Demand Uncertainty, Mismatch, and (Un)Employment*

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Abstract

We consider a finite number of heterogeneous firms which compete imperfectly for heterogeneous workers. Firms produce a homogeneous good sold on a competitive market. We first develop a model in which unemployment arises because of the mismatch between workers' skills and firms' job requirements. We then extend this model to the case in which firms face demand-induced price fluctuations. It is shown that unemployment may arise in equilibrium because of both uncertainty on product demand and job mismatch. However, unemployment does not arise when the variance of the demand shock is small enough and/or the cost of mismatch is sufficiently low. Finally, we show that instituting a minimum wage, decreasing unemployment benefits, or subsidizing training costs leads to a reduction in unemployment.

Keywords: mismatch, demand shock, unemployment, minimum wage, public policies.

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1 Introduction

There seems to be a large agreement in the economics profession to think that unemployment in European countries is due to the combination of several factors. Our purpose is therefore to investigate how such a combination may generate equilibrium unemployment.

It is widely accepted that one of the main explanations for unemployment is the larger uncertainty prevailing on product demand due to increases in consumers' idiosyncracies. This idea has been developed within the framework of implicit contract theory with the aim of explaining wage rigidity and, in turn, unemployment for some realizations of demand (see, e.g. Rosen, 1985; Stiglitz, 1986; Haley, 1990). Another reason for unemployment which has been put forward more recently is the possibility of mismatch between firms and workers. Indeed, since heterogeneous workers search for jobs and since heterogeneous firms post vacancies to recruit new employees, the matching is not perfect because of the heterogeneity on both sides of the labor market (see, e.g. Drèze and Bean, 1990; Pissarides, 1990; Layard, Nickell and Jackman, 1991). In this paper, we bring together some of the main ideas which can be found in these two strands of labor economics within a partial equilibrium framework.

When it is recognized that firms and workers are heterogeneous, it is reasonable to assume that the process of job matching drives the formation of wage in the labor market. Specifically, workers have heterogeneous skills while firms have differentiated requirements. Indeed, as shown by Stevens (1994), firms have an incentive to differentiate their skill requirements in order to obtain monopsony power in the labor market. Workers will be hired by firms for which the job matching is as good as possible because workers are able to observe firms' skill requirements and because they pay for the cost of training. As a result, firms have oligopsonistic power on the labor market which allows them to charge wages lower than the competitive level. *When the training costs are large and/or the volatility of price fluctuations high, we show that, in equilibrium, unemployment arises on the labor market.* Indeed, firms are able to set wages below marginal productivity because (i) they can use their monoposony power on workers who have a good match in the labor market and (ii) they insure workers against the risk inherent to the product market by paying them a wage independent of demand shocks.

In this way, we are able to uncover some of the microeconomic ingredients that belong to the 'black boxes' used by Pissarides and others. We also provide some reconciliation between the different approaches to unemployment mentioned above in that we show that *both mismatch and random shocks combine to increase unemployment.* The relative importance of both explanations is a matter of empirical analysis. For example, the work of Lillien and Hall (1986) and of Manacorda and Petrogonlo (1996) shows that both explanations are relevant, depending on the particular country under consideration.

Finally, we contemplate different policies financed by taxes on profits aiming at reducing unemployment. We show that imposing a minimum wage above the monopsony wage, subsidizing training costs and/or lowering unemployment benefits decreases unemployment.

The remainder of the paper is organized as follows. The model with skill mismatch is introduced in the next section. In section 3, we determine the labor market equilibrium when both skill mismatch and price fluctuations are introduced. Section 4 deals with policy implications while section 5 concludes the paper.

2 Labor market equilibrium under skill mismatch

Consider an industry with n firms producing a homogeneous good sold on a competitive market (we take this good as the numeraire). A firm is fully described by the type of worker it needs. This means that a job is a collection of tasks determined only by the technology used by the firm (Lazear, 1995). Firm i's $(= 1, ...n)$ skill requirement is denoted by x_i. Labor is the only input and production involves constant returns to scale.

There is a continuum of workers with the same level of general human capital but they have heterogeneous skills. Workers are heterogeneous in the type of work they are best suited for, but there is no ranking in any sense of these types of work. Hence, once unemployed, workers get the same level of unemployment benefit $b > 0$. Workers' skill types are denoted by x. The characteristics of a worker relevant to firms are summarized by her skill. Finally, each worker supplies one unit of labor provided that her wage net of training costs (her earnings) is greater than or equal to b.

Each firm has a specific technology such that workers can produce output only when they perfectly match the firm's skill needs. Since workers are heterogeneous, they have different matchings with the firm's job offer. Thus, if firm i hires a worker whose skill differs from x_i, the worker must get trained and her cost of training to meet the firm's skill requirement is a function of the difference between the worker's skill x and the skill needs x_i. We assume that workers pay for all the costs of training.

For simplicity, the skill space is described by the circumference C of a circle which has length L. Individuals' skills are continuously and uniformly distributed along this circumference; the density is constant and denoted by Δ. The density Δ expresses the thickness of the market, whereas L is a measure of the heterogeneity of workers. When the population of workers is heterogeneous, the extent of the labor market must be described by these two parameters. We will see that they play different roles in the labor market equilibrium. Firms' job requirements x_i are equally spaced along the circumference C so that L/n is the distance between two adjacent firms in the skill space.[1]

2.1 Full employment equilibrium

The working of the labor market is now described for the full employment case (the condition for such a situation to arise will be given below). As stated above, workers need to be trained within the firm. The aim of this training is to give the worker the skill corresponding to the firm's requirement. Thus, after this training, the matching is perfect and, without loss of generality, the worker then produces q units of the output. The more distant the skill of a worker from the firm's skill requirement, the larger the training cost. More precisely, the training cost is given by a linear function $s\,|x - x_i|$ of the difference between the worker's skill x and the firm's skill requirement x_i, where $s > 0$ is a parameter inversely related to the efficiency of the training process. After training, all workers are identical from the firm's viewpoint since their *ex post* productivity is observable and equal to q by convention.

[1] By analogy with what has been shown on a differentiated product market, the equidistant configuration of technologies is likely to be an equilibrium outcome of a game in which firms would choose their technologies prior to their wages (see, e.g. Economides, 1989; Kats, 1995).

Consequently, each firm i offers a wage to all workers, conditional on the worker having been trained to the skill x_i. Indeed, since all workers who have received specific training are *ex post* identical in the eyes of the firm, it offers them the same wage. Each worker then compares the wage offers of firms and the required training costs; she simply chooses to work for the firm offering the highest wage net of training costs. The training costs and the wages are in present value terms (we have collapsed the model to a single period for simplicity).

Firms choose simultaneously their wage level, $(w_1, .., w_i, .., w_n)$. The net wage is therefore equal to $w_i - s\left|x - x_i\right|$. Firms understand that workers choose to be hired by the firms which give them the highest net wage. As a result, they hire all the workers who want to work at the prevailing wages, $(w_1, .., w_i, .., w_n)$, since they know that these workers are willing to adjust to their skill requirement. Furthermore, w_i cannot exceed the productivity q for otherwise firm i would make a negative profit.

Let i be the representative firm. Given two wages w_{i-1} and w_{i+1}, firm i's labor pool is composed of two sub-segments whose outside boundaries are given by marginal workers $\overline{x}$ and $\overline{y}$ for whom the net wage is identical between firms $i-1$ and i, on the one hand, and firms i and $i+1$, on the other. In other words, $\overline{x}$ is the solution of the equation:

$$w_i - s(x_i - \overline{x}) = w_{i-1} - s(\overline{x} - x_{i-1})$$

so that

$$\overline{x} = \frac{w_{i-1} - w_i + s(x_i + x_{i-1})}{2s} \tag{1}$$

In this case, firm i attracts workers whose skills belong to the interval $[\overline{x}, x_i]$ because the net wage they obtain from firm i is higher than the one they would obtain from firm $i-1$. Clearly, workers belonging to the interval $[x_{i-1}, \overline{x}]$ are hired by firm $i-1$. In a similar way, we show that:

$$\overline{y} = \frac{w_i - w_{i+1} + s(x_i + x_{i+1})}{2s} \tag{2}$$

Firm i's labor pool thus consists of all workers with skill types in the interval $[\overline{x}, \overline{y}]$. Its profits are given by:

$$\Pi_i = \int_{\overline{x}}^{\overline{y}} \Delta(q - w_i)dx = \Delta(q - w_i)(\overline{y} - \overline{x}). \tag{3}$$

For an exogenous number of firms, wages and profits at the Nash equilibrium can be determined as follows.[2]

Proposition 1 *In an equilibrium with skill mismatch in which all workers are employed, the wage is given by*

$$w_m^F = q - sL/n \tag{4}$$

and profits per firm are

$$\Pi_m^F = \frac{sL^2}{n^2} \tag{5}$$

Such an equilibrium exists when $q > b + 3sL/2n$ holds.

Proof. The profit function is continuous in (w_{i-1}, w_i, w_{i+1}) and concave in w_i. We can therefore guarantee that there exists a Nash equilibrium in wages. With firms located symmetrically, we find the Nash equilibrium wages by taking the first-order condition for Π_i with respect to w_i:

$$\frac{\partial \Pi_i}{\partial w_i} = -(\overline{y} - \overline{x}) + (q - w_i)\left(\frac{\partial \overline{y}}{\partial w_i} - \frac{\partial \overline{x}}{\partial w_i}\right) = 0. \tag{6}$$

From (1), (2) and (6), and setting equilibrium wages equal to each other, we obtain our formula. This solution is unique, since the first-order conditions are a system of linear equations in the wage of each firm. When $q > b + 3sL/2n$ the worker with the worst match whose training cost is $sL/2n$ will choose to work so that all workers are employed. ■

When there is full employment, workers who receive less training also receive higher net wages. Though workers have the same level of general human capital and the same ex post productivity, they incur different training costs because of different matches. In addition, since firms do not discriminate between workers on the basis of their type, those with a better match end up with a higher net wage. This is easy to understand when it is recognized

[2] Superscripts F and U refer to the full employment case and the unemployment case respectively. Subscript m refers to the skill mismatch case whereas subscript f refers to the case where both skill mismatch and price fluctuations exist (section 3).

that both firms and workers are ex ante heterogeneous and that firms cannot discriminate in wages between workers of different skill types. Furthermore, as the number of firms increases, equilibrium wages rise because adjacent firms compete for workers who have better matches. When the number of firms becomes arbitrarily large, the wage tends to the competitive level q, while profits tend to zero. *The competitive model of the labor market is thus the limit of the spatial model of job assignment.* Finally, when q increases, gross productivity rises while the training cost of each worker decreases. As a result, the net wage increases with the level of general human capital, as supported by many empirical studies, while profits decrease because firms lose some of their monopsony power, an effect that overcomes the gain in productivity.

2.2 Unemployment equilibrium

We now consider an economic environment in which not all workers take a job (the condition for such a situation to arise will be given below). Consequently, each firm acts as a monopsony on the labor market. The corresponding outer boundaries of its labor pool $\widehat{x}$ and $\widehat{y}$ are such that $\widehat{y} - \widehat{x} = 2(w_i - b)/s$. The profit function of a monopsony firm i is thus given by:

$$\Pi_i = (q - w_i)\frac{2w_i}{s} \tag{7}$$

We have the following result:[3]

Proposition 2 *In an equilibrium with skill mismatch and unemployment, the equilibrium wage is given by*

$$w_m^U = \frac{q+b}{2} \tag{8}$$

profits per firm are

$$\Pi_m^U = \frac{q^2 - b^2}{2\,s} \tag{9}$$

[3]There is in fact another case in which $b + sL/n \leq q \leq b + 3sL/2n$. In this case, the labor pools just touch and all workers are hired at a wage $b + sL/2n$.

and the equilibrium level of unemployment is equal to:

$$u_m = \Delta \left[L - n \; S_m \right] = \Delta \left(L - n \frac{q-b}{s} \right) \tag{10}$$

Such an equilibrium exists when $q < b + sL/n$ holds.

Proof. The profit function is strictly concave in w_i so that there exists a unique solution. By taking the first order condition of (7), we easily obtain (8). Plugging (8) into (7) yields (9). Finally, the equilibrium labor supply of each firm is given by

$$S_m = \frac{2(w_0^M - b)}{s} = \frac{q-b}{s} \tag{11}$$

which leads to (10). ■

Since each firm acts as a monopsonist, the unemployment benefit affects positively both the monopsony wage and the unemployment level because, when b rises, workers are more reluctant to take a job and thus firms' monopsony power decreases. In the same manner, when the productivity q rises, wage increases but unemployment decreases. These results shed some light on the nature of unemployment. On the one hand, unemployment is voluntary because some workers refuse to take a job at the wage offered by firms. On the other, this unemployment can also be viewed as involuntary since the productivity of the unemployed is higher than their reservation wage which amounts here to their training costs (Lindbeck, 1993, p.47). In such a context, the concept of voluntary unemployment becomes ambiguous and fuzzy. In a perfectly competitive market, these workers would be employed since their net wage would be positive. Indeed, imagine that at each location x_i there is not one but two firms. This will obviously leads to a Bertrand competition so that wages equal marginal productivity, i.e., $w^C = q$ and profits are null (*perfect competition*). In this case, the labor supply is given by $S_c = 2(w^C - b)/s = 2(q-b)/s$ so that $S_c = 2.S_m$, i.e. the labor supply in the competitive case is exactly twice the one in the case of monopsony. In terms of unemployment, it is easily checked that[4] $u_c = u_m - \Delta \; n \; (q-b)/s$ so that unemployment is reduced when firms have less marker power. This

[4]Since we want to compare two situations with the same number of firms n, we assume in the competitive case that there two firms at each location x_i but only $n/2$ locations in the economy, i.e. $i = 1, ..., n/2$.

means that, first, *unemployment is principally due to firms' market power* and, second, some workers are *unemployable* because, *even at the competitive wage, they are too far away from firms' job requirements and prefer not to take a job.* In other words, it because workers' skills are too far from those needed and but also because firms exploit their monopsonistic power on the labor market that some workers are unemployed.

It is now interesting to extend this framework by allowing prices in the product market to randomly fluctuate. We will see how this will affect the unemployment level in this economy.

3 Labor market equilibrium under skill mismatch and price fluctuations

Consider the same industry formed by n firms and a continuum of workers. We now assume that firms which produce the homogeneous good sold in a competitive market *face demand-induced price fluctuations.* To express the resulting uncertainty, we suppose that the market price $\widetilde{p}$ is described by a random variable whose mean is chosen to be 1 (without loss of generality) while its variance is σ^2. As in Sandmo (1971), greater price uncertainty is measured by an increase in σ^2, that is, a mean preserving spread in prices.

Consequently, firm i's labor pool is still defined by the interval $[\overline{x}, \overline{y}]$ for the full employment case and $[\widehat{x}, \widehat{y}]$ for the unemployment case. For the full employment case, firm i's profits gross of fixed cost can the be written as:

$$\widetilde{\Pi}_i = \int_{\overline{x}}^{\overline{y}} \Delta(\widetilde{p}q - w_i)dx = \Delta(\widetilde{p}q - w_i)(\overline{y} - \overline{x}) \tag{12}$$

We assume that firms are risk averse. The main reason for this assumption is that firms make investments before producing (recall that firms bear some fixed cost) so that liquidity constraints may lead a risk neutral firm to behave as if it were risk averse (Drèze, 1987, ch. 15). This argument is supported by empirical studies showing that many firms have an imperfect access to the capital markets, especially when they are not large, and must therefore bear part of the risk associated with their production activity (Fazzari, Hubbard and Petersen, 1988; Evans and Jovanovic, 1989).

In order to obtain closed forms solutions, we further assume that firms have a mean-variance utility function given by:

$$V_i = E(\widetilde{\Pi}_i) - \frac{a}{2} Var(\widetilde{\Pi}_i) \tag{13}$$

where $a \geq 0$ is the degree of absolute risk aversion and where $\widetilde{\Pi}_i$ is defined by (12). Hence, we can rewrite (13) as follows:

$$V_i = \Delta(q - w_i)(\overline{y} - \overline{x}) - \frac{a}{2}\Delta^2(\overline{y} - \overline{x})^2\sigma^2 q^2 \tag{14}$$

Wages and employment are determined before the realization of product prices and are therefore *not* random variables. In this respect, workers are completely insured against price volatility.

As above, two cases may arise. In the former, all workers are employed at the labor market equilibrium. In the latter, some workers do not work in equilibrium.

3.1 Full employment equilibrium

Since all workers take a job, the outer boundaries of firm's labor pool are given by (1) and (2). Hence, the profit function (14) is continuous in (w_{i-1}, w_i, w_{i+1}) and concave in w_i. Therefore, there exists a Nash equilibrium in wages. First order conditions yield:

$$\frac{\partial V_i}{\partial w_i} = \Delta \left[-(\overline{y} - \overline{x}) + (q - w_i)\left(\frac{\partial \overline{y}}{\partial w_i} - \frac{\partial \overline{x}}{\partial w_i}\right)\right] - a\Delta^2\sigma^2 q^2 (\overline{y} - \overline{x})\left(\frac{\partial \overline{y}}{\partial w_i} - \frac{\partial \overline{x}}{\partial w_i}\right) = 0 \tag{15}$$

Combining (1), (2) and (15), and equalizing the equilibrium wages, we obtain (at the symmetric Nash equilibrium)

$$w_f^F = \Phi(q) - \frac{sL}{n} \tag{16}$$

where $\Phi(q) = q(1 - a\Delta\sigma^2 qL/n)$ is a quadratic function of q with $\Phi''(.) < 0$.

It will turn out to be useful to rewrite (16) as follows:

$$q = w_f^F + s\frac{L}{n} + a\Delta\sigma^2 q^2 \frac{L}{n} \tag{17}$$

In this expression, the LHS stands for the expected value productivity of a worker while the RHS is composed by three elements. The first one (w_f^F) is the marginal cost, the second one (sL/n) measures the monopsonistic exploitation of labor and the last one is the risk premium. It is worth noting that this premium increases with the worker productivity q and the density Δ while it decreases with n because the risk is spread over a larger number of firms.

The following comments are in order. First, the skill mismatch case (section 2) is equivalent to the case where firms are risk neutral ($a = 0$). Observe that, when firms are risk-neutral, the worker density Δ has no impact on the equilibrium wage while the equilibrium wage falls with the size of the skill space. However, when firms are risk averse ($a > 0$), increasing Δ has a negative impact on wage. Stated differently, *a larger labor market (both in terms of workers' density and skill space) leads to a lower wage.* This seemingly surprising result can be explained by the fact that each firm commits to hire $1/n$ of the labor force ΔL, regardless of its size, while facing the same uncertainty on the product market. It must then be that the risk premium rises with ΔL and decreases with n, thus implying our results. Second, when firms are risk averse, the equilibrium wage decreases with the degree of risk aversion. Similarly, wages depend on the variance σ^2 of the output price: the larger the variance of the price, the lower the equilibrium wage. In other words, *industries with greater price uncertainty (or more risk averse firms) are likely to charge lower wages* because the risk premium increases (see (17)). This means that, at the full employment equilibrium, firms share with workers the risk generated by price volatility. Last, changing n and s have the same impact as in the case of risk neutral firms. Indeed, w_f^F decreases with s because firms have more market power on the workers whose skills are close to their skill requirement, whereas it increases with n because the average matching is better when the number of firms is larger.

We must now check that at the equilibrium candidate (16) there is full employment and that this wage is always positive. For that, we need the following two expressions which are easy to derive:

$$\widehat{q} = \arg\max_q \Phi(q) = \frac{n}{2a\Delta\sigma^2 L} \tag{18}$$

$$\Phi(\widehat{q}) = \sup_q \Phi(q) = \frac{n}{4a\Delta\sigma^2 L} \tag{19}$$

Proposition 3 *There is full employment and the equilibrium wage (16) is greater than b if*

$$0 < \sigma^2 < \frac{n^2}{2a\Delta L(2nb + 3sL)} \tag{20}$$

Proof. The equilibrium wage (16) is greater than b if:

$$w_f^F > b \quad \Leftrightarrow \quad \Phi(q) > b + \frac{sL}{n} \tag{21}$$

and the condition ensuring that there is full employment at the equilibrium candidate (16) is given by:

$$w_f^F - \frac{sL}{2n} > b \quad \Leftrightarrow \quad \Phi(q) > b + \frac{3sL}{2n} \tag{22}$$

Clearly, (22) implies (21). A sufficient condition for (22) is (see Figure 1 for an illustration):

$$\sup_q \Phi(q) = \Phi(\widehat{q}) > b + \frac{3sL}{2n}$$

where $\Phi(\widehat{q})$ is defined by (19). After some manipulations, this inequality is equivalent to (20). ■

[*Insert Figure* 1 *here*]

Condition (20) insures that under the equilibrium wage (16), there will always be full employment. In other words, if the variance of $\widetilde{p}$ is not too large, then everybody will accept to work at the equilibrium wage, i.e., the net wage is strictly greater than b for all workers. The condition (20) is intuitive since each firm must set a sufficiently high wage to attract all workers in its labor pool. This is possible if the demand is not too volatile. On the other hand, *the existence of big random shocks in market demand necessarily leads to a labor market equilibrium with unemployment.* Observe also that, *ceteris paribus*, condition (20) is more likely to be satisfied if the number of firms n is large and if a, Δ, L, s and b are not too large. Stated differently, if firms are very risk averse or if there are many workers in the labor market or if the unit cost of mismatch is large or if the unemployment benefit is high, there will never be a wage equilibrium with full employment.

It is worth pointing out a difference between the cases of certainty (skill mismatch only) and of uncertainty (skill mismatch and price fluctuations). When $a = 0$, the condition reduces to $q > b + 3sL/2n$ (see Proposition 1), i.e., the productivity of workers must be large enough for the full employment configuration to arise. On the contrary, when $a > 0$, there is full employment for all the values of q such that $\Phi(q) > b + 3sL/2n$, that is, q must belong to the interval $[q_0, q_1]$ described in Figure 1 (the size of this interval depends on the value of the different exogeneous parameters $a, \sigma^2, n, \Delta, L, , s$ and b). This means that full employment occurs when the productivity of a worker takes intermediate values. Indeed, when q is sufficiently large, the risk premium becomes too high for the firms to be able to set wages that sustain full employment. This is rather surprising because one would expect that a rise in workers' productivity is favorable to full employment when the output market is competitive. Because of the uncertainty affecting the output price, firms become reluctant to hire more productive workers because they must pay them a higher wage. Though our model does not deal with differences in qualification across workers, this result seems to be in accordance with recent empirical analyses suggesting that employment of the most skilled workers is fairly sensitive to random shocks (see, e.g. Fonseca, 1996).

Finally, it is readily verified that:

$$V_f^F = \left(\frac{L}{n}\right)^2 \left(s\Delta + \frac{a}{2}q^2\Delta^2\sigma^2\right) > \Pi_m^F \tag{23}$$

Hence the expected utility of profits is higher than profits incurring to firms when there is no uncertainty. Therefore uncertainty endows firms with more market power on the labor market, thus allowing them to expect higher profits.

3.2 Unemployment equilibrium

Let us consider the case of unemployment. As above, the corresponding outer boundaries of its labor pool $\widehat{x}$ and $\widehat{y}$ are such that $\widehat{y} - \widehat{x} = 2(w_i - b)/s$. The expected utility of profit is now written as follows:

$$V_f^U = \Delta\left(q - w_i\right)\frac{2(w_i - b)}{s} - \frac{a}{2}\Delta^2 q^2 \left[\frac{2(w_i - b)}{s}\right]^2 \sigma^2 \tag{24}$$

By taking the first order condition of (24), we easily obtain:

$$w_f^U = \frac{qs + b\left(s + 2a\Delta q^2 \sigma^2\right)}{2s + 2a\Delta q^2 \sigma^2} \tag{25}$$

Observe first that the impact of a and σ^2 on the monopsony wage (25) is the same as for the Nash equilibrium wage (16) and for the same reason. However s now has a positive impact on w_f^U whereas it had a negative one on the Nash equilibrium wage (16). This is because firms no longer compete on the labor market. The training costs being borne by the workers, firms must compensate them when s increases in order to attract enough workers (the labor pool shrinks as s rises). On the contrary, as shown by (1) and (2), the size of the labor pool is independent of s at the full employment Nash equilibrium. Thus, *under uncertain product demand, when the unit the cost of mismatch becomes larger, monopsonistic firms are induced to rise their wages whereas oligopsonistic firms are induced to reduce their wages.* Moreover, as in section 3.1, the monopsony wage (25) also falls with Δ but for a different reason. Indeed, since each firm finds more suitable workers in its vicinity, it can afford to pay a lower wage because workers need a lower compensation for their training cost. Finally, the unemployment benefit affects positively the monopsony wage since, when b rises, workers are more reluctant to take a job and thus firms' monopsony power decreases.

Here also we have to check that for the equilibrium candidate (25) there is unemployment.

Proposition 4 *There is unemployment for the equilibrium wage (25) if*

$$\sigma^2 > \frac{n^2}{4a\Delta L(nb + sL)} \tag{26}$$

Proof. The condition that there is unemployment for the equilibrium candidate (25) is:

$$w_f^U - \frac{sL}{2n} < b \tag{27}$$

By using (25) and after some manipulations, it is easily checked that (27) is equivalent to:

$$q\left(1 - a\Delta\sigma^2 q\frac{L}{n}\right) < b + \frac{sL}{n}$$

where the LHS is exactly $\Phi(q)$, so that (27) reduces to:

$$\Phi(q) < b + \frac{sL}{n}$$

In this context, a sufficient condition for (27) to hold is thus given by (see Figure 2):

$$\sup_q \Phi(q) = \Phi(\widehat{q}) < b + \frac{sL}{n}$$

where $\Phi(\widehat{q})$ is defined by (19). It is readily verified that the condition above is equivalent to (26). ■

[*Insert Figure* 2 *here*]

Proposition 2 shows that the variance of $\widetilde{p}$ must be large enough to guarantee that in equilibrium there will always be unemployment.[5] Indeed, if the demand is not volatile, monopsonistic firms will set high wages and all workers will be willing to work. This captures the idea that both demand shocks and institutions (captured by too high unemployment benefits levels) are responsible for the equilibrium unemployment. Our result is in accordance with the recent literature that put forward 'economic turbulence' or demand shocks and institutions ('labor market rigidities') as the main cause for the rise of European unemployment between the sixties and now (Blanchard, 1999, Blanchard and Wolfers, 1999, Ljungqvist, 1999, Ljungqvist and Sargent, 1998, Marimon and Zilibotti, 1999).

Observe that condition (26) is likely to hold if a, Δ, L, s and b are not low and n not large. Inspecting (25), it is not clear that the equilibrium wage rises with the productivity of workers. Nevertheless, we have:

$$\frac{\partial w_f^U}{\partial q} > 0 \quad \text{iff} \quad q(q-2b) < \frac{s}{a\Delta\sigma^2} \Leftrightarrow \quad \Phi(q) < b + \frac{sL}{n} \tag{28}$$

By using the definition of $\Phi(q)$, it is easily verified that this condition is equivalent to:

$$\Phi(q) < b + \frac{sL}{n}$$

[5]As in the previous section, if the assumptions on the parameters in Propositions 3 and 4 are not satisfied (i.e., $b + sL/n \leq \Phi(q) \leq b + 3sL/2n$), there is still a Nash equilibrium: labor pools just touch and all workers are hired at the wage $b + sL/2n$.

which is always true by Proposition 4.

Since the certainty case is formally equivalent to the case of risk neutral firms, it follows directly from (25) that the equilibrium wage $w_m^U = (q+b)/2$ is strictly larger than w_f^U when a is positive. Moreover, it is easy to check that:

$$V_f^U = \frac{\Delta\,(q-b)^2}{2\,(s + a\Delta q^2\sigma^2)} < \Pi_m^U \tag{29}$$

Unlike the full employment case, the expected utility of monopsony profits is here *lower* than profits incurring to firms when there is no uncertainty. This is because there is no competition to attract workers, each firm being in pure monopsony in its local labor market. So when uncertainty in the product market is introduced, firms set lower (monopsony) wages because their report the risk on workers but, since there is no interaction with the other firms, the labor supply (see $\widehat{y} - \widehat{x}$) is also reduced. The second effect being stronger than the first one, firms end up with lower expected profits.

All this implies that *the level of unemployment rises with the volatility of demand.* Accordingly, this suggests that unemployment has here two different sources that are *not* independent. The former is due to the mismatch of firms and workers whereas the latter is due to the uncertainty affecting the level of price. The first source has already been discussed in the previous section. It was shown that firms' market power in the labor market was the main culprit for explaining unemployment. Let us now come to the second source. We have seen that demand uncertainty leads firms to lower their wages. Stated differently, firms use their market power to transfer the risk of price volatility on workers, thus worsening unemployment. Hence, in our model, it appears that market power gives rise to two forces which combine to raise unemployment.

In this context, it is easily verified by using (25) that the employed workers have 'rents' compared to the unemployed; these rents are comprised between 0 (for the last worker employed who is indifferent between working and not working) and $(q-b)s/[2s+2a\Delta q^2\sigma^2]$ (for the worker with the best match, i.e. no training costs). According to Mortensen (1989), as soon as the employed workers have rents, unemployment must be considered as involuntary. In such a context, the concept of voluntary unemployment becomes even more ambiguous and fuzzy.

When $a > 0$, the *global* labor supply is equal to:

$$S_f = \frac{2(w^M - b)}{s} = \frac{q-b}{s + a\Delta q^2\sigma^2} < S_m \tag{30}$$

Since the total active population is equal to ΔL, the global equilibrium level of unemployment u_f is given by:

$$u_f = \Delta \left[L - n.S_g\right] = \Delta \left(L - n\frac{q-b}{s + a\Delta q^2\sigma^2}\right) > u_m \tag{31}$$

As discussed above, the unemployment (31) is caused by both mismatch between firms and workers and by price fluctuations. If there is no uncertainty in the economy ($a = 0$ and $\sigma^2 = 0$), the (mismatch) unemployment is then given by (10). It is easy to verify that this unemployment level u_m is lower than the one caused by both mismatch and price fluctuation, i.e. u_f, even though wages are higher ($w_m^U > w_f^U$). This suggests that unemployment is amplified when larger uncertainty prevails on product demand because firms report their risk on workers' wages and on labor supply. This also suggests that *unemployment has a Keynesian flavor* because *it finds its origin in large demand shocks.*

Finally, as in the case of mismatch only, unemployment is not necessarily associated with monopsony and that even with perfect competition (i.e. wages equal marginal productivity) there will still be unemployment, even though u_f decreases. The results developed for the perfect competitive case at the end of section 2.2 are more pronounced when firms are risk averse, i.e. $a > 0$. This implies that, first, unemployment is principally due to firms' market power and to uncertainty in the product market, second, some workers are unemployable because, even at the competitive wage, they are too far away from firms' job requirements and prefer not to take a job.

Proposition 5 *When uncertainty in the product market is introduced in a market characterized by job mismatch, unemployment increases and wages and profits are reduced.*

To sum-up, in the case without uncertainty on price, unemployment u_m arises because monopsony firms set low wages, thus inducing some workers to refuse to work. When uncertainty on price rises, monopsonistic firms cut their wages further and hence less workers are induced to work. *Here, contrary to standard results in labor economics,*[6] *reducing the equilibrium wages increases the level of unemployment.*

4 Policy implications

In this section, we want to analyze policies aiming at reducing unemployment. We have seen in section 3 that unemployment arises when exogeneous parameters change values so that one switches from a market situation where $\sigma^2 > n^2/[4a\Delta L(nb+sL)]$ to another where $\sigma^2 < n^2/[2a\Delta L(2nb+3sL)]$. This means that in our model the values of these parameters are crucial for the formation of unemployment. In this context, the government can contemplate different policies that reduce unemployment: imposing a minimum wage, subsidizing the training cost, cutting the unemployment benefits.

4.1 Minimum wage

As stated above, the main cause of unemployment is due to the fact that firms set too low wages in equilibrium so that workers with large training costs prefer to stay unemployed. In this context, the implications of a minimum wage legislation are easy to trace. The government could institute a minimum wage above the monopsony one, w_m^U, defined by (25). This minimum wage would reduce firms' monopsony power and induce more workers to accept 'decent paid' jobs.

Our analysis can shed some lights on the recent debate on the positive effects of the minimum wage in the US (Card and Krueger, 1995) as well as in Europe (Dolado et al., 1996). In particular, Card and Krueger (1995) show empirically that, in some sectors such as the fast-food industry, unemployment would be due to too low wages set by firms exploiting monopsony power. Their conclusion is that a minimum wage should be imposed in these sectors. However, controversies have arisen on how can one measure firms' monopsony power.

Our model suggests a theoretical answer to this question. For a labor market to be characterized by monopsony with unemployment[7] it must be that

[6] See, for example Stiglitz (1986) and Haley (1990).

[7] Observe that we have shown above that unemployment can also prevail when firms

one switches from $\sigma^2 > n^2/\left[4a\Delta L(nb+sL)\right]$ to $\sigma^2 < n^2/\left[2a\Delta L(2nb+3sL)\right]$. In other words, our model gives us to think that when n (number of firms) and/or σ^2 (variance of prices) are relatively low and/or b (unemployment benefit), s (training cost), Δ (workers' density), L (length of the circle) and/or a (firms' degree of risk aversion) are relatively large, then the market structure is more likely to be monopsonistic. Therefore, if the government is able to estimate the size (n, Δ and L) and the characteristics (s, a, σ^2 and b) of a labor market, then one can recommend the institution of a minimum wages in the corresponding sectors.

Proposition 6 *To cut unemployment, the government should impose a minimum wage in sectors for which* $\sigma^2 < n^2/\left[2a\Delta L(2nb+3sL)\right]$ *holds.*

4.2 Training costs

So far, workers bear their entire training costs. Can the government reduce unemployment by subsidizing a part of it? Let us denote by α the share of the training cost borne by workers and by $(1-\alpha)$ that financed by the government $(0 < \alpha < 1)$.

For simplicity we assume that this subsidy α is financed by a lump-sum tax T_α paid by firms. In other words, firms pay T_α each time they hire a worker, irrespective of the worker's type x. In this context, the budget constraint of the government is given by (assume for simplicity that $x_i = 0$):[8]

$$\sum_{i=1}^{n} T_\alpha \ \Delta \ S_{i,\alpha} = 2 \ \sum_{i=1}^{n} (1-\alpha) \ s \int_0^{\widehat{y}_{i,\alpha}} \Delta \ x \ dx$$

where $S_{i,\alpha}$ is firm i's labor supply (which will be denoted by $S_{g,\alpha}$ at the symmetric labor market equilibrium). Out of equilibrium, this equality is equivalent to:

$$T_\alpha \sum_{i=1}^{n} S_{i,\alpha} = (1-\alpha) \ s \ \sum_{i=1}^{n} (S_{i,\alpha})^2 \tag{32}$$

It is shown in the appendix that when a firm maximizes its profit to set its wage, it can takes T_α as given when n is large enough. Since the labor

set competitive wages (i.e. equal to marginal productivity).

[8]Subscript α refers to the training cost analysis.

supply is given by:

$$S_{g,\alpha} = \frac{2(w^U_{f,\alpha} - b)}{\alpha s} \tag{33}$$

the monopsony wage is equal to:

$$w^U_{f,\alpha} = \frac{q\alpha s + b\left(\alpha s + 2a\Delta q^2\sigma^2\right)}{2\left(\alpha s + a\Delta q^2\sigma^2\right)} < w^M \tag{34}$$

so that the equilibrium values of the global labor supply and the global unemployment are respectively given by:[9]

$$S_{g,\alpha} = \frac{q-b}{\alpha s + a\Delta q^2\sigma^2} > S_g \tag{35}$$

$$u_{g,\alpha} = \Delta\left[L - n\frac{(q-b)}{\alpha s + a\Delta q^2\sigma^2}\right] u_g \tag{36}$$

Moreover, the equilibrium (expected) profits are:

$$V^M_\alpha = \frac{\Delta(q-b)^2}{2\left(\alpha s + a\Delta q^2\sigma^2\right)} - T_\alpha \gtreqless V^M \tag{37}$$

However, by using (32), in the symmetric labor equilibrium, the lump-sum tax is equal to:

$$\begin{aligned} T_\alpha &= (1-\alpha)\ s\ S_{g,\alpha} \\ &= (1-\alpha)\ s\ \frac{q-b}{\alpha s + a\Delta q^2\sigma^2} \end{aligned}$$

so that, by using (37), the equilibrium (expected) profit rewrites:

$$V^U_{f,\alpha} = \frac{q-b}{2\left(\alpha s + a\Delta q^2\sigma^2\right)}\left[\Delta\ (q-b) - 2\ (1-\alpha)\ s\right] < V^U_f$$

[9]It is easily checked that the condition ensuring that unemployment exists in equilibrium is:

$$\sigma^2 > \frac{n^2}{4a\Delta L(nb + \alpha sL)}$$

so that this condition is more likely to be verified when the government finances a small part $1-\alpha$ of the training costs. This is quite intuitive since if the government bears a very large part of the training costs then all workers will be willing to take a job and there will be no unemployment in equilibrium.

So when the government subsidizes workers' training costs by a lump-sum tax on firms, *(monopsony) wages, (expected) profits and (global) unemployment decrease whereas (global) labor supply increase.* These results are very intuitive and easy to understand. Observe, however, (in terms of utilities) workers who were previously employed bear a reduction in their wage while those who where previously unemployed have an increase in their utility. This means that *more workers take jobs but those who were employed before the policy change see their wage, and thus their utility, to decrease.* In such a context, there is a trade off between unemployment and the well-being of the employed workers.

Let $c_\alpha(d)$ denote an employed worker's equilibrium (net) wage as a function of the difference $d = |x - x_i|$ between his skill type and that of his employer for the training case:

$$c_\alpha(d) = \frac{q\alpha s + b\left(\alpha s + 2a\Delta q^2\sigma^2\right)}{2\left(\alpha s + a\Delta q^2\sigma^2\right)} - \alpha s d \tag{38}$$

A meaningful measure of wage dispersion is given by the difference between the highest net wage and the lowest one between employed workers:

$$WD_\alpha \equiv c_\alpha(0) - c_\alpha\left(\frac{S_{g,\alpha}}{2}\right) = \frac{\alpha s(q-b)}{2\left(\alpha s + a\Delta q^2\sigma^2\right)} \tag{39}$$

It is easily checked that WD_α is increasing in α so that when the government subsidizes a larger part of the workers' training cost, $(1-\alpha)$ increases, the wage dispersion among employed workers decreases. It is worth noting that wages are less disperse when b, s, a, Δ or σ^2 are large. When for example, there is more uncertainty in the product market or when the market is more dense, inequality decreases.

Proposition 7 *If the government subsidizes workers' training costs, it reduces unemployment and inequality among the employed workers but cuts firms' (expected) profits.*

4.3 Unemployment benefits

Should the government cut or not the unemployment benefits? The theoretical literature does not give a clear answer to this question (Atkinson and Micklewright, 1991); it depends strongly on the nature of unemployment.

As above, we assume that the unemployment benefit b is financed by a lump-sum tax T_b paid by firms. We first determine the labor market equilibrium under this condition and then examine how b varies with unemployment.

The budget constraint of the government is given by:[10]

$$\sum_{i=1}^{n} T_b \ \Delta \ S_{i,b} = u_{g,b} \ b$$

Since $u_{g,b} = \Delta \left[L - \sum_{i=1}^{n} S_{i,b}\right]$, this equation reduces to:

$$T_b \ \sum_{i=1}^{n} S_{i,b} = \left[L - \sum_{i=1}^{n} S_{i,b}\right] \ b \tag{40}$$

It is shown in the appendix that when a firm maximizes its profit to set its wage, it can takes T_b as given when n is large enough. Thus the monopsony wage is exactly the same as in section 3.2 and thus equal to:

$$w_{f,b}^{U} = w_{f}^{U} = \frac{q \ s + b\left(s + 2a\Delta q^2\sigma^2\right)}{2\left(s + a\Delta q^2\sigma^2\right)} \tag{41}$$

This also implies that the equilibrium values of both global labor supply and global unemployment are not altered and are thus given by:[11]

$$S_{g,b} = S_g = \frac{q - b}{s + a\Delta q^2\sigma^2} \tag{42}$$

$$u_{g,b} = u_g = \Delta \left[L - n\frac{(q - b)}{s + a\Delta q^2\sigma^2}\right] \tag{43}$$

However, the equilibrium (expected) profits change and are equal to:

$$V_b^M = \frac{\Delta (q - b)^2}{2\left(s + a\Delta q^2\sigma^2\right)} - T_b \ \gtreqless V^M \tag{44}$$

Then, by using (40), the lump-sum tax at the symmetric labor market equilibrium is:

$$T_b = \frac{b \ L}{n} \ \frac{s + a\Delta q^2\sigma^2}{q - b} - b \tag{45}$$

[10]Subscript b refers to the unemployment benefit analysis.

[11]The condition ensuring that unemployment exists in equilibrium is still given by (26).

which is positive by using (26). Furthermore, by using (26), it is easily checked that:

$$\frac{\partial T_b}{\partial b} = \frac{q}{n}\frac{L}{}\frac{s + a\Delta q^2\sigma^2}{(q-b)^2} - 1 > 0 \qquad (46)$$

In this context, the equilibrium (expected) profit is given by:

$$V_{f,b}^U = \frac{\Delta (q-b)^2}{2\,(s + a\Delta q^2\sigma^2)} - \frac{b\ L}{n}\frac{s + a\Delta q^2\sigma^2}{q-b} + b < V_f^U \qquad (47)$$

with

$$\frac{\partial T_b}{\partial V_{f,b}^U} < 0$$

It is now easy to see from (43) that the general level of unemployment u_g is positively affected by the unemployment benefit level: when b is reduced, u_g decreases. Indeed, since workers support the totality of their training costs, when b (the outside option) decreases, more workers are willing to take a job rather than to stay unemployed even though their 'mismatch' with firms is important. In fact, this effect reinforces firms' monopsony power. This result also highlights the involuntary nature of u_f in which firms report the risk of price fluctuations on their workers. Therefore, when b decreases, monopsony wages decrease and labor supply increase but this decreases firms' expected profits (because of the financing of b) and reduces global unemployment.

Let us now consider the impact of increasing the unemployment benefit on inequality. It is easily checked that wage dispersion is given by:

$$WD_b \equiv c_b(0) - c_b\left(\frac{S_{g,b}}{2}\right) = \frac{s(q-b)}{2\,(s + a\Delta q^2\sigma^2)} \qquad (48)$$

which shows that reducing b raises inequality.

Proposition 8 *A reduction in unemployment benefits in monopsonistic sectors decreases the general level of unemployment and firms' (expected) profits but increases inequality.*

The model thus predicts that, in monopsonistic sectors, unemployment benefits should be lower than in non-monopsonistic ones. Hence, by Proposition 5, *in sectors in which there is a minimum wage, unemployment benefits should be low.*

5 Concluding remarks

Though the model used in this paper may seem quite stylized, we believe that it captures some basic features of the interaction between heterogeneous workers and firms' differing in their skill requirement and facing demand uncertainty. Our main result is that unemployment can be attributed to imbalance in demand and supply of skills as well as to random shocks in product demand. To obtain this conclusion, we have assumed that the labor market is imperfectly competitive because both firms and workers are heterogeneous. Demand uncertainty and mismatch reinforce each other in generating unemployment but none of them is really necessary to create it if the intensity of the other factor is sufficiently large. Therefore, any policy that allows one to reduce the cost of mismatch and/or the magnitude of demand fluctuations should lead to a fall in unemployment.

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APPENDIX

A1. The training cost policy

We want to show that when the number of firms n is large enough, firms takes the lump-sum tax T as given when determining their wage. The profit function is still given by (24) but we have to add $-T$. Thus, we must show that:

$$\frac{\partial T}{\partial w} = 0 \text{ when } n \to +\infty$$

T is given by (32) so that it can be written as:

$$T = (1-\alpha)\ s \frac{\sum_{i=1}^{n}\ (S_{i,\alpha})^2}{\sum_{i=1}^{n}\ S_{i,\alpha}}$$

When firm i maximizes its profit to set its wage w_i we have (only for the last term concerning T):

$$\frac{\partial T}{\partial w_i} = \frac{\partial S_{i,\alpha}}{\partial w_i}(1-\alpha)\ s \frac{[2\ S_i \sum_{i=1}^{n}\ S_{i,\alpha} - \sum_{i=1}^{n}\ (S_{i,\alpha})^2]}{\left(\sum_{i=1}^{n}\ S_{i,\alpha}\right)^2}$$

However, since

$$\frac{\partial S_{i,\alpha}}{\partial w_i} = \frac{1}{\alpha\ s}$$

we obtain:

$$\frac{\partial T}{\partial w_i} = \left(\frac{1-\alpha}{\alpha}\right)\ \frac{[2\ S_i \sum_{i=1}^{n}\ S_{i,\alpha} - \sum_{i=1}^{n}\ (S_{i,\alpha})^2]}{\left(\sum_{i=1}^{n}\ S_{i,\alpha}\right)^2}$$

At the symmetric equilibrium, $w_i = w$ and $S_{i,\alpha} = S_{g,\alpha}$, so it is easily verified that this equation is equivalent to:

$$\frac{\partial T}{\partial w_i} = \left(\frac{1-\alpha}{\alpha}\right)\ \frac{1}{n}$$

We have therefore our result:

$$\frac{\partial T}{\partial w} = 0 \text{ when } n \to +\infty$$

A2. The unemployment benefit policy

As for A1, we want to show that:

$$\frac{\partial T}{\partial w} = 0 \text{ when } n \to +\infty$$

However, T is now given by (40) so that it can be written as:

$$T = \left[\frac{L}{\sum_{i=1}^{n} S_{i,b}} - n \right] \; b$$

When firm i maximizes its profit to set its wage w_i we have (only for the last term concerning T):

$$\frac{\partial T}{\partial w_i} = -\frac{\partial S_{i,b}}{\partial w_i} \frac{L}{\left(\sum_{i=1}^{n} \; S_{i,b}\right)^2} \; b < 0$$

But since $S_{i,b} = (w_i - b)/\alpha s$, we have:

$$\frac{\partial S_{i,b}}{\partial w_i} = \frac{1}{\alpha \; s}$$

so that we obtain:

$$\frac{\partial T}{\partial w_i} = -\frac{b}{\alpha \; s} \frac{L}{\left(\sum_{i=1}^{n} \; S_{i,b}\right)^2}$$

At the symmetric equilibrium, $w_i = w$ and $S_{i,b} = S_{g,b}$, so that it is easily verified that this equation is equivalent to:

$$\begin{aligned} \frac{\partial T}{\partial w} &= -\frac{b}{\alpha \; s} \frac{L}{n^2 (S_{g,b})^2} \\ &= -\frac{\alpha \; s \; b \; L}{n^2 (w - b)^2} \end{aligned}$$

We have therefore our result:

$$\frac{\partial T}{\partial w} = 0 \text{ when } n \to +\infty$$

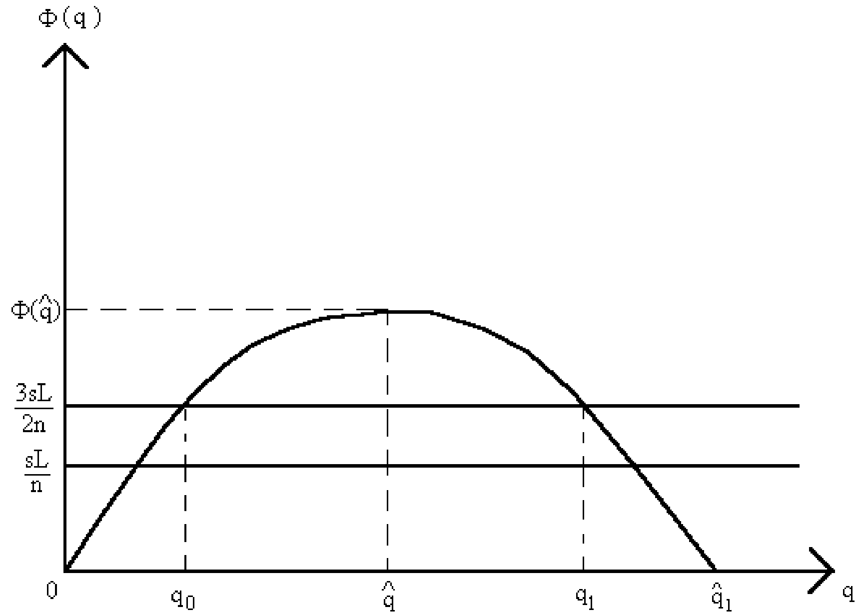

Figure 1: Condition for full employment

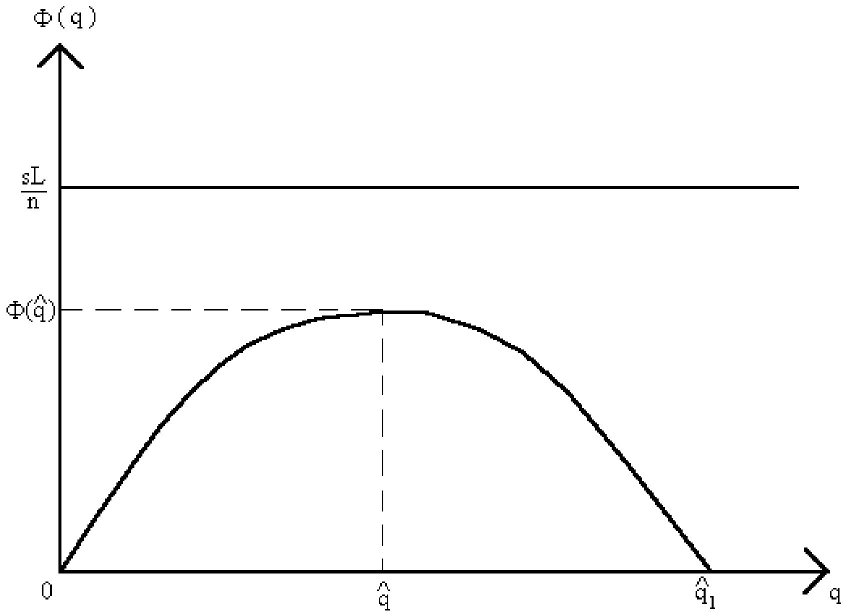

Figure 2: Condition for unemployment