Public Markets Tailored for the Cartel
- Favoritism in Procurement Auctions -*

Ariane Lambert-Mogiliansky†, Gregoryi Kosenok‡

January 6, 2006

Abstract

In this paper, we investigate the interaction between two firms engaged in a repeated procurement relationship modelled as a multiple criteria auction, and an auctioneer (a government employee) who has discretion in devising the selection criteria.

A first result is that in a one-shot context, favoritism turns the asymmetric information private cost procurement auction, into a symmetric information auction (in bribes) for a common value prize. In a repeated setting we show that favoritism substantially facilitates collusion. It increases the gains from collusion and contributes to solving basic implementation problems for a cartel of bidders that operates in a stochastically changing environment. A most simple allocation rule where firms take turn in winning independently of stochastic government preferences and firms’ costs is optimal. In each period the selection criteria is fine-tailored to the in-turn winner: the "environment" adapts to the cartel. This result holds true when the expected punishment is a fixed cost. When the cost varies with the magnitude of the distortion of the selection criteria (compared with the true government’s preferences), favoritism only partially shades the cartel from the environment. Collusion is sustainable in a simple but contingent scheme that under some conditions approaches full cartel efficiency. We thus find that favoritism generally facilitates collusion and that this occurs at a high cost for society. Some policy implications of the analysis are suggested.

Keywords: auction, collusion, favoritism, procurement

JEL: D44, D73, H57

*We would like to thank Olivier Compte, Paul Milgrom and Eric Rasmusen as well as seminar participants at INSEAD for insightful comments.
†PSE, Paris-Jourdan Sciences Economiques (CNRS, EHESS, ENS, ENPC), Paris, alambert@pse.ens.fr.
‡New Economic School, Moscow, gkosenok@nes.ru
1 Introduction

Many cartels operate in a stochastically changing environment. In particular, this is the case of firms involved in public procurement. The public demand for e.g., construction works typically depends on a number of factors that are difficult to predict. They include social needs, elected representatives’ political agenda, internal budget concerns etc... In addition, firms’ technology changes with time. Altogether this implies a significant uncertainty about the profitability of future contracts. In face of such an uncertain environment, a cartel of firms must devise a mechanism that while being responsive to changes does not open up for gaming opportunities. In this paper we propose that favoritism can contribute to solving key problems for a cartel of bidders that operate in a stochastically changing environment. A main motivation for the paper is the mounting body of evidence that collusion and corruption often go hand in hand in public procurement.

In France, practitioners and investigators in courts of accounts, competition authorities, and in the judiciary have long been aware of the close links between collusion and corruption in public procurement. The testimony of J. C. Mery provides suggestive evidence of those links (Le Monde, September 22 and 23, 2000).\(^1\) A recent judgment in ‘Les Yvelines’ (Cour d’Appel de Versaille, January 2002) provides a vivid illustration as well. According to a judge investigating a major corruption case in Paris, there exists in France, almost not a single case of large stake collusion in public procurement without corruption.\(^2\) Beside empirical motivations, there are theoretical motivations for investigating the links between favoritism and collusion. In particular, a cartel typically faces a tension between the efficiency goal and the need to provide firms with incentives to reveal private information. A fair amount of attention has been given to the theoretical problems facing a cartel that operates within an imperfectly or privately observable environment. Recently, Athey and al. (2004) show that it can simply be too costly for a price cartel to provide the right incentives for firms to reveal private information about shocks to costs so the optimal mechanism entails price rigidity (see also Green and Porter (1984) for the analysis of a price cartel on a market with a demand

\(^1\)J. C. Mery, a City Hall official, admitted that for ten years (1985-94) he organized and arbitrated collusion in the allocation of most construction and maintenance contracts for the Paris City Hall. In exchange, firms were paying bribes used to finance political parties.

\(^2\)The case concerns the procurement of a 4.3 billion euros market for the reconstruction of Paris’s lycees (see Le Monde April 23 2005).
subject to shocks). Our analysis is concerned with a cartel of bidders that face both incomplete information about demand i.e., government preferences and asymmetric information about shocks to firms’ costs. The central result is that favoritism shades the cartel from hazards in the environment so the cartel achieve full cartel efficiency. A non-contingent in-turn rule is optimal: firms take turn to win in a pre-determined manner The cartel needs not adapt to the environment. Instead the environment adapts to the cartel: the contract is fine-tailored to the pre-determined in-turn winner. This result is established for the case punishment cost is independent of the magnitude of government preferences. When the expected punishment varies with that magnitude the optimal mechanism is a contingent yet simple scheme that approaches full cartel efficiency. Favoritism generally exacerbates the social costs of collusion: the selected specification is socially inefficiency and the price paid by the government is higher than in the absence of favoritism.

We model the procurement procedure as a “first score auction”. Two firms characterized by a vector of cost parameters compete in scores with offers that include a specification of the project and a price. The true scoring rule reflecting public preferences is a stochastic variable. The procedure is administered by an auctioneer who is a government employee. At the beginning of the period the auctioneer, privately observes a signal of the public preferences. His duty is to devise and announce a scoring rule that reflects the (current) public preferences. In the absence of favoritism, the procedure selects the socially efficient specification of the project.

The presence of asymmetric information between the government and its auctioneer implies that the auctioneer has some discretion when deciding over the scoring rule. We call favoritism the act of biasing the scoring rule in favor of one of the firms. Corruption is modeled as an auction-like procedure that takes place before the official auction. Firms compete in (menus of) corrupt “deals” including a bribe and a demanded scoring rule. We find that with favoritism the procedure selects a non-standard specification of the project. The intuition is that the associated scoring rule induces minimal competition and thus maximal profit-if-win in the official auction. In the one-shot setting favoritism turns the asymmetric information private cost procurement auction, into a symmetric information auction (in bribes) for a common value prize corresponding to the gain associated with winning a fine-tailored contract.

We then consider a situation when firms meet repeatedly, each period on a new market
(the auctioneers are short-run players). We show that favoritism fully solves the cartel’s problems related to stochastic government’s preferences and costs. Provided each firm is efficient at producing some specification of the project, the cartel can earn the maximal income in a scheme that selects the winner independently of the true preferences and of firms’ costs. The intuition is that at the corruption stage firms submit corrupt deals that truthfully reveal private information about costs to the auctioneer. This is because a corrupt auctioneer has own incentives to use that information to fine-tailor the scoring rule to the in-turn winner. Firms’ main concern is to contain competition in bribes. That is achieved by opting for a fixed in-turn allocation rule which makes any defection immediately observable.

In an extension we investigate a case where the expected punishment for favoritism is a function of the magnitude of the distortion between the announced scoring rule and the true preferences. We find that the central insights from the fixed punishment case carry over. However, in the stage game competition in bribes does not dissipates all the firms’ rents in the basic model. And in the repeated setting the cartel faces a problem of imperfect public information. The optimal scheme is contingent on the true government preferences which are never observed. We show however that with favoritism a simple scheme approaches full cartel efficiency. With asymmetric information about firms’ costs the scheme become more complex and is strictly bounded away from efficiency.

The equilibrium allocation patterns emerging from the analysis is consistent with empirical findings. There exists ample evidence e.g., in developing countries of problems of maintenance of construction objects due to the non-standard design that was selected in the international procurement procedure. Evidence from corruption scandals in France also show that the tender winner is the most efficient firm and that its profits often are larger than the average in the branch (30% contra 5%) as in the case with the court case concerned with the series of constructions contracts in Paris.

This paper contributes to a rapidly growing literature on corruption in auction. The auctioneer’s abuse of discretion to devise the selection rule has been studied in Che and Burget (2004) in the context of a single auction. The present article is most closely related to Compte, Lambert-Mogiliansky and Verdier (2005) and Lambert-Mogiliansky and Sonin (forthcoming 2006). Both articles are concerned with links between corruption and collusion in a single auction context. They address a cartel’s enforcement problem and focus on the impact of the

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3 Add references.
auctioneer’s abuse of discretion to let firms readjust their bid. In Compte et al. the auctioneer sells an illegal opportunity to resubmit, which is shown to permit sustaining collusion in a single object auction. In Lambert-Mogiliansky and Sonin, the auctioneer of a multiple-object auction abuses a legal right. He sells his decision to let all firms simultaneously readjust their offer. As a consequence collusive market-sharing becomes sustainable. The contribution of the present paper is to demonstrate corruption’s role with respect to another central problem of a cartel: how to achieve efficiency in a stochastically changing environment.

The paper is organized as follows. The model is described in section 2. Section 3 offers an analysis of the one-stage game. In Section 4 we derive our central results. Section 5 proposes an extension to the case with varying punishment cost. Central assumptions are discussed in section 6 where we also suggest policy implications of the analysis for procurement and control agencies.

2 The model

In each time period a project is allocated. A project allows for a multiplicity of specifications. A specification is a vector \( \mathbf{q} = (q_1, \ldots, q_k) \) where \( q_j \) represents the level of the \( j \) (quality) component. There are two firms indexed \( i, \ i = 1, 2 \), which are characterized by their cost function

\[
c(\mathbf{q}; \theta_i^t) = \sum_{j=1}^{k} \frac{\theta_{ij}^t q_j^2}{2}
\]

where \( \theta_{ij}^t \in \{\bar{\theta}, \bar{\theta}\} \), \( j = 1, \ldots, k \) is firm \( i \)'s cost parameter associated with quality component \( q_j \) in period \( t \). We denote \( D \) the cost differential: \( D = \bar{\theta} - \bar{\theta} \). The vector of cost parameters \( \theta_i^t = (\theta_{i1}^t, \ldots, \theta_{ik}^t) \) is firm \( i \)'s private information. In each period there is a new draw of \( (\theta_1, \theta_2) \). Parameters \( \theta_{ij}^t \) are i.i.d. with \( \text{prob}(\theta_{ij}^t = \bar{\theta}) = \rho \) for all \( j = 1, \ldots, k \). For the sake of convenience we remove the realizations \( \theta_i^t = (\bar{\theta}, \bar{\theta}, \ldots, \bar{\theta}) \), \( i = 1, 2 \) from the support. So the probability of a specific realization with \( k_1 \) components \( \bar{\theta} \) and \( k_2 \) components \( \bar{\theta} \) (clearly \( k_1 + k_2 = k \)) is equal to \( \rho^{k_1} (1 - \rho)^{k_2} / [1 - (1 - \rho)^k] \).

The government derives utility from the realization of a project in period \( t \):

\[
W(\mathbf{q}^t, p^t; \alpha^t) = \alpha_{q1}^t q_1^t + \ldots + \alpha_{qk}^t q_k^t - p^t;
\]  
(1)

with \( \alpha_{qj}^t \geq 0, \ \forall j = 1 \ldots k, \ \sum_{j=1}^{k} \alpha_{qj}^t = 1, \)  
(2)
where $p^t$ is the price paid to the firm that delivers the project and $\alpha^t = (\alpha^t_1, ..., \alpha^t_k)$ is a vector of parameters representing the true social preference in period $t$. $\alpha^t_j = 0$ is understood as no social value of $q_j$ above a *minimal level* that defines a “basic good”. The $\alpha^t$ are i.i.d. with a uniform measure on $\Delta^{k-1}$. The government does not know the true $\alpha^t$. It hires an auctioneer who privately observes a signal of the true $\alpha_t$ at the beginning of each period. For simplicity we assume that the signal is fully informative.\footnote{This is not a crucial assumption.}

**The auction rule**

At the beginning of each period the auctioneer announces a selection criteria which is a function of both price $p$ and quality $q = (q_1, ..., q_k)$. We consider a class of selection criteria similar to the government’s utility function:

$$S(q, \hat{\alpha}) = s(q, \hat{\alpha}) - p = \sum_j \hat{\alpha}_j q_j - p, \quad \sum \hat{\alpha}_j = 1,$$

where $\hat{\alpha}$ is the vector of parameters *announced* by the auctioneer (see *Timing* below). Throughout the paper we refer to $\hat{\alpha}$ as the “scoring rule” which is a slight abuse of language since the score of an offer is determined by the selection criterion which includes the price bid. The firms simultaneously submit in a sealed envelop an offer including a project specification $q^t_i$ and a price $p^t_i$. The contract is awarded to firm $i^*^t$ whose offer maximizes (among submitted offers) the announced selection criteria subject to a “reserve score” normalized to zero:

$$i^*^t = \arg \max_{i=1,2} S(q^t_i, p^t_i, \hat{\alpha}) \quad \text{s.t.:} \quad S(q^t_i, p^t_i, \hat{\alpha}) \geq 0. \quad (3)$$

The winner is due to deliver the specification $q^t_{i^*}$ at price $p^t_{i^*}$. In case of tie in scores the project is awarded to the firm whose “quality score” (i.e., $s(q, \hat{\alpha})$) is highest. In case of tie in both price and quality the auctioneer randomizes. We refer to this procedure as a First Score Auction (FSA).

The firm $i$’s per-period profit-if-win is

$$\pi^t_i = p^t_i - c(q^t_i; \theta^t_i). \quad (4)$$

Profit-if-lose is zero. The game is infinitely repeated with the same two firms but with a different auctioneer in each period. The firms discount future gains with a common factor $\delta$. 
Corruption

The auctioneer is opportunistic. He accepts bribes in exchange for announcing a scoring rule i.e., some \( \alpha \). The auctioneer’s utility is

\[
U = w + b - m,
\]

where \( w \) is a wage that we normalize to 0, \( b \) is a bribe and \( m \geq 0 \) is a term that captures moral and other costs e.g. expected punishment for distorting the government preferences and for taking bribes.\(^5\) In the basic model expected punishment is a fixed cost. This is consistent with e.g., French legislation (Code Penal 432-14, 432-11). In an extension we consider the special case where \( k = 2 \) and the expected punishment depends on the magnitude of the distortion of social preferences so \( U = b - m (\alpha_1 - \alpha_1)^2 \). Such a model can be relevant when the magnitude of the distortion significantly affects the probability of detection. We discuss these assumptions in section 6.

Corruption is modelled as a procedure whereby the firms compete in corrupt “deals” where a deal is an offer to pay a bribe in exchange for a specific scoring rule. The two firms simultaneously and secretly submit a menu of deals \( M_i = \{ (\alpha_{ij}, b_{ij}) \mid j = 1, \ldots, n_i \} \), where \( n_i \) is freely chosen by firm \( i \). The bribe is only paid by the official auction’s winner if the announced scoring rule corresponds to one of the ones he demanded and the offers submitted in the official auction are not identical i.e., include both the same specification and the same price. This rule is not crucial to the main results but it greatly simplifies the analysis as it contributes to aligning firms’ and the auctioneer’s incentives. It is also consistent with the presumption that fully identical offers reflect fierce competition so favoritism brought no gain to the winning firm.

3 The stage game

The stage game is defined by the following Timing:

\begin{enumerate}
\item \textit{Step 0:} Firms learn privately their cost parameters \( \theta_1 \) and \( \theta_2 \).
\item \textit{step 1:} The auctioneer learns \( \alpha \), the firms submit \( M_1 = \{ (b_{1j}, \alpha_{1j}) \mid j = 1, \ldots, n_1 \} \) and \( M_2 = \{ (b_{2j}, \alpha_{2j}) \} \);
\item \textit{step 2:} The auctioneer makes an announcement \( \hat{\alpha}, \hat{\alpha} \in \Delta^{k-1} \);
\item \textit{step 3:} The firms submit \( (q_i, p_i) \);
\end{enumerate}

\(^5\) The government can engage a procedure to find out its true preferences and punish the auctioneer if he distorted them in his announcement.
step 4: The auctioneer publicly opens the envelopes and he selects the firm whose offer maximizes the selection criteria corresponding to the announced scoring rule. The winner pays the bribe he offered whenever \( \alpha = \alpha_{i^*j} \) for some \( j \). Otherwise no bribe is paid.

We first establish a result applying to the First Score Auction described by the Timing above when deleting step 1 and as we show later applying to any subgame starting from step 2.

**Lemma 1** The equilibrium offers of the FSA are characterized by specification efficiency:

\[ q_i^* = \arg \max s(q, \alpha) - c(q; \theta_i). \]

All proofs are gathered in the appendix.

The result in Lemma 1 greatly simplifies the forthcoming analysis. Lemma 1 allows us to, at step 3, separate between firms’ offer of project specification and their price bid.\(^6\) The equilibrium values of the components are the efficient ones corresponding to the announcement \( \alpha \)

\[ q_i^* = \arg \max s(q, \alpha) - c(q; \theta_i) \]

\[ q_{ij}^* (\theta_{ij}, \alpha_{ij}) = \frac{\alpha_j}{\theta_{ij}}, \; j = 1, ..., k, \; i = 1, 2. \]

The result exploits separability between quality and price in the selection criteria. It can be shown that for any offer not including the efficient values for the components, we can find another offer with the same score but that yields a higher expected profit. When the announcement corresponds to the true government preferences, Lemma 1 implies social efficiency in project specification.

### 3.1 Favoritism

We now proceed to investigate the game described in Timing above.

**Proposition 1** In any Bayes-Nash equilibrium we have that

i. The equilibrium scoring rule is \( \alpha^* = (0, ..., 1_j, ..., 0) \) for some \( \theta_j; \theta_{1j} \neq \theta_{2j} \) and \( b_{1j}^* = b_{2j}^* = \frac{D}{2 \theta_j} \) for all \( j \). When \( m > \frac{D}{2 \theta_j} \) or \( \theta_1 = \theta_2 \) (\( \theta_{1j} = \theta_{2j} \) for all \( j \)) there is no favoritism .

ii. The equilibrium offers are the competitive offers relative to the announced scoring rule.

\(^6\)A similar result can be found in Che (1993).
A first result is that whatever the true government preferences, favoritism always entails an extreme (single-peaked) scoring rule $\hat{\alpha}^* = \hat{\alpha}^j = (0, \ldots, 1_j, \ldots 0)$ for some $j$. We show in section 5 below (Extensions) that this result is robust to other specification of the punishment costs. The general argument is similar to that in Lemma 1. It rests on the separability between bribe and punishment in the auctioneer’s utility function (see Lemma 2 in the appendix). The intuition for the single-peakness result is that favoritism is an opportunity to weaken competitive pressure. The winner’s profit is maximal when the scoring rule emphasizes a single component for which the winner has a comparative advantage.

The interpretation of this result is that with favoritism the scoring rule tends to drive to a minimum the weight given to most components while emphasizing quite exclusively a component characterized by weak competition in production.\(^7\) This means that the winning project has a specification that tends to be “non-standard”. The true preference have minimal impact on the announced scoring rule. In case of ties in the corruption game, the auctioneer may choose the deal that is most congruent with the true preferences.

Quite remarkably we find that asymmetric information is a minor concern in our context. The intuition is that unlike an honest auctioneer who uses information about costs to minimize firms’ rents, the corrupt auctioneer has incentives to use that information to devise a scoring rule that maximizes the winning firm’s rents. In equilibrium the firms’ cost structure is revealed in the submitted menus of deals. Favoritism biases competition in the FSA, but competition does not disappear. The equilibrium offers are the (unique) competitive Nash equilibrium offers of the symmetric information FSA defined to the announced scoring rule. In equilibrium firms infer all relevant information about each other’s cost structure from the announced scoring rule. The specification is efficient relative to $\hat{\alpha}^*$ and the equilibrium price is determined by the second score.

Competition for favors drives up the bribe to $\frac{P}{2m^2}(\geq m)$. This is the profit that yields with a scoring rule that is most favorable to the winner. The auctioneer privately captures the totality of the winning firm’s rents. In the remaining of the paper we assume that $m < \frac{P}{2m^2}$ so the stage game is characterized by favoritism.\(^8\)

\(^7\)Strictly speaking the interpretation of zero weight as a minimal level is equivalent to assuming that firms are (more or less identical) in the production of a ”basic good” while they differ in the production of specifications of the project in excess of the requirement defining the ”basic good”.

\(^8\)When reviewing court cases, it appears quite clear that the cost of favoritism is very low. The only instances of conviction for favoritism in France, pertain to cases where the auctioneer explicitly required a
In effect, favoritism turns the asymmetric information auction, into a common value auction (in bribes) with symmetric information. The common value prize is the gain when winning the official auction when the selection criteria favors the winner maximally. This gain is common knowledge and identical for both firms. The social cost of favoritism is twofold. First, a socially inefficient project specification is selected. Second, the price paid by the government is higher than in the absence of favoritism. The bias in project specification due to favoritism minimizes competition between firms. The equilibrium depicted in Proposition 1 will serve a threat point in the collusive schemes we study next.

4 Repeated interaction

We now proceed to investigate a situation when the two firms interact repeatedly. In each period they meet on a public market administered by a new auctioneer, e.g., different local governments. In each period there is a new draw of \((\theta_{1t}, \theta_{2t}, \alpha^t)\). We are interested in collusion between the two firms under the assumption that transfers between them are precluded.

Information assumptions

At the end of each period the submitted contract offers are publicly observed by the two firms and the active auctioneer. The corrupt deal offers remain private information to the involved parties. The true value of \(\alpha\) is never revealed. The public history of the game up to period \(t\) is denoted \(H_t\). There is no communication between auctioneers from different periods.

4.1 Collusion

As a benchmark we characterize the optimal collusive scheme under symmetric information and in the absence of favoritism. The timing of the stage game is as follows.

\textit{Step 0:} Firms learn the cost parameters \(\theta_1\) and \(\theta_2\).

\textit{Step 1:} The auctioneer learns \(\alpha\) and makes an announcement \(\hat{\alpha}, \hat{\alpha} \in \Delta^{k-1}\);

\textit{Step 2:} The firms submit \((q_i, p_i)\);
step 3: The auctioneer selects the firm whose offer maximizes the announced selection criteria.

The two firms play this game repeatedly an infinite number of periods each time with a new auctioneer. They discount the future with a common factor $\delta$.

**Proposition 2** There exists $\delta_0 \leq 1$, such that for $\delta \geq \delta_0$ collusion is a Bayes-Nash equilibrium of the repeated game. Any optimal collusive scheme entails an allocation rule contingent on the true government preferences and on the firms’ costs.

Proposition 2 establishes that in any optimal collusive scheme the winner’s identity depends on the government true preferences and on the firm’s cost. This is not surprising since for any given selection criteria, the cartel’s income is maximized when the most efficient firm relative to that criteria implements the contract. With symmetric information about costs and in the absence of favoritism the optimal collusive cartel can be implemented. We shall not investigate the asymmetric information optimal cartel, which is a serious enterprise aside the main focus of this paper. We content ourselves with remarking that the dependence of the optimal symmetric information scheme on firms’ costs implies that under asymmetric information any collusive scheme is likely to be plagued by inefficiency. This is due to firms’ incentive to distort information and mimic other cost structures to increase the probability to win. Similar issues have been thoroughly investigated in Athey et al. (2004).

#### 4.2 Collusion and favoritism: A Strategic complementarity

We now consider a repetition of the game described in **Timing** (p. 7-8). Proposition 3 constitutes the central result of this paper.

**Proposition 3** i. There exists $\delta_1 < 1$ such that for $\delta \geq \delta_1$ full cartel efficiency is achievable in a Bayes-Nash equilibrium of the repeated game.

ii. In the official auction firms take turn in winning independently of government preferences and firms’ costs.

Full cartel efficiency is defined for the official auction as follows. i. In each period the winner is the firm that can realize the highest profit from the project paid at the reserve score; ii. The selection criteria that applies yields the highest gains to the winning firm from among all possible selection criteria. Proposition 3 establishes that with favoritism full cartel
efficiency is achievable in spite of asymmetric information. The cartel needs not adapt to the "environment" i.e., to the current cost structure and the government preferences. Instead the environment adapts to the cartel: in each period the auctioneer is bribed to fine-tailor the scoring rule to the in-turn winner. The optimal allocation rule is extremely simple: firm take turn for winning in a non-contingent manner. A main concern for the cartel is to contain competition in bribes which can be very costly as we know from proposition 1. Proposition 3 shows that competition for favors can be eliminated by opting for the simplest non-contingent in-turn allocation rule. At the corruption stage both firms offer a menu of deals each with a single-peaked scoring rule as in proposition 1. The out-of-turn firm offers a zero bribe while the in-turn winner offer a bribe that just covers the expected punishment cost: \( m \). The out-of-turn firm may deviate and (unobserved) bribe the auctioneer to announce a scoring rule favorable to itself. This is immediately detected however - the pre-determined in-turn rule is violated - and punished by reverting to the equilibrium of proposition 1 which yields zero payoff to the firms from next period on. This explains why the bribe can be kept to a minimum of \( m \geq 0 \). In the official auction the out-of-turn firm submits an offer that scores at most zero. Since contracts offers become public information any defection at that stage is detected after the official opening and punished similarly.

Favoritism facilitates collusion in several ways. The gains from collusion are higher than with an honest auctioneer: the scoring rule is fine-tailored to maximize the winner’s profit. And the threat payoffs are lower than in the absence of corruption because competition in bribes dissipates the rents. Most importantly we find that favoritism solves key problems for a repeated cartel in a stochastic environment. The auctioneer’s self-interested determination of the scoring rule effectively shades the cartel from fluctuations in the profitability of projects due to stochastic government preferences and changing costs. The “environment adapts” to the cartel and ex-post efficiency i.e., efficiency relative to the \( \textit{announced} \) scoring rule, is secured. But this comes at a cost, the bribe. The fixed in-turn rule eliminates competition in bribes. In our setting only firms are repeated players while the agent is short-run. This is key to the results since a long-run auctioneer would have an incentive to disrupt collusion to collect the higher stage game bribe. For \( k \) sufficiently large firms are better off with favoritism.
Remark 1 The social cost of collusion is generally higher with favoritism than without.

This follows from the fact that favoritism induces a socially inefficient project specification while simple collusion does not (see Proposition 2 above). Moreover with favoritism the selected project is always non-standard, which also implies a higher price paid by the government than in the case of simple collusion. On the other hand with favoritism ex-post efficiency is secured. The firms that actually implements the contract is cost efficient. This is also true with simple collusion but under symmetric information only. We have not characterized the asymmetric information optimal collusive scheme but as earlier mentioned it is likely to be plagued by some ex-post inefficiency.

5 Extensions

In this section we extend the analysis by considering the case when the expected punishment for favoritism depends on the magnitude of the distortion of social preferences: $U_0 = b - m (\alpha_1 - \alpha_1)^2$. We do that in a simpler setting with $k = 2$ so $\alpha_1 = \alpha$ and $\alpha_2 = (1 - \alpha)$. It is common knowledge that firms are anti-symmetric in costs: $\theta_1 = (\bar{\theta}, \bar{\theta})$ and $\theta_2 = (\tilde{\theta}, \tilde{\theta})$. The time line of events in stage game is as follows:

step 1: The auctioneer learns $\alpha$, the firms submit $M_1 = (b_1, \alpha_1)$ and $M_2 = (b_2, \alpha_2)$;
step 2: The auctioneer makes an announcement $\tilde{\alpha}$, $\tilde{\alpha} \in [0, 1]$;
step 3: The firms submit $(q_i, p_i)$;
step 4: The auctioneer selects the firm whose offer maximizes the selection criteria corresponding to the announced scoring rule. The winner pays the bribe he offered whenever the announced scoring rule is as demanded $\tilde{\alpha} = \alpha_i\ast$. Otherwise no bribe is paid.

Proposition 4 characterizes symmetric Bayes Nash equilibria of the stage game described above. We show that for $m \leq \frac{4}{5} \left( \frac{D}{2\bar{\theta}} \right)$

Proposition 4 Any symmetric Bayes-Nash equilibrium is characterized by

\footnote{For $k = 3$ it can be shown that the per period expected payoff is larger with favoritism than without provided $\bar{\theta} < 3\bar{\theta}$ for all $m \leq \frac{D}{2\bar{\theta}}$.}
i. The equilibrium scoring rule is 

\[ \tilde{\alpha}^*(m, \alpha) = \begin{cases} 
1 & \text{for } \alpha \geq 1/2 \\
0 & \text{for } \alpha < 1/2 
\end{cases} \]

ii. The equilibrium bribe is 

\[ b_1^* = b_2^* = b^*(m) = \frac{D}{2m} - m; \]

iii. The contract offers are the competitive equilibrium offers relative to the announced scoring rule.

A first important result is that the equilibrium scoring rule is single-peaked as in the fixed punishment case. In the appendix we prove this as a Lemma. The intuition is that in the corruption game firms compete in the auctioneer's utility levels. This utility is separable in bribe and expected punishment cost. We show that any deal that achieves a given utility to the auctioneer yields firms a lower expected profit from the whole game than a deal that achieves the same utility to the auctioneer while including a single-peaked scoring rule. We also note that, as in proposition 1, the official auction offers are the (efficient) competitive equilibrium offers.

In contrast with earlier results competition for favors does not dissipate all rents. The intuition is that contingent punishment costs introduces an asymmetry between firms: the firm whose demanded scoring rule is closer to the true one is more attractive to the auctioneer. Firms' incomplete information about the true preferences therefore induces a continuity of the probability to win in the submitted bribe. As a result competition for favor is mitigated.

The results in proposition 4 apply for 

\[ m \leq \frac{4}{5} \left( \frac{D}{2m} \right). \] 

Since \( \frac{D}{2m} \) is the competitive profit-if-win associated with the most favorable scoring rule, this range covers most interesting cases.\(^{10}\)

Summing up, with an expected punishment that is a function of distortion, the scoring rule always induces a "non-standard" project but a downright reversal of the true preferences is precluded. The same outcome obtains with fixed punishment when assuming auctioneer's weak preference for avoiding downright reversal. What government expenditure concerns there is no advantage in the more sophisticated punishment rule. Finally, because it mitigates competition in favors, some of the rents stay with the firms. We conclude that in the stage game, the sophisticated punishment scheme offers no advantage from a social efficiency point of view.

\(^{10}\)In an earlier version we investigated the whole solution. For \( m > \frac{4}{5} \left( \frac{D}{2m} \right) \) the equilibrium bribe is equal to the expected punishment cost and favoritism occurs less often until as the cost grows it gets fully prohibitive.
We now consider a repeated version of the game described above. As in the case with \( k \)-components, the two firms meet in each period with a new (short-run) auctioneer. At the end of each period, the submitted contracts offers are publicly observed by the two firms and the active auctioneer. The corrupt deal offers remain private information to the involved parties. The true value of \( \alpha \) is never revealed. The public history of the game up to period \( t \) is denoted \( H_t \). There is no communication between auctioneers from different periods.

**Proposition 5**

i. For \( \delta \geq \delta_2 \), there exists a Public Perfect Equilibrium equilibrium of the repeated game with collusion in contract offers and in corruption deals.

ii. The optimal collusive scheme is a contingent allocation rule. In each period the firm with the most congruent cost structure wins and it pays \( b^* = \frac{1}{4}m \).

iii. The scheme approaches full cartel efficiency as the discount factor tends to one.

This game is an instance of repeated game with imperfect public information. First we note that once the winner has been designated collusion in the official auction is sustainable relying on a standard folk theorem argument. This is because offers are ex-post public information. In the scheme of Proposition 5, the announcement resulting from competition in corruption deals determines the winner. The main issue for the cartel is therefore to sustain collusion in deals in order to contain competition in bribes and make an efficient use of stochastic government preferences as an allocation rule. The problem however is that firms don’t know the true scoring rule and do not observe the submitted bribe deals. They only observe the announced scoring rule which is an imperfect public signal of firms’ action in the corruption game. There are only two public outcomes whether firms comply or defect: \( \widehat{\alpha} = 1 \) and \( \widehat{\alpha} = 0 \). Therefore firms must sometime be “punished” even when complying (cf. Green and Porter 1986, Radner, Myerson and Maskin 1986). In the appendix we show that collusion is sustainable in a Public Perfect Equilibrium (PPE) but bounded away from efficiency for discount factor strictly smaller than 1. A PPE is a profile of public strategies that, beginning any date \( t \) and given any public history \( H_{t-1} \), form a Nash equilibrium.\(^{11}\) Deterrence from defection at the bribing stage is achieved by competition in the official auction. In case a firm wins twice in a row, it is “punished” by the other firm which then submits an offer that scores more than zero. This reduces the cartel’s revenue.

\(^{11}\)A strategy for player \( i \) is public if, at each time \( t \), the strategy depends only on the public history \( H_{t-1} \) and not on \( i \)’s private history.
The main insight from Proposition 5 is that the collusive scheme remains reasonably simple. In order to prevent defection in bribes, firms win a lower than the full collusive payoff when they win for the second period in a row. Only the preceding period’s announced scoring rule matters. So unlike in the case with a fixed punishment cost, favoritism here does not fully shade firms from future hazards in preferences and costs. Yet, matters are greatly simplified for firms. With favoritism the profit-if-win is fully known by force of single-peakness and depends minimally on the environment. As in the fixed punishment case the cartel’s gain is nearly maximized.

The results in proposition 5 apply to a symmetric information context. We conclude this section with a few remarks showing that the main insights generalize to an asymmetric information context where there is a new draw of $\theta_i$ in each period and it is privately observed by firms. As before we delete the realizations when any one of the firms is inefficient at the production of both components.

First we note that collusion in contract offers and in bribes is achievable in a simple pre-determined in-turn scheme at $b^* = m$. The reasoning is similar to that in proposition 3. However, such a scheme is plagued by inefficiency. For $m$ relatively large, the simple scheme implies a significant loss in revenue for the cartel compared with that of Proposition 5. The bribe always covers the maximal punishment cost. In a working paper we show that a more sophisticated scheme that blends features of the pre-determined in-turn rule with features of the contingent scheme can achieve collusion in bribes and contract offers while keeping the bribe to $b^* = \frac{1}{2}m$. Although the scheme is not truly complex, the proof is rather laborious and lengthy which is why we chose to leave it out. In order to be able to sustain this scheme the winning firm’s profit from the FSA can never be the maximal gain which suggests that for small $m$ the simple predetermined in-turn rule can be optimal.

Our conclusion is that the central insight from proposition 3 that favoritism facilitates collusion and contributes to resolving implementation problems in face of demand uncertainty (incomplete information about the true government preferences) and privately observed shocks to costs is robust to introducing varying punishment cost.

6 Discussion and Policy implications

The main insights of the analysis can be summarized as follows:
• Favoritism facilitates collusion because

− Favoritism induces the revelation of private information as information is used by the corrupt auctioneer to reduce competition;

− Bribery shades firms from fluctuations in government preferences. The selected contract specifications reflect the cartel’s interests instead of social preferences;

• Favoritism exacerbates the cost of collusion for society. The contract specification is socially inefficient and the price is higher price than with collusion alone.

The analysis thus reveals that favoritism fundamentally perverts the auction mechanism. Firms are willing to reveal their private information because the auctioneer uses it to maximize the winning firm’s rent. In the one-shot setting favoritism turns the asymmetric private value auction into a common value auction with symmetric information. the prize is the obtention of the contract under minimal competition. In the repeated setting it shades the cartels from the hazards of government preferences and shocks to costs.

A central intermediary result is that the equilibrium scoring rule is "single-peaked". The substantial content of that result is that favoritism tends to minimize competition in the official auction which implies a bias toward non-standard specifications of projects. Single-peakness as the expression for minimal competition obtains from the conjunction of a series of assumptions most of them are standard or reasonable. Two assumptions deserve some comments: separability in costs between components and separability in bribes and punishment cost.

We may consider relaxing the assumption of separability in costs. There is a natural way to reinterpret the single-peakness result then. If we have complementarities in costs, we could group components that are complementary in production into a composite component that is given full weight in a proper manner. Clearly, a more involved cost structure would entail more complex computation of the demanded scoring rule(s) and a more involved operation to compute the competition minimizing scoring rule (used in the stage game). A conjecture is that the menu of deal offers is sufficiently rich a message language to allow for quite sophisticated information to be revealed so the auctioneer can minimize competition as in the basic case. With (ex-ante) symmetric firms the prize i.e., winning the contract with minimal competition is the same for both firms in which case most of the results carry over. Relaxing the assumption about separability in bribes and punishment may require some more
analysis. In particular to investigate the case when the auctioneer is not willing to take a bribe so high that it covers the expected cost of all distortions. However, evidence suggests (see footnote 8 and policy implications below) that the expected cost is rather low in which case those problems would not arise.

Our conjecture is thus that the main insights of the analysis do not depend on the fine details of the model but express key features of the reality of corruption in procurement as revealed by empirical evidence. First there exists ample evidence e.g., from developing countries of problems of maintenance of construction objects due to the non-standard project specification that was selected by the international procurement procedure (add references). Second, the allocation pattern emerging from the analysis: a pre-determined in-turn rule allocates the contract to the most efficient firm while generating large profits is very close to the patterns observed in Paris Hall case mentioned in the introduction. Interestingly people have argued that the fact that the contract were allocated to the most efficient firm, was an indication that there was no collusion. The present analysis shows that it is sufficient that a firm has an advantage in some component to obtain this outcome in a collusive equilibrium with favoritism.

**Policy implications**

A central message of the analysis is that the risks of collusion and favoritism are linked and must be addressed simultaneously. Yet, the investigation of collusion is often the jurisdiction of Competition Authorities while that of corruption is the jurisdiction of criminal courts. A first recommendation is to institute structures to overcome this institutional separation so as to secure better condition for the prosecution of cases that involve both corruption and collusion.

An important insight of the analysis is that the discretion to devise the scoring rule or similarly to define the technical specification is very sensitive to capture as it is connected with large rents. An immediate recommendation is to limit that discretion. This can be achieved in various way. A first simple measure is to move toward a standardization wherever it is possible. When that is too costly (or not feasible), the auctioneer’s decision should be submitted to close scrutiny. An interesting idea stems from the observation that firms often have a superior information about each other than the government has. They can be in a
position to recognize when a scoring rule is fine-tailored to some other firm. In a one-shot situation or when some firms are excluded from a cartel, one should consider devising a mechanism to reveal this information e.g., by performing an anonymous consultation prior the official submission.

The results suggest that over a significant interval, an increase in $m$ i.e., stricter controls and/or more severe punishments, has no effect on the cost of favoritism to society. Hence, we find that to be any effective the expected punishment has to be very severe. This contrast with the current legislation in France and Europe that makes it very difficult to convict for favoritism. A central reason for this is that favoritism is difficult to prove. Indeed, generally any selection criteria would favor some firm(s) at the expense of others. "Deciding to build a swimming pool rather than a stadium is good for firms that have a comparative advantage in building swimming pools." The problem is thus to compare between selection criteria that favor different firms. The honest auctioneer picks up the one that is congruent with public preferences while the corrupt selects another one. But public preferences are seldom so well-defined that congruence can be measured in a way that is non-controversial (which suggests that a fixed punishment cost model maybe the relevant one). The analysis suggests that a study of the allocation patterns can be very useful to detect favoritism in particular to detract the view that when the most efficient firm wins "things are fine". Unfortunately courts tend to focus on bribery and few cases of favoritism are brought to court. We thus suggest that high economic expertise be given more power in cases of collusion and favoritism.
References


A Proof of lemma 1

for any announcement \( \hat{\alpha} \), the efficient specification for firm \( i \) is defined: \( q_i^* = \arg \max s(q, \hat{\alpha}) - c(q, \theta_i) \). We claim that in the equilibrium of the FSA both firms offer the efficient specification corresponding to their cost structure. Assume that this was not the case i.e., that, in equilibrium, firm \( i \) offers \((\hat{q}_i, \hat{p})\) with \( \hat{q} \neq q_i^* \). We first note that under asymmetric information \( \text{prob}\{\text{win} | (\hat{q}, \hat{p})\} > 0 \). To show this we order the firms type according to the number of components for which they have high cost \( \theta^0 < ..., < \theta^{k-1} \), where \( \theta^{k-1} \) is the highest cost type. Let \( S^*(\theta) = \max_q s(q, \hat{\alpha}) - c(q, \theta) \), in our model we have that \( \text{prob}\{\text{win} | S^*(\theta)\} \geq \frac{\rho (1-\rho)^{k-1}}{2(1-(1-\rho)^2)} > 0 \). If the offer \((q, p)\) for some \( \theta^j < \theta^{k-1} \) was such that \( \text{prob}\{\text{win} | (q, p)\} < \text{prob}\{\text{win} | S^*(\theta^{k-1})\} \), it could not be an equilibrium offer since the score in decreasing in cost i.e., there could exist another offer that would yield higher score with a positive probability to win a positive profit.

We now show that offer \((q_i^*, p')\) with \( p' = \hat{p} + s(q_i^*, \hat{\alpha}) - s(\hat{q}, \hat{\alpha}) \) dominates \((\hat{q}, \hat{p})\). Note that \( S(q_i^*, p') = S(\hat{q}, \hat{p}) \) so in particular \( \text{prob}\{\text{win} | (\hat{q}, \hat{p})\} = \text{prob}\{\text{win} | (q_i^*, p')\} \). Now

\[
\pi_i(q_i^*, p'; \theta_i) = \left[ p' - c(q_i^*, \theta_i) \right] \text{prob}\{\text{win} | (q_i^*, p')\}
= \left[ \hat{p} - c(q_i^*, \theta_i) + s(q_i^*, \hat{\alpha}) - c(q_i^*, \theta_i) \right] \text{prob}\{\text{win} | (\hat{q}, \hat{p})\}
> \left[ \hat{p} - c(q_i, \theta_i) \right] \text{prob}\{\text{win} | (\hat{q}, \hat{p})\} = \pi_i(\hat{q}, \hat{p}; \theta_i).
\]

The argument applying to the symmetric information case which we also use below (in Proposition 1, 2 and 4) is even simpler. In any equilibrium firms submit an offer that yields the same score. Consider the case when firm 1 has a cost structure that is more congruent with the announced scoring rule than firm 2. Firm 1 is sure to win when submitting the second highest score (corresponding to firm 2’s efficient specification associated with a price bid equal to its cost) because the tie breaking rule favors quality. Suppose firm 2 submits an offer that does not include the efficient specification and firm 1 matches that score. Then firm 2 could switch to an offer that includes the efficient specification to achieve a higher score and win. Suppose now that firm 1 matches firm 2’s score with an offer that does not include the efficient specification. Appealing to the argument above (setting the winning probability equal to 1), we see that it cannot be optimal. Firm 1 could earn a higher profit with an offer that scores the same but includes the efficient specification. Similar reasoning applies.
when firms are identically efficient. Hence, in equilibrium firms submit offers that include the efficient specification. \textit{QED}

\section*{B Proof of proposition 1}

Firms’ strategy:

At \textit{step 1} submit a menu of deals \( \left\{ \left( \alpha^j, b_{ij} \right) \right\} \) with a deal for \textit{each} component \( j \); \( \theta_{ji} = \theta \) with \( \alpha^j = (0, \ldots, 1_j, \ldots, 0) \). The same bribe is offered in each one of the deals. Both firms offer \( b^* = \frac{D}{200} \).

A \textit{step 3} When the announced scoring rule is single-peaked, firms submit the competitive equilibrium offers assuming that they are anti-symmetric in cost. When the scoring rule is not single-peaked, they submit the competitive offers under the assumption that their cost structures are identical.

The auctioneer’s strategy:

At \textit{step 2}

The auctioneer selects from among the submitted corrupt deals that include scoring rules \textit{only demanded by one firm}, a one associated with the highest bribe provided the bribe covers the costs. He announces the associated scoring rule. Otherwise he announces the true preferences.

We below show that the strategies above form a Bayes-Nash equilibrium of the game. For that purpose we first derive the competitive offers that form the (unique) symmetric information Nash equilibrium of the First Score Auction described by step 3 and 4 with no bribes.

Consider the cost structures \( \theta_1 \) and \( \theta_2 \). Define the efficient firm: \( i^* = \arg \max s(\mathbf{q}_i^*, \tilde{\alpha}) - c(\mathbf{q}_i^*, \theta_i) \) where \( \mathbf{q}_i^* = \arg \max \mathbf{q} s(\mathbf{q}, \tilde{\alpha}) - c(\mathbf{q}, \theta_i) \) i.e., \( q_{ij}^*(\theta_i) = \frac{\tilde{\alpha}_j}{\tilde{\theta}_j} \), \( i = 1, 2 \), \( j = 1, \ldots k \). We refer to \( i^* \) as firm 1 and appealing to Lemma 1, we focus on the price bids. By a standard argument, firm 2 bids the lowest price that just secures non-negative profit

\[ p_2^* = c(\mathbf{q}_2^*, \theta_2) = \sum_j \frac{\tilde{\alpha}_j^2}{2 \tilde{\theta}_j}. \]

Denote \( \theta^*_{ij}; \theta^*_{ij} \neq \theta^*_{j2} \). Let \( \beta_{11} = \sum \alpha^2_j (\theta^*_1 = \theta) \) and \( \beta_{21} = \sum \alpha^2_j (\theta^*_1 = \theta) \) and \( \beta_0 = \sum \alpha^2_j (\theta^*_1 = \theta^*_2) \). We can rewrite expression (5) \( p_2^* = \frac{\beta_{11}^*}{2\theta} + \frac{\beta_{21}^*}{2\theta} + \frac{\beta_0^*}{2\theta} \) where we do not specify the cost parameter for the cases when firms have identical cost. Firm 2’s score can be
computed $s(q_2^*, \alpha) - c_2(q_2^*, \theta_2) = \frac{\beta_{11}}{2\theta} + \frac{\beta_{21}}{2\theta} + \frac{\beta_0}{2\theta}$. Firm 1’s best response is to bid the lowest price that secures win: The winner’s payoff is

$$\pi_1 = \frac{D}{2\theta \theta} \left( \beta_{11} - \beta_{21} \right).$$  \hfill (6)

$$p_1^* = c(q_2^*, \theta_2) + s(q_1^*) - s(q_2^*) = p_2^* + \frac{D}{2\theta} \left( \beta_{11} - \beta_{21} \right).$$  \hfill (7)

We call the bids $(q_1^*, p_1^*), (q_2^*, p_2^*)$ defined in Lemma 1 and (5) and (7) the (symmetric information equilibrium) competitive bids.

We now proceed to investigate the game by backward induction. Step 4 is neglected throughout the appendix since players’s moves are fully determined by the rules of the game. At Step 3 firms make their offers. Consider first the subgames when $\alpha = \alpha^j$ for some $j = 1, \ldots, k$ and say it favors firm 1. Because the auctioneer’s strategy calls for not announcing a scoring rule demanded by both, firms infer from $\alpha = \alpha^j$ that they are anti-symmetric in cost with respect to $\theta_j$ so in particular when a firm is not favored (firm 2), it infers that its opponent is favored. Assume now that firm 2 believes that $b_1 < p_1^* - c(q_1^*(\alpha^j), \theta_1)$ where $p_1^*$ is defined in (7) for $\alpha^j$. We check firm 2’s incentives to bid $p_2^*$ defined in (5) for $\alpha^j$. Assume by contradiction that $p_2 = c(q_2^*(\alpha^j), \theta_2) + x > p_2^*$. Firm 1’s best response is $p_1 = p_2 + \frac{D}{2\theta} > p_1^*$. Firm 2 could then lower its price and win a positive payoff. Where $b_1$ is so large that $p_2^* + s(q_1^*) - s(q_2^*) < c(q_1^*, \theta_1) + b_1$, firm 1 cannot win with positive profit. We below show that this cannot happen in equilibrium so $b_1 < p_1^* - c(q_1^*, \theta_1)$ is an equilibrium belief for firm 2. In a subgame where $\alpha \neq \alpha^j$, firms infer that $(q_1^*, p_1^*, \alpha) = (q_2^*, p_2^*, \alpha)$ so profit-if-win is zero no bribe will be paid and by definition the proposed offers are best response to each other. Hence, the Nash equilibrium offers of FSA without corruption described in Lemma 1 and (5) and (7) are part of an equilibrium.

At step 2 the auctioneer chooses a deal among the submitted menus $(M_1, M_2)$. The auctioneer expects firms to ask for scoring rules that emphasize components in which they have low cost. Assume both ask for the same scoring rule, which can happen since they don’t know each other’s cost. If that scoring rule is announced firms make their offer under the assumption that costs are asymmetric, they submit identical offers. But then the auctioneer receives no bribe. So he never selects a scoring rule demanded by both: $\alpha_k \in \{\alpha_k : \alpha_k \notin M_i \text{ and } \alpha_k \notin M_{-i}\}$. Next since $U = b - m$, he selects $\alpha_k$ such that $b_k \geq \max \left\{ b_{1j}, b_{2j} \right\}$ and $b_k \geq \max \left\{ b_{1j}, b_{2j} \right\}$.
m. If no such deal has been submitted, there is no way for auctioneer to earn a bribe sufficient to cover the expected punishment costs and so it is optimal to announce the true $\alpha$.\footnote{We do not analyze the case of coincidence of true $\alpha$ with the one in some corrupt deal. Because it has a zero probability and hence it has no effect on a choice of corrupt deals by firms at Step 1.}

At step 1 We know from step 2 that the auctioneer selects a deal associated with the highest bribe among those with scoring rules demanded by one firm only. Firms can without risk demand scoring rules that maximize the profit-if-win $\pi_1 = \frac{D}{2\theta^2} (\beta_{11} - \beta_{21})$ which is decreasing in $\beta_{21}$ so firm 1 sets $\beta_{21} = 0$ i.e., all the weight is be put on components $\theta_{1j} = \theta$. Next, the single term $\beta_{11} = \sum \alpha_{j}^2 (\theta_{1} = \theta)$ must be such that $\sum \alpha_{j}^2 (\theta_{1} = \theta) = 1$, implying $\frac{\partial \beta_{11}}{\partial \alpha_{j}} > 0$ so $\pi_1$ is maximized with any $\alpha_{j}^2 = (0, ..., 1_j, ..., 0); \theta_{1j} = \theta$. Since the firms don’t know each other’s cost, the probability that the auctioneer finds a deal with a scoring rule demanded by only one firm increases with the number of submitted deals. So it is optimal to submit a deal on each low cost component. A firm’s profit-if-win with any of the $\alpha_{j}$ it demands is equal to $\frac{D}{2\theta^2}$. The corruption game boils down into a symmetric information auction with identical firms. By a standard argument, firms submit their value $b_{1j}^* = b_{2i}^* = \frac{D}{2\theta^2}$ for all $j$ and $i$. QED

C Proof of proposition 2

We first introduce a function $G(\alpha, \theta_i)$ representing the social gain corresponding to the efficient specification. By Lemma 1, firms choose the efficient specification profile. When firm $i$ implements the contract in period $t$ the gain is

$$G(\alpha^t, \theta^t_i) = \sum_{j=1}^{k} \frac{(\alpha^t_{ij})^2}{2\theta^t_{ij}}.$$  \hfill (8)

The object of proposition 2 is a repeated FSA game of complete information, we propose a simple Grim Trigger (GT) strategy for sustaining collusion. As usual it is composed of a punishment phase and a cooperative phase. In the punishment phase each firm gets the payoff of the stage game Nash equilibrium (defined in the proof of proposition 1). These non-cooperative payoffs can be expressed as the difference between the values of the $G$ function above, for firm 1

$$E_{\pi}^{\text{ne}} = E_{\alpha^t, \theta_1^t, \theta_2^t} \{ G(\alpha^t, \theta^t_1) - G(\alpha^t, \theta^t_2) | G(\alpha^t, \theta^t_1) > G(\alpha^t, \theta^t_2) \},$$ \hfill (9)

where the notation $E_{\alpha, \theta_1, \theta_2} \{ | \}$ stands to taking the conditional expectation over random variables $\alpha$, $\theta_1$ and $\theta_2$. Notice that due to symmetry, firm 2 gets the same payoff $E_{\pi}^{\text{ne}}$. 


In the cooperative phase, firms collude to collect the highest feasible expected profit. This profit is achieved when the firm, say firm $i$, with the highest value of $G$ given by (8) (the firm with the cost structure $\theta$ the most congruent to $\alpha$) wins the auction and retains the entire social gains from the contract i.e., it earns $G(\alpha, \theta_i)$. The expected per period payoff of firm 1 (as well as of firm 2) in the cooperative phase is

$$E\pi^c = E_{\alpha^t, \theta_1^t, \theta_2^t} \{ G(\alpha^t, \theta_1^t) |  G(\alpha^t, \theta_1^t) > G(\alpha^t, \theta_2^t) \}.$$  

We note that $E\pi^c > E\pi^{ne}$ which follows from $E_{\alpha^t, \theta_1^t, \theta_2^t} \{ G(\alpha^t, \theta_1^t) |  G(\alpha^t, \theta_1^t) > G(\alpha^t, \theta_2^t) \} > 0$.

We now describe a GT strategy. The game starts in the cooperative phase. In the cooperative phase in each period $t$ the firm with the highest value of $G(\alpha^t, \theta^t)$ is designated as the current winner. The winner makes an offer that scores zero while the loser makes an offer that scores negative. In case of tie in $G(\alpha^t, \theta^t)$, they toss a coin, so with the probability .5 each firm is designated as the in-turn winner. If the actual winner is different from the designated winner in some period $t$, from the next period on the firms revert to the play of the Nash equilibrium of the stage game (punishment phase).

We now define the range of the discount factor $\delta$ for which the GT strategy is an equilibrium strategy. According to the one-stage-deviation principle we check for the highest gain from deviation within a period. This gain is maximal in a period when the designated loser has the highest value of $G$. This, for example, takes place when at some time period $\alpha$ has only one nonzero component with weight equal to 1, and for instance all components of $\theta_1$ and $\theta_2$ are equal to $\theta$. Here $G(\alpha, \theta_1) = G(\alpha, \theta_2) = 1/(2\theta)$, the tie breaking rule ruled against firm 2. Firm 2 can then achieve a one period profit close to $1/(2\theta)$ by slightly overbidding firm 1’s equilibrium offer. However from the next period on the play yields the non-cooperative Nash payoffs. The IC constraint securing that such a deviation is not profitable is

$$\frac{E\pi^c}{1-\delta} \geq \frac{1}{2\theta} + \frac{\delta(E\pi^c - E\pi^{ne})}{1-\delta}.$$  

so the GT strategy form an equilibrium with collusion of the repeated FSA for $\delta \geq \delta_0$ where

$$\delta_0 = \frac{1/(2\theta) - E\pi^c}{1/(2\theta) - E\pi^c + E\pi^{ne}} < 1$$

QED
D Proof of Proposition 3

We show that the collusive equilibrium of proposition 3 can be supported by a Trigger strategy with a punishment phase corresponding to the play of the equilibrium of proposition 1. The cooperative phase is characterized by the following:

Firms’ strategy

At step 1 the in-turn-firm submits a menu \( M_{in} = \{ (b^*, \alpha^j) \} \) with \( \theta_{in,j} = \emptyset, \quad b^* = m \). The out-of-turn firm submits \( M_{out} = \{ (0, \alpha^j) \} \) for some \( \theta_{out,j}, \quad b^* = 0 \).

At step 3 For any \( \hat{\alpha}^j \) the in-turn firm submits an offer that scores zero. The out-of-turn firm bids to score strictly less than zero.

The auctioneer’s strategy

At step 2

The auctioneer selects from among the submitted corrupt deals a one associated with the highest bribe. If that bribe covers the costs, he announces the associated scoring rule.

Let \( H_{t-1} = H^* \) denote a public history of the game when it is in a cooperative phase i.e., in all \( t' = 1, ..., t - 1 \) the outcome is characterized by the firm winning in alternance every second period.

The trigger strategy entails that in any subgame following \( H_{t-1} \neq H^* \), the firms move to (stay in) the punishment phase. Since it is a Nash equilibrium, conforming is by construction a best response for all players.

We now consider a subgame following \( H_{t-1} = H^* \) to show that cooperating according to the strategies defined above is optimal. We proceed by backward induction.

At step 3 whatever \( \hat{\alpha}^j \), the in-turn firm expects the out-of-turn firm to bid less that zero. The maximal payoff \( \pi^c = \frac{1}{2g} \) yields when the in-turn firm offers the efficient specification and a price so its offer scores just zero. So the proposed strategy is optimal. The out-of-turn firm may deviate. The most profitable deviation occurs when the announced scoring rule is single-peaked and the out-of-turn firm also has low cost on the emphasized component and submits \( p = \frac{1}{g} - \varepsilon \). Its gain is \( \pi^d = \frac{1}{2g} - \varepsilon \). However the in-turn rule is violated and from the next period on the firms revert to the zero payoff competitive equilibrium of proposition 1.

So the out-of-turn firm complies with the collusive strategy whenever
\[ IC : \delta \frac{1}{1 - \delta} \left( \frac{1}{2\theta} - m \right) \geq \frac{1}{2\theta} - \varepsilon \]  

(11)

which is satisfied for \( \delta \geq \delta_1 < 1 \) with \( \frac{\partial IC}{\partial m} > 0 \).

At step 2 Since the auctioneer is a short-run player, the argument developed in the proof of proposition 1 carries over. A distinction is that the auctioneer needs not care about avoiding scoring rules submitted by both because firms never submit identical offers in response to single peaked scoring rules.

At step 1 the firms submit their menu of deals. Since the auctioneer only cares about the bribe the argument of proposition 1 carry over and firms always propose deals with single-peaked scoring rules. The in-turn firm expects the out-of-turn firm to offer \( b = 0 \). It is sufficient to offer \( b = m \) to cover the auctioneer's cost so he announces one of the in-turn firm's preferred scoring rule. The out-of-turn firm can defect and offer \( b = m + \varepsilon \) associated with a menu including a most preferred scoring rule \( \alpha_{out} \). It knows that the auctioneer would respond by announcing that \( \hat{\alpha} = \alpha_{out} \). But such a deviation only brings profit if the out-of-turn wins the official auction. Since we know that such a win triggers a punishment phase, under (11) defection is not profitable. Hence for \( \delta \) satisfying 11 the proposed strategies do form a Bayes-Nash equilibrium of the repeated game. The cartel's gain is maximized. In each period, the scoring rule is the most favorable to the winner, the price is given by the reserve score and the bribe is the lowest possible. \( QED \)

**E Proof of Proposition 4**

Firms' strategy:

At step 1 The firms offer a deal for its low cost component with a single-peaked scoring rule emphasizing that component and the bribe \( b^* (m) \) defined below.

A step 3, the firms submit the competitive offers relative to the announced scoring rule.

The auctioneer's strategy

At step 2

The auctioneer selects from among the submitted deals, the one that maximizes his payoff. He announces the associated scoring rule provided the bribe covers the cost.

We below show that the strategies described above form a symmetric Bayes-Nash equilibrium with favoritism. We develop the proof in terms of firm 1 which has its advantage in
the production of $q_1$. Firm 2 is symmetric with advantage in component 2. We proceed by backward induction.

At Step 3, In the present context firms know each other costs yet we can apply the same reasoning as in Proposition 1 for $k = 2$, $\alpha_1 = \alpha$ and $\alpha_2 = (1 - \alpha)$ and with $\theta_1 = (\bar{\theta}, \bar{\theta})$ and $\theta_2 = (\bar{\theta}, \bar{\theta})$. In Proposition 1 firms correctly believe they are asymmetric in cost. here they simply know that.

At step 2 the auctioneer’s utility function is $b_i - m(\alpha_i - \alpha)^2$. So it is optimal to choose a deal among the submitted ones as follows: $
abla_i = \arg \max (b_1, \alpha_1)(b_2, \alpha_2) b_i - m(\alpha_i - \alpha)^2$ s.t. $b_i \geq m(\alpha_i - \alpha)^2$. In case of ties, he selects each firm with equal probability. The auctioneer announces $\alpha_i^*$ if $i^*(\alpha_i) = \nabla_i$. If no bribe deal can secure win in the official auction or if $b_i < m(\alpha_i - \alpha)^2$, the auctioneer announces the true alpha.

At step 1 We start with a Lemma. First a definition. The "cartel efficient" scoring rule is defined for firm 1, $\alpha_1^* = \arg \max_{\alpha_1 \in [0,1]} \left\{ \frac{D}{2m}\left(2\alpha_1 - 1\right) - m(\alpha_1 - \alpha)^2 \right\}$. It is the scoring rule that maximizes the cartel’s payoff when firm 1 implements the contract given that there is a cost associated with deviations from the true scoring rule. The cartel efficient scoring rule for firm 2 is defined similarly.

**Lemma 2** In a symmetric equilibrium firms always demand the "cartel efficient" scoring rule contingent on its cost structure i.e., $\alpha_1^* = 1$ and correspondingly $\alpha_2^* = 0$.

**Proof.** We know that the auctioneer selects the firm whose deal maximizes $U(b_i, \alpha_i, \alpha) = b_i - m(\alpha_i - \alpha)^2$. And that in any symmetric equilibrium he is indifferent between firms when $\alpha = 1/2$ (= $E(\alpha)$) or $U(b_1, \alpha_1, \alpha = 1/2) = U(b_2, \alpha_2, \alpha = 1/2)$. Now suppose by contradiction that firm 1’s the equilibrium deal offer $(b_1, \alpha_1)$ with $\alpha_1 \neq \alpha_1^*$. We now construct offer $(b_1', \alpha_1^*)$ with $b_1' = b_1 - m(1/2 - \alpha_1)^2 + m(1/2 - \alpha_1^*)^2$. By construction $U(b_1', \alpha_1^*, \alpha = 1/2) = U(b_1, \alpha_1, \alpha = 1/2)$ and hence

$$prob\left(U\left(b_1', \alpha_1^*, \alpha\right) > U\left(b_2, \alpha_2, \alpha\right)\right) = prob\left(U\left(b_1, \alpha_1, \alpha\right) > U\left(b_2, \alpha_2, \alpha\right)\right) = 1/2.$$
Now

\[
E\pi \left( b'_1, \alpha^*_1 \right) = \left[ \frac{D}{2\theta \theta} - b'_1 \right] \text{prob} \left( U \left( b'_1, \alpha^*_1, \alpha \right) > U \left( b_2, \alpha_2, \alpha \right) \right)
\]

\[
= \left[ \frac{D}{2\theta \theta} - b_1 + m(1/2 - \alpha_1)^2 - m(1/2 - \alpha^*_1)^2 \right] \frac{1}{2}
\]

\[
= \left[ \left( \frac{D}{2\theta \theta} (2\alpha_1 - 1) - b_1 \right) + \left( 2 \frac{D}{2\theta \theta} - m\alpha_1 \right) (1 - \alpha_1) \right] \frac{1}{2}
\]

\[
> \left[ \frac{D}{2\theta \theta} (2\alpha_1 - 1) - b_1 \right] \text{prob} \left( U \left( b_1, \alpha_1, \alpha \right) > U \left( b_2, \alpha_2, \alpha \right) \right)
\]

where the inequality holds because \( m < \frac{D}{2\theta \theta} \).

Hence \( \alpha^*_1 = 1 \) and similarly for firm 2: \( \alpha^*_2 = 0 \).

We now consider the determination of \( b_1 \) and \( b_2 \)

\[
E\pi_1 \left( b_1 \right) = \left( \frac{D}{2\theta \theta} - b_1 \right) \text{prob.} \left\{ U \left( b_1, 1 \right) > U \left( b_2, 0 \right) \right\}
\]

\[
= \left( \frac{D}{2\theta \theta} - b_1 \right) \text{prob.} \left\{ b_1 - m \left( 1 - \alpha \right)^2 > b_2 - m \left( \alpha \right)^2 \right\}
\]

\[
= \left( \frac{D}{2\theta \theta} - b_1 \right) \text{prob.} \left\{ \alpha > \frac{1}{2} + \left( \frac{b_2 - b_1}{2m} \right) \right\}
\]

\[
= \left( \frac{D}{2\theta \theta} - b_1 \right) \left( \frac{1}{2} - \frac{(b_2 - b_1)}{2m} \right)
\]

Taking the derivative with respect to \( b_1 \) an interior solution satisfies

\[
b^*_1 = \frac{1}{2} b_2 + \frac{1}{2} \left( \frac{D}{2\theta \theta} - m \right)
\]

In a symmetric equilibrium we obtain

\[
b^* = b^*_1 = b^*_2 = \frac{D}{2\theta \theta} - m,
\]

In a symmetric equilibrium the auctioneer never distorts more than .5 so the highest cost for distortion is \( \frac{1}{4} m \). Hence, for \( \frac{D}{2\theta \theta} - m > \frac{1}{4} m \Leftrightarrow m < \frac{4}{5} \frac{D}{2\theta \theta} \), the strategies described above including the deal offers \( (b^*, 1) \) and \( (b^*, 0) \) form a Bayes-Nash equilibrium of the FSA with favoritism. \( QED \)

\section*{F \ Proof of proposition 5}

We use the notation: \( w_{1(2)}(1) \) and \( w_{1(2)}(0) \) to denote firm 1(2) continuation payoff-if-win after a period when the announcement is \( \widehat{\alpha} = 1 \) and \( \widehat{\alpha} = 0 \) respectively. We focus on a smaller
set of strategies including \( b \in \{ \frac{1}{4}m, m \} \) i.e., we only consider a defection that secures win. This is not crucial to the result but it simplifies the presentation.

Let \( H_{t-1} = H^* \) denote a history of the game where in all periods preceding \( t \), where \( \bar{\alpha}^{t'} \in \{0, 1\}, t' = 0, \ldots, t - 1 \) and \( S\left( p^{*t'}, q^{*t'} \right) = 0 \) for \( t' \); \( i^{*t'} \neq i^{*t'-1} \) and \( S\left( p^{*t'}, q^{*t'} \right) > 0 \) otherwise.

We propose the following strategies for the players:

i. If \( H_{t-1} \neq H^* \), the firms and the auctioneer play the equilibrium depicted in proposition 4.

ii. If \( H_{t-1} = H^* \),

Firms’ strategy

At step 1 firm 1 submits a deal including \( \alpha_1 = 1 \) and firm 2 \( \alpha_2 = 2 \) together with \( b_c = \frac{1}{4}m \).

At step 3 If \( \bar{\alpha}^{t} = 1 \) and \( \alpha^{t-1} = 0 \) firm 1 submits an offer with the efficient specification such that it scores zero, while firm 2 submits an offer that scores at most zero. If \( \alpha_{t-1} = 1 \), firm 2 submits an offer that scores more than zero such that it yields a payoff \((1 - \delta)w_1(1)\) to firm 1 when it exactly matches firm 2’s score. Firm 1 strategy is, for \( \alpha_{t-1} = 1 \), is to exactly matches that score. When the scoring rule favors firm 2 the strategies are similarly defined with \( \alpha_{t-1} = 1 \) leading to zero score bids and \( \alpha_{t-1} = 0 \) to bids that yield \((1 - \delta)w_2(0)\).

At step 2, the auctioneer selects the bribe deal that maximizes his utility provided the bribe covers expected costs and announces the corresponding scoring rule. In case of ties he randomizes.

We below show that these strategies form a Perfect Public Equilibrium of the repeated game with the stage game as described in section 5. Collusive bidding at step 3 is sustainable relying on an argument similar to the one developed in Proposition 3 when setting \( k = 2 \). The IC constraint defining the minimal discount factor is similar to (11): \( \frac{e}{(1-\delta)} \frac{1}{2} \left( \frac{1}{2} - \frac{1}{4}m \right) \geq \frac{1}{2\delta} - \frac{1}{4}m - \varepsilon \) defining \( \delta_2 < \delta_1 \) because defection is less profitable that in the context of proposition 3. In particular, in the symmetric information context we assume firms to be asymmetric in cost so the defector has high cost. We also note when a firm is designated winner the second time in a row, the opponent submits an offer that score more that zero according to the rule below. Since firm 1 expect firm 2 to submit the non-zero score offer defined below, it is optimal for firm 1 to match. If it does not it loses and firms revert to the non-cooperative equilibrium. So firm 1 matches and it is optimal for firm 2 to submit the punishment offer.
At step 2 the short-run auctioneer’s strategy is optimal by the same argument as in proposition 4. We proceed to investigate firms’ strategy at step 1. We first note that single-peakness of demanded scoring rules is secured by the Lemma 2 (Proposition 4). We now focus on bribe bids using technics exploited in Fudenberg et al. (1994) in particular we decompose the firms’ payoff in current payoff and a continuation payoff. Let $\pi_c$ denote the collusive payoff from the official auction. In each period firms’ expected payoff from complying with the collusive strategy is

$$E\pi_{fc} = \frac{1}{2} \left[ \pi_c - \frac{1}{4} m \right]$$

while the defection payoff $E\pi_d = \pi_c - m$. We first note that for $m \geq \frac{4}{7} \pi_c$ there is no incentive to defect. But for $m < \frac{4}{7} \pi_c$ the following incentive constraint, expressed in average payoff, applies in any period $t$ preceded by $t_{t-1} = 0$:

$$(1 - \delta) \frac{1}{2} \left[ \pi_c - \frac{1}{4} m \right] + \delta \left[ \frac{1}{2} w_1 (1) + \frac{1}{2} w_1 (0) \right] > (1 - \delta) (\pi_c - m) + \delta w_1 (1)$$

$$\delta \left[ \frac{1}{2} w_1 (0) - \frac{1}{2} w_1 (1) \right] > (1 - \delta) \left( \frac{1}{2} \pi_c - \frac{7}{8} m \right)$$

$$\delta [w_1 (0) - w_1 (1)] \geq (1 - \delta) \left( \pi_c - \frac{7}{4} m \right)$$

(12)

So as $\delta \to 1$, $w_1 (1) \to w_1 (0) = \frac{1}{2} (\pi_c - \frac{1}{4} m)$ and the allocation approaches efficiency. When $\delta < 1$ the continuation payoff following an announcement of $\alpha = 1$ must be lower that the one following $\alpha_{t-1} = 0$. This payoff is achieved by letting firm 2 submit an offer in the official auction that induces a lower profit to firm 1. Note that when (12) holds incentives to comply in a period following an announcement of $\alpha = 1$ also are satisfied. This is because the gain from defection are lower then. Hence, for $\delta \geq \frac{5}{7}$ and $w_1 (1) = w_2 (0)$ satisfying (12) the proposed strategies form a Perfect Public Equilibrium of the repeated game. QED