and 3, where player 2 is taken to be relatively more impatient than player 3:* $\delta_2 < \delta_3$. *If the two candidates were isolated in the communication network (meaning that they only have one neighbor and that their only common neighbor is the promoter) player 1 would bargain with the most impatient neighbor player 2. Player 2, though, has an additional neighbor, player 4. Assume that player 4 is relatively more impatient than the promoter:* $\delta_4 < \delta_1$. *Suppose that* $\tau_g(1) = 2$ *that is, player 1 initially makes a proposal to player 2. In case of refusal it is then optimal for player 2 to switch over to player 4 instead of replying to the promoter by a counter-offer. Indeed, player 2 obtains* a_{24} *in the continuation game in the former case whereas she ends up with* a_{21} *in the latter. Clearly* $a_{24} > a_{21}$ *as* $\delta_4 < \delta_1$. *Therefore, if player 1 bargains with player 2 at some round and the respondent rejects the offer, player 2 and player 4 initiate at the next round a standard two-player game of alternating offers. Formally,* $\tau_g(2) = 4$ *and* $\tau_g(4) = 2$. *The equilibrium outcome for player 1 is then* $1 - \delta_2 a_{24} = a_{42}$.[11] *If on the contrary* $\tau_g(1) = 3$ *meaning that player 1 chooses to bargain bilaterally with her relatively more patient neighbor player 3, she ends up with* a_{13}. *Therefore,* $\{\tau_g(1) = 3\} \underset{1}{\succ} \{\tau_g(1) = 2\}$ *if and only if* $a_{13} > a_{42}$. *We can check that, conditional on* $\delta_4 < \delta_1$ *and* $\delta_3 < \delta_2$, *this inequality holds for a large range of parameter values. With these values, player 1 obtains a higher payoff when she bargains with the relatively more patient but isolated* neighbor *(player 3) than when she bargains with the relatively more impatient but well-connected* neighbor.[12]

Example 4. $N = \{1, 2, 3, 4, 5\}$ *and* g *is a line (see Figure 5.1). Suppose that* $\max\{\delta_4, \delta_5\} < \delta_1$. *Following a similar reasoning than that of the previous example, it is easy to show that the optimal (selected) partners for players 2, 3, 4 and 5 are respectively* $\tau_g(2) = 4$, $\tau_g(4) = 2$, $\tau_g(3) = 5$ *and* $\tau_g(5) = 3$. *In words, players 2 and 4 (resp. 3 and 5) play a standard two-player game of alternating offers. We now have to determine what neighbor yields player 1 a higher payoff at equilibrium to determine the identity of her selected partner. If* $\tau_g(1) = 2$ *(resp.* $\tau_g(1) = 3$*)*

[11] It thus coincides with the unique subgame perfect equilibrium outcome a_{42} obtained by player 4 when she bargains bilaterally with player 2.

[12] It suffices that $\delta_2 - \delta_3 + \delta_1\delta_3 - \delta_2\delta_4 + \delta_2\delta_3(\delta_4 - \delta_1) > 0$. For instance, if $\delta_1 = \delta_3 = 0.9$, $\delta_2 = 0.8$ and $\delta_4 = 0.7$ we get $a_{13} = 0.5263\ldots$ and $a_{42} = 0.4545\ldots$. Therefore, $a_{13} > a_{42}$. Player 1 (the promoter) selects as her potential associate the relatively more patient neighbor.

player 1 gets at equilibrium $1 - \delta_2 a_{24} = a_{42}$ *(resp.* $1 - \delta_3 a_{35} = a_{53}$*). Therefore,* $\{\tau_g(1) = 2\} \underset{1}{\succ} \{\tau_g(1) = 3\}$ *if and only if* $a_{42} > a_{53}$*. Suppose for instance that* $\delta_4 = \delta_5$*. Then,* $\{\tau_g(1) = 2\} \underset{1}{\succ} \{\tau_g(1) = 3\}$ *if and only if* $\delta_2 < \delta_3$*. In words, when the peripheral players (players 4 and 5) have the same time preferences, the optimal bargaining partner for the central player is her relatively* more impatient *neighbor. Suppose on the contrary that* $\delta_2 = \delta_3$*. Now,* $\{\tau_g(1) = 2\} \underset{1}{\succ} \{\tau_g(1) = 3\}$ *if and only if* $\delta_4 > \delta_5$*. Hence, if the intermediate players (player 2 and 3) have the same time preferences, the best bargaining partner for the central player is the neighbor connected to the relatively* more patient *peripheral player.*

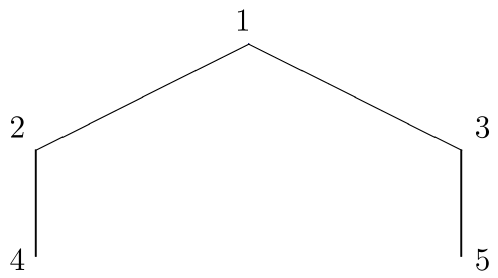

Figure 5.1: ve players on a line.

The two previous examples show that *selecting the most impatient neighbor is not always an equilibrium strategy.* In most cases, both individual time preferences and each player location matter. We can nonetheless state a general result. Without loss of generality, we assume that $\delta_1 < \cdots < \delta_n$.[13] Given a graph $g \in \mathcal{G}^*$ we denote by τ_g^* the selection function where all players select their most impatient neighbor that is,

$$\tau_g^*(i) = \arg\min \{\delta_k \mid k \in N_i(g)\}, \forall i \in N.$$

Proposition 2. *When the population is heterogeneous in time preferences,* τ_g^* *is a Nash equilibrium of the rst stage game if* $\tau_g^*(j) = \tau_g^*(k)$, $\forall i \in N, \forall j, k \in N_i(g)$.

This proposition establishes a sufficient condition guaranteeing that the behavioral rule-of-thumb selecting the most impatient neighbor is a subgame perfect

[13] The impatience rates are taken to be different two by two just to avoid ties. The speci c ordering of the impatience rates of the players does not impose any restriction up to a relabeling of the players.

equilibrium of the selection and bargaining game. This condition imposes restrictions on the communication structure that concern not only the geometry of links but also the identity (the discount factor) of the connected players. It is required that any two players having a neighbor in common must also share their relatively more impatient neighbor.

We know from the previous section that when the population is homogeneous in time preferences all players are identical from the bargaining game viewpoint and location does not matter. On the contrary, when the population is heterogeneous in time preferences, the two players attributes together −namely, their relative discount factors and their relative locations− govern the optimal selection of a bargaining partner and are tightly related. Indeed, Proposition 2 provides a criterium on the distribution of the discount factors (and their relative values) within neighborhoods. Heterogeneity in time preferences triggers an impact of the asymmetry of links which itself depends on how individual discount factors are located in the graph. The communication network matters not only because it tells us how players are connected but also in determining who is connected to whom. One may now ask what kind of graphs, if any, satisfy the sufficient condition stated in Proposition 2 and ensuring that selecting the most impatient neighbor is an equilibrium strategy. Star-shaped communication structures are trivial examples ful lling this requirement.

Corollary 2. *When the population is heterogeneous in time preferences, selecting the most impatient neighbor is a subgame perfect equilibrium of the two-stage selection and bargaining game for all star-shaped graphs composed of a central player and $n-1$ peripheral players.*

The sufficient condition in Proposition 2 is appealing by its apparent simplicity. Moreover, familiar graphs such as stars satisfy this condition (see Figure 2.1 for an example). But players select their most impatient neighbor at equilibrium with many other con gurations of links apart from stars. It suffices indeed that any two players having a neighbor in common also have in common their most impatient neighbor. Players thus have to be located cautiously in the network so that individual discount factors satisfy locally (within neighborhoods) this condition. For instance, the requirement of Proposition 2 is true with four players on a line (see Figure 4.1) provided that $\delta_2 < \delta_3$ and $\delta_1 < \delta_4$. Indeed, player 1 belongs both to player 2 and to player 3 neighborhood ($N_2(g) = \{1, 4\}$ and $N_3(g) = \{1\}$) and is also their most impatient neighbor ($\tau_g^*(2) = \tau_g^*(3) = 1$). Similarly, player 2 belongs both to player 1 and to player 4 neighborhood ($N_1(g) = \{2, 3\}$ and $N_4(g) = \{2\}$) and is their most impatient neighbor ($\tau_g^*(1) = \tau_g^*(4) = 2$). The following example provides another con guration of links such that this criterium applies.

Example 5. *$N = \{1, 2, 3, 4, 5, 6, 7\}$ and g a tree on N (see Figure 5.2). Suppose that $\delta_1 < \min\{\delta_i \mid i = 4, 5, 6, 7\}$. Selecting the most impatient neighbor is then a subgame perfect equilibrium of the selection and bargaining game on g.*

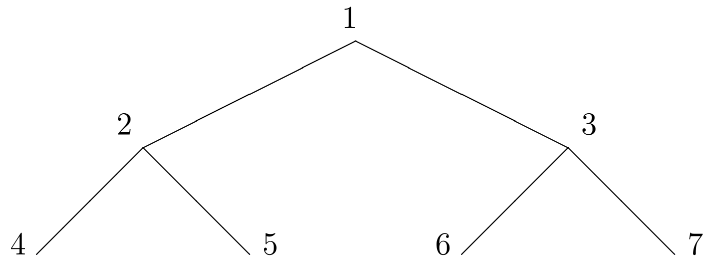

Figure 5.2: seven players on a tree.

In particular, we can always assign players to nodes so that selecting the most impatient neighbor is an equilibrium strategy for certain types of graphs called trees. Trees are connected and acyclic graphs. Connected graphs are such that there is a path between every pair of nodes in the graph. Stated differently, $g \in \mathcal{G}$ is connected if any two players i and j in N either communicate directly ($ij \in g$) or are connected through a path of neighbors in g.[14] A cycle is a path on the graph containing at least three nodes in which all lines are distinct and all nodes except the beginning and ending nodes are distinct. For instance, the network in Figure 3.1 contains cycles. Acyclic graphs contain no cycles. Trees are also minimally connected graphs containing exactly $n-1$ links. For instance, the communication structures depicted in Figures 2.1, 4.1, 5.1 and 5.2 are trees.[15]

Proposition 3. *If $g \in \mathcal{G}^*$ is a tree we can always locate players in the communication structure so that selecting the most impatient neighbor is an equilibrium strategy.*

Remark 1. *The assumption of connectedness implicit for a tree is not essential. If $g \in \mathcal{G}^*$ is a forest, meaning that g is acyclic and disconnected (g has more than one component and each component is a tree), the result still holds.*

[14] Formally, this means that for all $i, j \in N$, there exists $m \geq 1$ and a sequence $\{i_0, \ldots, i_m\} \subset N$ such that $i_0 = i$, $i_m = j$, $i_{k-1} \neq i_k$ and $i_{k-1}i_k \in g$, $\forall 1 \leq k \leq m$.

[15] There are exactly n^{n-2} possible trees.

Although quite general, the criterium stated in Proposition 2 yields, in practice, to tedious and systematic link-by-link examination. But we can identify a (geometric) subclass of networks −including the stars− for which this criterium holds irrespective of how players (and their individual discount factors) are assigned to the nodes of the graph. We say that two disjoint subsets of N are collectively connected by some communication structure $g \in \mathcal{G}^*$ if any player in one subset is linked with any other player in the other subset, while players within subsets are not linked. In other words, all players in a subset are adjacent to players from the other subset, but no player is adjacent to any player in its own subset. We then introduce a particular subclass of graphs on N.

De nition 3. *A network $g \in \mathcal{G}^*$ is a $p-$strati ed graph if there exists $p \in \mathbb{N}$ and a partition $S_1, \ldots, S_p$ of N such that S_i and S_{i+1} are collectively connected by g for all $1 \leq i \leq p-1$.*[16]

The most simple example of a strati ed graph is a star-shaped communication structure centered on $i \in N$ where $S_1 = \{i\}$ and $S_2 = N \backslash \{i\}$. Strati ed graphs can have more than two strata as illustrated by the following example.[17]

Example 6. $N = \{1, 2, 3, 4, 5, 6, 7\}$ *and g is a three$-$strati ed graph with strata $S_1 = \{1\}$, $S_2 = \{2, 3\}$ and $S_3 = \{4, 5, 6, 7\}$ (see Figure 5.3).*

Corollary 3. *When the population is heterogeneous in time preferences, selecting the most impatient neighbor is a subgame perfect equilibrium of the two-stage selection game for all $p-$strati ed graphs $g \in \mathcal{G}^*$ such that $p \leq 3$.*

The situations where selecting the most impatient neighbor is an equilibrium strategy are not limited to $p-$strati ed graphs with $p \leq 3$ (see Proposition 3, for

[16] In graph theory, strati ed graphs with two strata are denominated *complete bipartite graphs*. Strati ed graphs with p strata are a particular case of *p-partite graphs*.

[17] Two-sided graphs (among which *two*−strati ed graphs) have been the object of recent research on buyer-seller networks. See for instance Chatterjee and Dutta (1998), Corominas-Bosch (1999) and Kranton and Minehart (1998). In particular, Chatterjee and Dutta (1998) analyze the effect of competition for bargaining partners on bargaining agreements (or prices) in thin two-sided markets with two sellers and two buyers. They describe three particular bargaining procedures (or trading processes) where players choose whom they are matched with and discuss how bargaining agreements and endogenously determined matches depend upon the current trading institution. Our model is more general in some respects (we allow for any nite population of players connected through an arbitrary communication network) and particular in others (we stick to a unique −nontrivial− extension of Rubinstein s original model with a single price, so to speak).

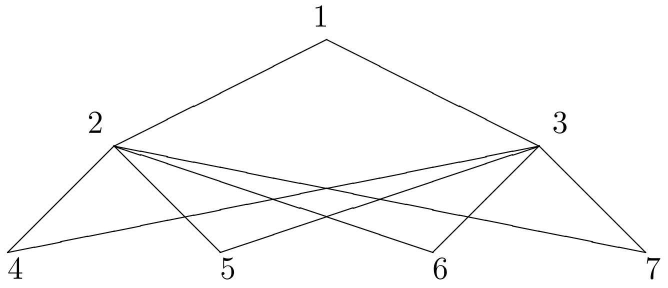

Figure 5.3: a graph with three strata.

instance). For strati ed graphs, though, the rationale for this behavioral rule-of-thumb is guaranteed no matter the particular location of players (equivalently, discount factors) in the graph. In other words, the geometry of strati ed graphs is such that the criterium stated in Proposition 2 holds whatever the identity and the individual time preferences of any player at any node.

5.2. Bargaining duplication, rm multiplication and competition

As we have already done previously for the case of homogeneous time preferences, we now modify the selection and bargaining procedure analyzed so far and allow both the initial promoter and any potential associate rejecting a proposal to act separately as different promoters with different bargaining partners. As before, we conduct this analysis for the particular case where four players are located on a line, as depicted in Figure 4.1. We consider the more general case where players have different discount factors.

We already know from Example 3 (Figure 4.1) that in the extreme case where the initial promoter is doomed to inactivity when spoliated of her own idea, and assuming that player 2 is more impatient than player 3 ($\delta_2 < \delta_3$), player 1 may obtain a higher payoff when bargaining with her relatively *more patient but isolated* neighbor (player 3) than with her relatively *more impatient but well-connected* neighbor (player 2).[18] We now show that allowing for rm multiplication and *ex post* competition can reverse this result and reestablish the validity of the rule-of-thumb criterium that consists on selecting as a bargaining partner the relatively more impatient neighbor.

Suppose indeed that player 1 initially makes a proposal to player 2. Player 2 may either accept and get $\delta_2 a_{21} \pi_M$ or refuse, switch over to player 4 and get a share $\delta_2 a_{24}$ of the duopoly pro t π_D, thus ending up with a payoff $\delta_2 a_{24} \pi_D$. If

[18] Conditional on $\delta_4 < \delta_1$ and on $\delta_3 < \delta_2$ is suffices that $a_{13} > a_{42}$ where $a_{ij} = \frac{1-\delta_j}{1-\delta_i \delta_j}$, $i \neq j$. See Example 3 for more details.

player 4 is more impatient than player 1 ($\delta_4 < \delta_1$), $a_{24} > a_{21}$ but when *ex post* competition is tough enough ($\pi_D < \pi_M$) we may still have $\delta_2 a_{24}\pi_D < \delta_2 a_{21}\pi_M$, implying that player 2 never switches over to her alternative partner player 4. The promoter player 1 then clearly selects as bargaining partner her relatively *more impatient and well-connected neighbor* player 2.[19] Therefore, the selection of a bargaining partner is governed both by players preferences, their location in the communication structure and the effect of market competition. Most of the time, it requires a case-by-case study.

6. Conclusion

This paper analyzes the optimal selection of a bargaining partner when communication among players is graph-restricted. Co-bargainer selection is a priori governed by two players attributes: the *relative rates of impatience* of the potential partners and their *relative positions* in the network of communications.

When the population is homogeneous in time preferences, the pairs of players that realize a common project always share the corresponding bene ts according to a standard half-half partition, irrespective of their relative positions in the communication network. All players are identical from the bargaining game viewpoint and promoters do not discriminate among potential neighbors who are worth the same *ex post* whatever their location in the graph. Allowing for rm multiplication and introducing oligopoly competition among the several rms created on the basis of the same initial idea apparently does not alter this result.

When discount factors vary within the population, the equilibrium agreed-upon shares depend both on players relative positions and on their time preferences. The two players attributes matter for the optimal selection of a bargaining partner in a tightly related way. We can nonetheless determine general conditions such that selecting the most impatient neighbor is an equilibrium strategy. For a particular class of networks called strati ed graphs this condition is always met no matter the particular location of players in the network. Unfortunately, this behavioral rule-of-thumb is not always a valid selection criterium. Also, allowing for multiple rm creation may signi cantly alter the equilibrium behavior.

This paper can be seen as a contribution to the literature on outside options. Here players can opt out by switching over to an alternative partner and initiating a new negotiation. The value of this outside option thus corresponds to

[19] Let for instance $\delta_1 = \delta_3 = 0.9$, $\delta_2 = 0.8$ and $\delta_4 = 0.7$. We have seen in Example 3 that in this case player 1 selects her relatively more patient neighbor player 3. Assume now that rms compete à la Cournot with duopoly pro ts $\pi_D = \frac{4}{9}\pi_M$. Then, $\delta_2 a_{24}\pi_D = 0.2424\ldots\pi_M < \delta_2 a_{21}\pi_M = 0.2857\ldots\pi_M$. Therefore, player 2 has no incentives to quit the negotiation table with player 1 who then selects her relatively more impatient neighbor (player 2) as associate.

a bargaining agreement that depends itself on other outside alternatives. The outside options available to one player are related to her set of potential partners or neighborhood, hence to players location in the communication structure. We show that behavior governed by a rule-of-thumb based upon local (neighborhood) characteristics is valid when the population is homogeneous in time preferences, but may be misleading in a more general context. In this latter case players have incentives to select their associate strategically and, apart from some particular situations that we describe, optimal selection requires a full knowledge of the time preferences and the bilateral relationships within the population that is, the structure of interaction.

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A. Proofs

Proof of Proposition 1. The proof is decomposed into the following lemmata:

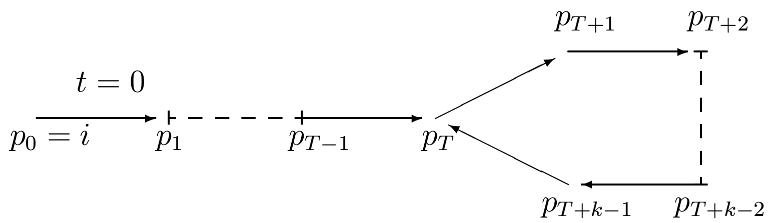
Lemma 1. *For all $g \in \mathcal{G}^*$ and for all $\langle \tau_g, i \rangle$, where τ_g is a selection on g and $i \in N$, $\exists 0 \leq T \leq n$ and $1 \leq \ell \leq n+1-T$ such that $\tau_g^{\ell}(p_T) = p_T$.*

Proof. Suppose not. Then, for all $0 \leq T \leq n$ and for all $1 \leq \ell \leq n+1-T$, $\tau_g^{\ell}(p_T) \neq p_T$. Fix $T = 0$: by letting ℓ vary between 1 and $n+1$ we obtain that $p_1 = \tau_g^1(p_0), ..., p_{n+1} = \tau_g^{n+1}(p_0)$ are all different of p_0 (where $p_0 = i$). Reiterating the process for all $0 \leq T \leq n$ and with ℓ taking all the values between 1 and $n+1-T$, we obtain that $p_0, ..., p_n, p_{n+1}$ are all different two by two, which implies that $|\{p_0, ..., p_{n+1}\}| = n+1$. Impossible because $p_t \in N$ for all $t \in \mathbb{N}$. ■

Lemma 2 (Decomposition). *For all $g \in \mathcal{G}^*$ and for all $\langle \tau_g, i \rangle$, where τ_g is a selection on g and $i \in N$, there exists a unique $0 \leq T^* \leq n$ and a unique $1 \leq \ell^* \leq n+1-T^*$ such that the bargaining sequence $\langle \tau_g, i \rangle$ can be decomposed on:*

1. *a nite-horizon bargaining game on a line: nite succession of proposals from p_t to p_{t+1} for $0 \leq t \leq T^*-1$ where players $p_0, ..., p_{T^*}$ are all different two by two (with $p_0 = i$);*

2. *an in nite-horizon bargaining game on a cycle: in nite succession of proposals from p_t to p_{t+1} for $t \geq T^*$, where $p_t = p_{T^*+(t-T^*) \bmod \ell^*}$ and $p_{t+1} = p_{T^*+1+(t-T^*) \bmod \ell^*}$.*[20]



[20]In other words, the in nite sequence of proposers and respondents from round T on is: $p_T, p_{T+1}, \cdots, p_{T+\ell-1}, p_T; p_{T+1}, \cdots, p_{T+\ell-1}, p_T, \cdots$

Proof. From the previous lemma we know that $\tau_g^{\ell}(p_T) = p_T$ for some $0 \leq T \leq n$ and $1 \leq \ell \leq n+1-T$. Take the smaller T satisfying this property and the corresponding ℓ, denoted respectively by T^* and ℓ^*. Clearly $p_0, ..., p_{T^*}$ are all different two by two because otherwise T^* is not the smaller T satisfying the property of the previous lemma. We also know that $\tau_g^{\ell^*}(p_{T^*}) = p_{T^*}$. Let $t \geq T^*$: $t = T^* + k\ell^* + (t - T^*) \bmod \ell^*$ for some $k \in \mathbb{N}$. Then $p_t = p_{T^*+k\ell^*+(t-T^*) \bmod \ell^*} = \tau_g^{k\ell^*}\left(p_{T^*+(t-T^*) \bmod \ell^*}\right) = p_{T^*+(t-T^*) \bmod \ell^*}$. ■

We analyze the subgame perfect equilibria of this two type of games. Let g be a connected graph on N, and let $\langle \tau_g, i \rangle$ be a bargaining sequence, where τ_g is a selection on g and $i \in N$. Let T and ℓ given by the lemma of decomposition: T is the length of the nite-horizon bargaining game from round $t = 0$ until round $t = T - 1$, and ℓ is the length of the cycle supporting the in nite-horizon bargaining game beginning at round $t = T$. Note that necessarily $\ell \geq 2$. Indeed, ℓ is such that $\tau_g^{\ell}(p_T) = p_T$ and by de nition of the selection function $\tau_g(p_T) \subset N_{p_T}(g) \subset N \backslash \{p_T\}$, implying that $\tau_g(p_T) \neq p_T$. Note also that when $\ell = 2$, the in nite-horizon bargaining game is a standard 2−player alternating offers game between the players labeled p_T and p_{T+1}, beginning at round $t = T$.

We introduce some useful notations. For any two rounds t and s ($t \geq T$, $s > 0$), $\omega^{\tau_g}(t, t+s)$ denotes the product of the discount factors of the players that respond to offers from round t to round $t+s-1$ (or, equivalently, that make offers from round $t+1$ to round $t+s$):

$$\omega^{\tau_g}(t, t+s) = \delta_{p_{t+1}} \times \cdots \times \delta_{p_{t+s}} = \prod_{i=1}^{s} \delta_{p_{t+i}}.$$

Let also $a_t(g) = \frac{1}{1-\omega^{\tau_g}(0,2\ell)} \left[1 + \sum_{i=1}^{2\ell-1} (-1)^i \omega^{\tau_g}(t, t+i) \right]$, for all $t \geq T$. Finally, denote by $\Delta_n = \{(x_1, \ldots, x_n) \mid x_1 + \cdots + x_n = 1; x_1, \ldots, x_n \geq 0\}$ the unit simplex of $\mathbb{R}^n$.

Lemma 3. *The in nite-horizon bargaining game on the cycle of length ℓ beginning at round T has a unique perfect equilibrium outcome supported by the following set of stationary strategies, where offers made at any stage correspond to the perfect equilibrium outcome and players agree to any offer of at least this amount: at round $t \geq T$, player $p_t (= p_{T+(t-T) \bmod \ell})$ proposes the share $1 - a_t(g)$ to player p_{t+1} and accepts any offer $x \in \Delta_n$ from player p_{t-1} as long as $x_{p_t} \geq a_t(g)\delta_{p_t}$.*

Proof. It is easy to check that the strategies described above are a subgame perfect equilibrium. We follow Shaked and Sutton (1984) method to show uniqueness, i.e. we show that the in mum and the supremum of the set of equilibria payoffs coincide. For all $t \geq T$, let $\mathfrak{S}_t$ be the set of perfect equilibrium payoffs of player p_t at period t for the subgame where p_t is the initiator of the bargaining procedure (subgame beginning at round t). Let $S_t = \sup \mathfrak{S}_t$ and $s_t = \inf \mathfrak{S}_t$, $\forall t = 0, 1, \ldots$. We establish the proof in four steps.

Step 1. $S_t \leq 1 - \delta_{p_{t+1}} s_{t+1}$, $\forall t \geq T$

Proof. At round $t+1$, the proposer p_{t+1} can get at least s_{t+1} which corresponds to a discounted payoff of $\delta_{p_{t+1}} s_{t+1}$ at round t. Hence, at round t the proposer p_t can not offer less than $\delta_{p_{t+1}} s_{t+1}$ to her bargaining partner (respondent) p_{t+1} for the offer to be accepted. Therefore, at round t the proposer p_t can not get more than a share of $1 - \delta_{p_{t+1}} s_{t+1}$ that is, $S_t \leq 1 - \delta_{p_{t+1}} s_{t+1}$. *Q.E.D.*

Step 2. $s_t \geq 1 - \delta_{p_{t+1}} S_{t+1}$, $\forall t \geq T$

Proof. Similar argument than in the previous step. *Q.E.D.*

Step 3. $S_t \leq \frac{1}{1-\omega^{\tau_g}(0,2\ell)} \left[1 + \sum_{i=1}^{2\ell-1} (-1)^i \omega^{\tau_g}(t, t+i) \right]$, $\forall t \geq T$

Proof. Let $t \geq T$. We show by induction on k that:

$$S_t \leq 1 + \sum_{i=1}^{2k-1} (-1)^i \prod_{s=1}^{i} \delta_{p_{t+s}} + \prod_{s=1}^{2k} \delta_{p_{t+s}} \cdot S_{t+2k}, \forall k = 1, 2, \ldots \tag{A.1}$$

Combining the results of the two previous steps we get:

$$S_t \leq 1 - \delta_{p_{t+1}} \left(1 - \delta_{p_{t+2}} S_{t+2}\right) \Leftrightarrow S_t \leq 1 - \delta_{p_{t+1}} + \delta_{p_{t+1}} \delta_{p_{t+2}} S_{t+2}$$

Hence, the result is true for $k = 1$. Suppose then that it is true for some $k \geq 1$. We show that it still holds at $k + 1$. Indeed, combining again the results of steps 1 and 2, $S_{t+2k} \leq 1 - \delta_{p_{t+2k+1}} + \delta_{p_{t+2k+1}} \delta_{p_{t+2(k+1)}} S_{t+2(k+1)}$. We then obtain a new upper bound for S_t:

$$S_t \leq 1 + \sum_{i=1}^{2k-1} (-1)^i \prod_{s=1}^{i} \delta_{p_{t+s}} + \prod_{s=1}^{2k} \delta_{p_{t+s}} \left(1 - \delta_{p_{t+2k+1}} + \delta_{p_{t+2k+1}} \delta_{p_{t+2(k+1)}} S_{t+2(k+1)}\right)$$
$$\Leftrightarrow S_t \leq 1 + \sum_{i=1}^{2k+1} (-1)^i \prod_{s=1}^{i} \delta_{p_{t+s}} + \prod_{s=1}^{2(k+1)} \delta_{p_{t+s}} \cdot S_{t+2(k+1)}$$

Hence, if the inequality is true at k it still holds at $k+1$. Therefore, by induction, the inequality $(A.1)$ holds for all integer $k \geq 1$, and for all integer $t \geq T$. We know from the description of the bargaining procedure that for all $t \geq T$, $p_{t+\ell} = p_t$. Indeed, after ℓ rounds all the players on the cycle have been successively a proposer and a responder, and a new complete tour of bilateral bargaining contests begins. Also, for all $t \geq T$, $p_{t+2\ell} = p_t$.[21] Therefore, $S_{t+2\ell} = S_t$, $\forall t \geq T$. Then, if we let $k = \ell$ in $(A.1)$ we obtain:

$$S_t \left(1 - \prod_{s=1}^{2\ell} \delta_{p_{t+s}}\right) \leq 1 + \sum_{i=1}^{2\ell-1} (-1)^i \prod_{s=1}^{i} \delta_{p_{t+s}}$$

And with the notation introduced at the beginning of this section:

$$S_t \leq \frac{1}{1-\omega^{\tau_g}(0,2\ell)} \left[1 + \sum_{i=1}^{2\ell-1} (-1)^i \omega^{\tau_g}(t,t+i)\right], \forall t \geq T. \qquad \text{Q.E.D.}$$

Step 4. $s_t = S_t = \frac{1}{1-\omega^{\tau_g}(0,2\ell)} \left[1 + \sum_{i=1}^{2\ell-1} (-1)^i \omega^{\tau_g}(t,t+i)\right], \forall t \geq T.$

Proof. With a similar argument than in the previous step we show that:

$$s_t \geq \frac{1}{1-\omega^{\tau_g}(0,2\ell)} \left[1 + \sum_{i=1}^{2\ell-1} (-1)^i \omega^{\tau_g}(t,t+i)\right], \forall t \geq T. \qquad \text{Q.E.D.}$$

The result follows immediately as, by de nition, $s_t \leq S_t$, $\forall t \geq T$. ■

Lemma 4. *The nite-horizon bargaining game on the line from round* 0 *to round* $T-1$ *has a unique perfect equilibrium outcome. At this equilibrium players are indifferent between their share as a respondent and their share as a delayed proposer.*

Proof. By backward induction from round $T-1$. ■

Proof of Corollary 1. First, when the population is homogeneous in time preferences, at the unique perfect equilibrium of the in nite-horizon bargaining game on a cycle, all players make the same standard cake division proposal $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ to their bargaining partner. Indeed, for all $t \geq T$ and $s > 0$, $\omega^{\tau_g}(t, t+s) = \delta^s$. Therefore, $a_t(g) = \frac{1-\delta+\cdots-\delta^{2\ell-1}}{1-\delta^{2\ell}} = \frac{1}{1+\delta}$. Second, at the unique

[21] If ℓ is even, we do not need to make two complete tours of the cycle ($k = \ell$). One such tour (i.e. $k = \ell/2$) suffices and we end up with the reduced expression $a_t(g) = \frac{1}{1-\omega^{\tau_g}(0,\ell)} \left[1 + \sum_{i=1}^{\ell-1} (-1)^i \omega^{\tau_g}(t,t+i)\right]$, for all $t \geq T$.

perfect equilibrium of the nite-horizon bargaining game on a line, all players make the same standard cake division proposal $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ to their bargaining partner. Indeed, the player labeled p_T can ensure $\frac{1}{1+\delta}$ at the round T by initiating the in nite-horizon bargaining game on the cycle. Therefore, at round $T-1$ the player labeled p_{T-1} offers p_T a share $1 - a_{T-1} = \frac{\delta}{1+\delta} \Leftrightarrow a_{T-1} = \frac{1}{1+\delta}$. The proof then follows by backward induction. ■

Proof of Proposition 2. We now consider the general case of a population of players heterogeneous in time preferences. We assume without loss of generality that $\delta_1 < \cdots < \delta_n$. Given a connected graph g on N, we denote by τ_g^* the selection function where all players select their most impatient neighbor that is, $\tau_g^*(i) = \arg\min\{\delta_k \mid k \in N_i(g)\}$, $\forall i \in N$.

Lemma 5. *For all $i \in N$, the in nite-horizon bargaining game component of the bargaining sequence $\tau_g^*, i\rangle$ is such that $\ell = 2$ (standard 2−player alternating offers game between the players labeled p_T and p_{T+1}).*

Proof. Suppose not. Then $\ell \geq 3$. Assume for simplicity that $\ell = 3$. Then $\tau_g^*(p_T) = p_{T+1}$, $\tau_g^*(p_{T+1}) = p_{T+2}$ and $\tau_g^*(p_{T+2}) = p_T$, implying that $p_{T+1} \in N_{p_T}(g)$, $p_{T+2} \in N_{p_{T+1}}(g)$ and $p_T \in N_{p_{T+2}}(g)$. Therefore, by transitivity of the neighborhood relationship, $\{p_{T+1}, p_{T+2}\} \subset N_{p_T}(g)$, $\{p_T, p_{T+2}\} \subset N_{p_{T+1}}(g)$ and $\{p_T, p_{T+1}\} \subset N_{p_{T+2}}(g)$. By de nition of τ_g^* we then get $\delta_{p_{T+1}} < \delta_{p_{T+2}}$, $\delta_{p_{T+2}} < \delta_{p_T}$ and $\delta_{p_T} < \delta_{p_{T+1}}$, which is impossible. ■

Lemma 6. *For all $i \in N$, the nite-horizon bargaining game component of the bargaining sequence $\tau_g^*, i\rangle$ is such that for all $t \leq T$, the unique perfect equilibrium outcome of the proposer (player labeled p_t) decreases with the discount factor of her respondent (player labeled p_{t+1}) that is, $\frac{\partial a_t}{\partial \delta_{p_{t+1}}} < 0$.*

Proof. Let $i \in N$. Let T and ℓ given by the lemma of decomposition. We know from the previous Lemma that $\ell = 2$ that is, players p_T and p_{T+1} play a standard alternating offers game. At the unique perfect equilibrium of this game player p_T gets a share $a_T = \frac{1-\delta_{p_{T+1}}}{1-\delta_{p_T}\delta_{p_{T+1}}}$. This payoff satis es $\frac{\partial a_T}{\partial \delta_{p_{T+1}}} = -\frac{1-\delta_{p_{T+1}}}{\left(1-\delta_{p_T}\delta_{p_{T+1}}\right)^2} < 0$. Hence, the result is true for $t = T$ (and, trivially, for all $t \leq T$ when $T = 0$). Suppose that $T \geq 1$. We have to show that $\frac{\partial a_t}{\partial \delta_{p_{t+1}}} < 0$, $\forall t \leq T - 1$. We rst show that for all $t \leq T-1$, $a_t = f(\delta_{p_{t+1}}, \ldots, \delta_{p_T}, \delta_{p_{T+1}})$. Indeed, at round $t = T-1$ the unique perfect equilibrium share a_{T-1} of player p_{T-1} is such that this player (the proposer) concedes to her bargaining partner p_T (the respondent) the expected equilibrium outcome a_T (appropriately discounted) this respondent can get in the continuation game. Formally, $1 - a_{T-1} = \delta_{p_T} a_T \Leftrightarrow a_{T-1} = 1 - \delta_{p_T} a_T$. But $a_T = \frac{1-\delta_{p_{T+1}}}{1-\delta_{p_T}\delta_{p_{T+1}}} = f(\delta_{p_T}, \delta_{p_{T+1}})$. Therefore, $a_{T-1} = f(\delta_{p_T}, \delta_{p_{T+1}})$. Reiterating the process at $t = T-2$, we get $a_{T-2} = f(\delta_{p_{T-1}}, \delta_{p_T}, \delta_{p_{T+1}})$. By backward induction $a_t = f(\delta_{p_{t+1}}, \ldots, \delta_{p_T}, \delta_{p_{T+1}})$, $\forall t \leq T-1$. At round $t = T-1$, $a_{T-1} = 1 - \delta_{p_T} a_T$ with $a_T = \frac{1-\delta_{p_{T+1}}}{1-\delta_{p_T}\delta_{p_{T+1}}}$. Therefore $a_{T-1} = \frac{1-\delta_{p_T}}{1-\delta_{p_T}\delta_{p_{T+1}}}$, implying $\frac{\partial a_{T-1}}{\partial \delta_{p_T}} = -\frac{1}{\left(1-\delta_{p_T}\delta_{p_{T+1}}\right)^2} < 0$. Assume that $T \geq 2$ and let $t \leq T-2$. We know that $1 - a_t = \delta_{p_{t+1}} a_{t+1} \Leftrightarrow a_t = 1 - \delta_{p_{t+1}} a_{t+1}$. Moreover, $a_{t+1} = f(\delta_{p_{t+2}}, \ldots, \delta_{p_T}, \delta_{p_{T+1}})$. Therefore, $\frac{\partial a_t}{\partial \delta_{p_{t+1}}} = -a_{t+1} < 0$, $\forall t \leq T-2$. Hence, $\frac{\partial a_t}{\partial \delta_{p_{t+1}}} < 0$, $\forall t \leq T-1$. ■

Let $g \in \mathcal{G}^*$ such that for all $i \in N$, $\tau_g^*(j) = \tau_g^*(k)$, $\forall j, k \in N_i(g)$. Let $i \in N$ such that $n_i(g) \geq 2$. We want to show that, conditional on all players in the population selecting their relatively more impatient neighbor, i prefers to do so. Formally, let $\tau_g(i)$ be any remaining neighbor in $N_i(g) \backslash \{\tau_g^*(i)\}$. By de nition, both $\tau_g^*(i)$ and $\tau_g(i)$ belong to the neighborhood of player i that is, $\left\{\tau_g^*(i), \tau_g(i)\right\} \subset N_i(g)$, these two players are different that is, $\tau_g^*(i) \neq \tau_g(i)$, and $\tau_g^*(i)$ is relatively more impatient than $\tau_g(i)$ that is, $\delta_{\tau_g^*(i)} < \delta_{\tau_g(i)}$. Denote by a_i^* (resp. a_i) player i s payoff at the unique subgame perfect equilibrium outcome of $\tau_g^*, i\rangle$ (resp. $\tau_g(i), \tau_g^*(-i), i\rangle$). The connected graph g is such that for all $i \in N$, $\tau_g^*(j) = \tau_g^*(k)$, $\forall j, k \in N_i(g)$. Therefore, $\tau_g^*(\tau_g^*(i)) = \tau_g^*(\tau_g(i))$, implying that the bargaining sequences $\tau_g^*, i\rangle$ and $\tau_g(i), \tau_g^*(-i), i\rangle$ coincide from period $t = 1$ onwards. Therefore the equilibrium outcome a_i of the initiator corresponding to $\tau_g(i), \tau_g^*(-i), i\rangle$ is simply deduced from the equilibrium outcome a_i^* of the same player corresponding to $\tau_g^*, i\rangle$ replacing $\delta_{\tau_g^*(i)}$ in the latter expression by $\delta_{\tau_g(i)}$. We know from the previous Lemma that $\frac{\partial a_i}{\partial \delta_{\tau_g(i)}} < 0$. Moreover, $\delta_{\tau_g^*(i)} < \delta_{\tau_g(i)}$. Therefore, $a_i^* < a_i$. In other words, if every in the population selects her more impatient neighbor when g satis es the property stated in the proposition, it is not pro table for any player to deviate from this pattern of partner selection. The selection function τ_g^* is thus a subgame perfect equilibrium of the selection game. ■

Proofs of Corollaries 2 and 3. Straightforward from Proposition 2. ■

Proof of Proposition 3. The proof is constructive. Assume that $\delta_1 < \cdots < \delta_n$ and let $g \in \mathcal{G}^*$ a tree. We recursively partition the set N of players in the following way. Let $N_0 = N$, $g_0 = g$ and $L_0 = \{i \in N_0 \mid n_i(g_0) = 1\}$: the set of peripheral players in N_0 having only one neighbor according to the tree g_0. This set is non empty. Indeed, $L_0 = \emptyset$ implies that $n_i(g_0) \geq 2$, $\forall i \in N_0$ which in turn implies that g_0 has at least n_0 different links. But g_0 is a tree, equivalent to having exactly $n_0 - 1$ distinct links. Thus, $L_0 \neq \emptyset$.

Let now $N_1 = N \backslash L_0$ and let g_1 deduced from g_0 by cutting the $|L_0|$ links connecting the peripheral players in L_0 to players in N_1. By construction, g_1 is a tree and $g \subseteq g^{N_1}$. If $n_1 \geq 2$ let $L_1 = \{i \in N_1 \mid n_i(g_1) = 1\}$. As before, $L_1 \neq \emptyset$. If $n_1 \leq 1$ the decomposition algorithm stops. Note that $n_1 = 1$ is equivalent to $g_0 = g$ being a star-shaped communication structure with $n_0 - 1 = n - 1$ peripheral players in L_0, while $n_1 = 0$ is only possible if $n_0 = n = 2$. We proceed recursively until we reach a step $d \geq 1$ where $n_d \leq 1$ (equivalent to g_{d-1} being a star-shaped communication structure with $n_d - 1 \geq 2$ peripheral players or to $n_{d-1} = 2$). Such a d exists because while $n_k \geq 2$ the set of peripheral players $L_k \neq \emptyset$ and thus the reduced set $N_{k+1} = N_k \backslash L_k$ is strictly included in N_k, that is $N_{k+1} \subset N_k$ and $n_{k+1} < n_k$.

By construction, $N = \underset{k \leq d}{\cup} L_k$ where the $\{L_k\}_{k \leq d}$ are disjoint two by two. In other words, $\{L_k\}_{k \leq d}$ is a partition of the nite population N of players. Let $l_k = |L_k|$, $\forall k < d$.[22] We now locate players in the tree in the following way. The l_0 players with the higher discount factors $\delta_n, \ldots, \delta_{n-l_0+1}$ are assigned to nodes in L_0; the l_1 forthcoming players with higher discount factors $\delta_{n-l_0}, \ldots, \delta_{n-l_0-l_1+1}$ are assigned to nodes in L_1; and so on until we reach L_d that is either empty -in which case all players in N have already been located- or contains a single node to which we assign the (remaining) player with the lowest discount factor δ_1.

Now, two different players i and j have a neighbor in common if and only if $i, j \in L_k$ for some $k < d$ when $l_d = 1$, or some $k < d - 1$ when $l_d = 0$. Suppose that this is the case. By construction, this common neighbor belongs L_{k+1}. But g_k is a tree and no cycle is permitted. Therefore, i and j have a unique common neighbor in L_{k+1}. Moreover, also by construction, both $N_i(g) \subseteq L_{k-1} \cup L_{k+1}$ and $N_j(g) \subseteq L_{k-1} \cup L_{k+1}$. But, $\min \{\delta_i \mid i \in L_{k-1}\} > \max \{\delta_i \mid i \in L_{k+1}\}$. Therefore, i (resp. j) most impatient neighbor $\tau_g^*(i) \in L_{k+1}$ (resp. $\tau_g^*(j) \in L_{k+1}$), implying that $\tau_g^*(i) = \tau_g^*(j)$. Consequently, any two players that have a neighbor in common also have in common their most impatient neighbor. ■

[22] L_d is such that either $l_d = 0$ (when $l_{d-1} = 2$) or $l_{d-1} = 1$ (when g_{d-1} is a non-degenerated star meaning that $l_{d-1} > 2$).